

Analysis of 3-Phase Symmetrical and Unsymmetrical Fault on Transmission Line using Fortescue Theorem

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Abstract: This paper investigates the major faults affecting the transmission of electrical energy after power has been generated from the power generating station. In a 3-phase transmission line, faults arise due to numerous causes such as aircraft, line breaks due to the excessive loading, heavy winds, trees falling across the lines etc. The faults faced in a 3-phase transmission line are broadly categorized into two main parts, namely: unsymmetrical faults and symmetrical faults. Furthermore, there is another classification of faults in 3-phase transmission lines such as: shunt type of faults and series type of faults, but this paper discusses the shunt type of faults which create short circuit on single line to ground (L-G) faults between two conductors or line to line (L-L) faults, or double line to ground (LL-G) or (triple) three line to ground (LLL-G) faults. This was achieved using the Fortescue Theorem on MATLAB software. The results show that the single L-G faults occur more frequently followed by the L-L faults, LL-G faults, and LLL-G faults. This study is essential to evaluate the power reliability and stability of power transmission lines.

Keywords: Transmission line, Line to ground, Double line ground, three line to ground, 3-phase fault

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1 Introduction

Electric power is transmitted and distributed through transmission lines. In many cases, the voltage level of the transmission line is raised by a transformer before electric power is transmitted through the transmission line. Basically, electric power is proportional to the product of current and voltage, in transmission lines, high voltage is transmitted in order to reduce the line current i^2r losses. Operation of a 3-phase AC power system has equal distribution of current and voltage magnitudes in each phase when operated in normal condition; however, faults may occur to disrupt this condition. The types of faults created in a transmission line may be balanced or unbalanced. Balanced faults involve all phases while unbalanced faults involve only 1 or 2 phases. Unsymmetrical and symmetrical fault analysis is performed to determine the value of the fault current in KVA or in MVA, [1]. Faults in transmission lines are caused by circuit failure which interferes with the normal flow of current. It is the undesirable creation of conducting path for

short circuit or open circuit fault which blocks the flow of current, [2]. When a fault occurs in a transmission line, the short circuit current is high, usually six to ten times more than the normal full load current in the system, [3]. The growth of power systems with increasing load demand has brought about the need for speed and accuracy of power transmission equipment. Transmission line faults which are not detected early and removed cause blackout or wide spread damage of power system equipment, [4].

This paper presents the analysis of a 3-phase transmission line with resistive, inductive, and capacitive loads (RLC) during the L-G faults, L-L faults, LL-G faults, and 3-line to ground (LLL-G) fault.

2 Proposed Technique

2.1 Proposed Test System

In this section, we present the test system which consist of a 3-phase voltage source and 3-phase

load as shown in Figure 1. The loads active and reactive power variation with the positive sequence component of voltage are given by (1) and (2), [5].

$$P = P_0 \left(\frac{V}{V_0}\right)^{np} (1 + T_{p1})(1 + R_{p2})$$

where P is active power, P₀ is reference active power, T_{p1} and T_{p2} are time constants, V is the voltage, and V₀ is the reference voltage, [6].

$$Q = Q_0 \left(\frac{V}{V_0}\right)^{nq} (1 + T_{q1})(1 + T_{q2})$$

where Q is reactive power, Q₀ is the reference reactive power, T_{q1} and T_{q2} are time constants, V is the voltage, and V₀ is reference voltage. In (1) and (2), n_q = 1 if V > V_{min} and n_q = 2 if V < V_{min}.

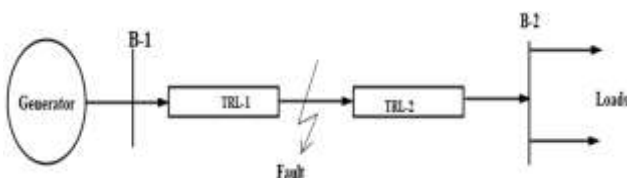


Fig. 1: Test System for the Study of Fault Analysis

a. Unsymmetrical and Symmetrical Fault Analysis using Fortescue's Theorem

A symmetrical fault is a fault where all the 3-phases are affected equally, thereby making the system balanced. This type of fault usually arise from symmetrical currents or short circuit currents, [7]. Figure 2 gives an illustration of a current fault in a 3-phase line in a short circuit condition. Hence only positive sequences are needed to analysis the fault, [8].

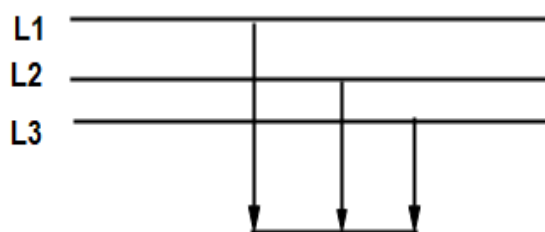


Fig. 2: Symmetrical fault.

Any three unbalanced set of voltage or current can be resolved into three balanced sets of voltage or current. This condition indicates that the system is symmetrical.

Positive sequence components: three phasor of equal magnitude displaced by 120 degree from each other following the positive sequence.

Negative sequence components: three phasors of equal magnitude displaced by 120 degree from each other following the negative sequence.

Zero sequence components: three parallel phasors having the same magnitude and angle.

For 3-phase systems, three unbalanced phases can be resolved into three balanced systems of three phasors each.

The transmission 3-phase voltage can be expressed as V_a, V_b, and V_c. According to the Fortescue Theorem these can be transformed. Positive sequence voltage are supplied by the power generator within the system and are always present and are displaced 120 degree apart from the other lines, but display a counter clockwise rotation sequence of A-B-C. The representation of positive voltage is expressed in terms of V_{a1}, V_{b1}, and V_{c1}, while negative sequence voltage is expressed in terms of V_{a2}, V_{b2}, and V_{c2}, and zero sequence are given as V_{a0}, V_{b0} and V_{c0}.

Thus:

$$\begin{aligned} V_a &= V_{a1} + V_{a2} + V_{a0} \\ V_b &= V_{b1} + V_{b2} + V_{b0} \\ V_c &= V_{c1} + V_{c2} + V_{c0} \end{aligned}$$

Figure 3 shows the representation of positive, negative, and zero sequence voltages.

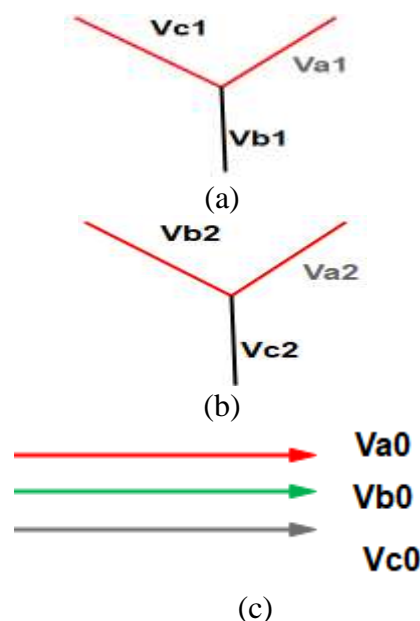


Fig. 3: Representation of (a) positive, (b) negative, and (c) zero sequence.

The 'a' operator of 3-phase transmission line is given as:

$$\begin{aligned} a &= 1 \angle 120^\circ = -0.5 - j0.866 \\ a &= |1| \text{ rotates by } 120^\circ \\ a^2 &= 1 \angle 240^\circ = -0.5 - j0.866 \\ a^3 &= 1 \angle 360^\circ = 1 \angle 0^\circ = 1 + j0 \end{aligned}$$

Therefore this summation is $1+a+a^2 = a^3+a^2+a$.
Figure 4 presents the representation of the 'a' operators for the 3-phase transmission line.

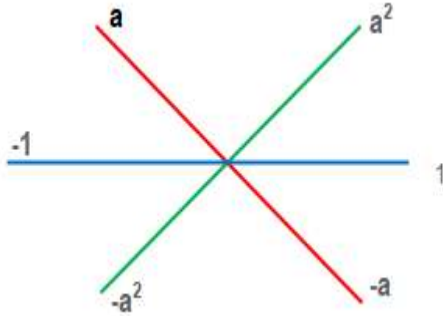


Fig. 4: 'a' operators of transmission line.

From the above equations,

$$\begin{aligned} V_{b1} &= a^2 V_{a1}, & V_{c1} &= a V_{a1}, \\ V_{b2} &= a V_{a2}, & V_{c2} &= a^2 V_{a2}, \\ V_{b0} &= V_{a0}, & V_{c0} &= V_{a0}. \end{aligned}$$

Finally, the positive sequence voltage is given as:

$$\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ V_c &= V_{a0} + a V_{a1} + a^2 V_{a2} \end{aligned}$$

This relationship is given in matrix form as:

$$\begin{aligned} V_p &= \begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix}; V_s = \begin{bmatrix} Va0 \\ Va1 \\ Va2 \end{bmatrix}; A \\ &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{ invers of } A \\ &= 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \end{aligned}$$

Then the matrix expression is given as:

$$\begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} Va0 \\ Va1 \\ Va2 \end{bmatrix}$$

Therefore, V_p can be expressed in matrix forms:

$$\begin{aligned} V_p &= AV_s \text{ and } V_s = A^{-1}V_p \\ V_{a1} &= 1/3(V_a + aV_b + a^2V_c) \\ V_{a2} &= 1/3(V_a + a^2V_b + aV_c) \\ \begin{bmatrix} Va0 \\ Va1 \\ Va2 \end{bmatrix} &= 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Va \\ Vb \\ Vc \end{bmatrix} \end{aligned}$$

Finally the power of the 3-phase using symmetrical component analysis is given as:

$$\begin{aligned} S &= V_p^T I_p^* = [AV_s]^T [AI_s]^* \\ &= V_s^T A^T A^* I_s^* = 3V_s^T I_s^* \\ &= 3V_{a0} I_{a0}^* + 3V_{a1} I_{a1}^* \\ &\quad + 3V_{a2} I_{a2}^* \end{aligned}$$

Note that $A^T = A^*$ =

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{ and } A^T A^* = 3 * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In Figure 5, we show the flowchart of our proposed method which we used for the analysis of symmetric faults. However, the major faults created in transmission lines are unsymmetrical in nature. These include single L-G faults, LL-G faults, and L-L faults as shown in Figures 6, 8, and 9, respectively. In the 3-phase system, single L-G faults are observed to occur more frequently followed by the L-L faults, LL-G faults, and 3-phase faults. At the time of electrical storms, the above types of faults do occur, which ultimately affect transmission lines, [9].

Single L-G faults: Single L-G fault is the most common type of shunt fault. Single line to ground fault accounts for 70 to 80 % of faults that occur on transmission lines which cause interruptions in power supply, [10], [11].

This fault occurs when a conductor is in contact with the ground or a neutral terminal. A brief illustration is given in Figure 6. Suppose that phase 'a' is connected to ground at the fault point 'F', where, the fault impedance is given as Z_f . By convention, the fault current is taken as positive when flowing out of the fault point.

From Figure 6, phase 'a' is connected to ground at the fault, phase 'b' and 'c' are in open circuit mode and carry no current.

Therefore $I_b = I_c = 0$ at 'F', $V_a = Z_f I_a$ and the sequence of current at the fault is given as:

$$\begin{aligned} \begin{bmatrix} Ia0 \\ Ia1 \\ Ia2 \end{bmatrix} &= 1/3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Ia \\ 0 \\ 0 \end{bmatrix} \\ \text{where } Ia0 &= \frac{Ia}{3}, Ia1 = \frac{Ia}{3}, \text{ and } Ia2 = \frac{Ia}{3} \\ \text{so the fault current becomes } Ia &= 3 * Ia1 \\ \text{Furthermore, } V_a &= I_a Z_f = 3I_{a1} Z_f \text{ and} \\ \text{therefore } I_{a1} &= \frac{V_a}{3Z_f}. \end{aligned}$$

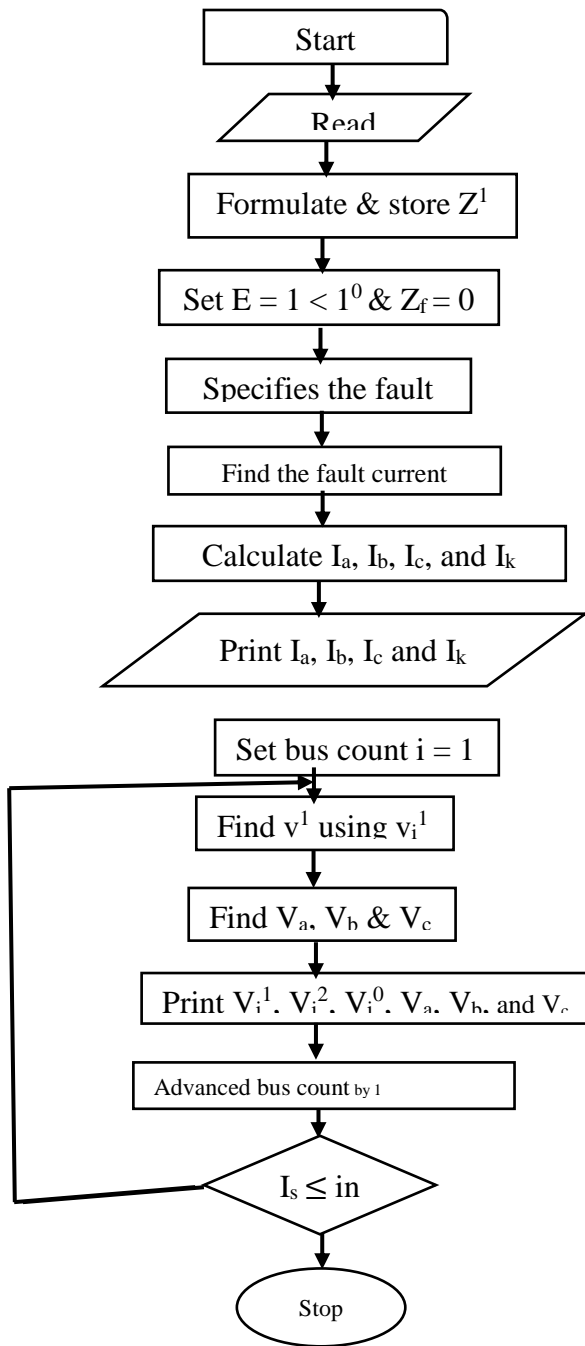


Fig. 5: Flowchart of symmetric faults.

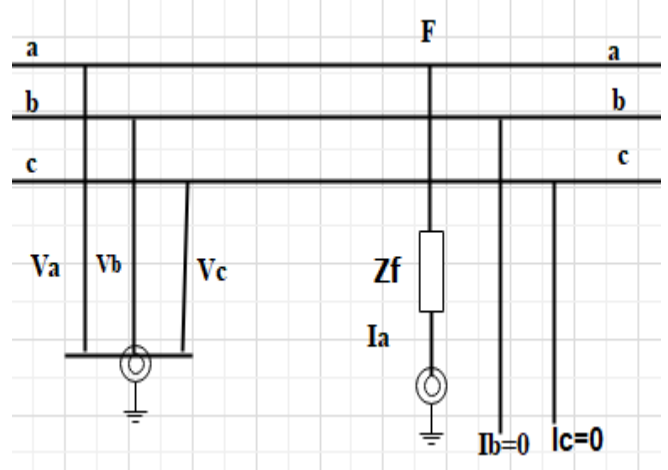


Fig. 6: Single L-G fault.

The sequence component connection for the single L-G fault is shown in Figure 7.

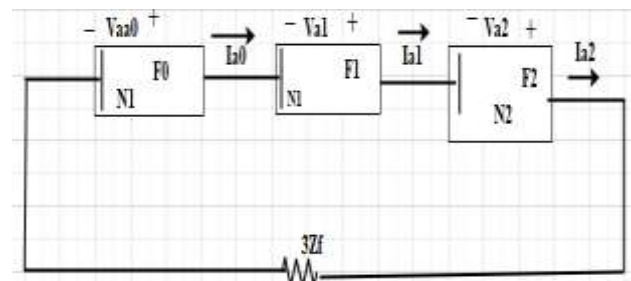


Fig. 7: Sequence component of the single L-G fault.

Generally, the equation of current is:

$$I_{a0} = I_{a1} = I_{a2} = \frac{E}{Z_1 + Z_2 + Z_3 + 3Z_f}$$

The summation of these currents is given as:

$$I_a = I_{a0} + I_{a1} + I_{a2} = \frac{3E}{Z_1 + Z_2 + Z_3 + 3Z_f}$$

$$V_{a1} = E - I_{a2}Z_1, \quad V_{a2} = -I_{a2}Z_2 \text{ and } V_{a0} = -I_{a0}Z_0$$

L-L fault is the second most prominent type of fault that occurs in a transmission line when the two lines are shorted together. This is shown in Figure 8 and further explained in [12], [13].



Fig. 8: L-L fault.

The overall line current equations are given as:

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}$$

$$I_{a2} = -I_{a1}$$

$$I_{a0} = 0$$

LL-G is the type of fault which occurs when two phases of a power system fall on the power line. This is depicted in Figure 9, [14], [15].



Fig. 9: LL-G fault.

The overall value of currents in this configuration is given as follows:

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 \cdot Z_2}{Z_0 + Z_2}}$$

$$I_{a0} = -\frac{Z_2}{Z_0 + Z_2} I_{a1}$$

$$I_{a2} = -\frac{Z_0}{Z_0 + Z_2} I_{a1}$$

The flowchart used for the analysis of unsymmetrical faults is shown in Figure 10.

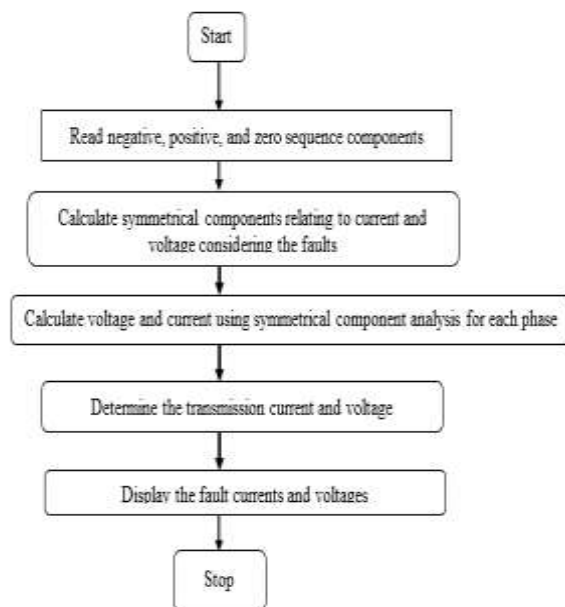


Fig. 10: Flowchart for the analysis of unsymmetrical faults.

3 Simulation Results and Discussion

The proposed approach analysed a load connected to a 3-phase generator using a single transmission line. The transmission voltage and current is measured at the load terminals. The outputs of current and voltage wave forms for all the 3-phases are provided for each case of the study in the following sections.

a. Single L-G Faults

A L-G fault is one in which a short circuit occurs between one phase (line A) of the system and the earth. The simulation result of the voltage during the L-G fault is given in Figure 11. The results show that the value of voltage for the asymmetrical fault line to ground decreases in amplitude, while the voltage is stable or rise slowly during the time of fault occurrence.

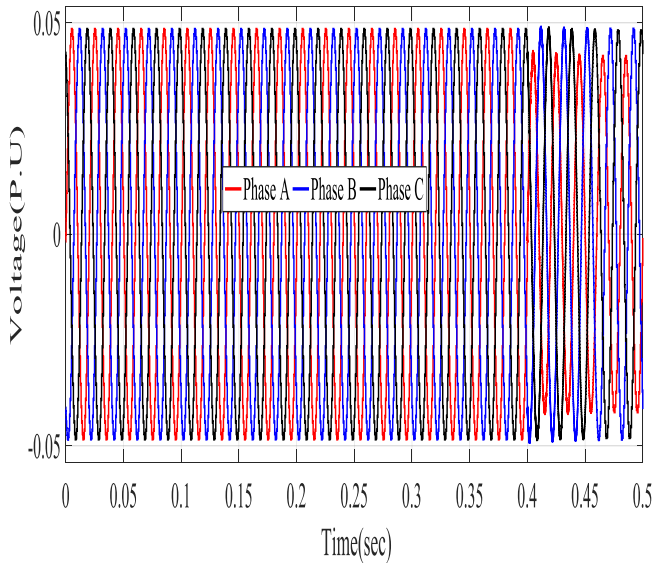


Fig. 11: 3-phase voltages during L-G fault with R-L load.

In Figure 12, the value of current rises above its amplitude which produce a high current during the fault events. The amplitude of the fault current is observed to increase above the normal per unit current after 0.4 seconds. In single L-G faults, the current flows through the ground from the phase that is faulty. The current drawn by the load during the faulty conditions is depicted in Figure 13.

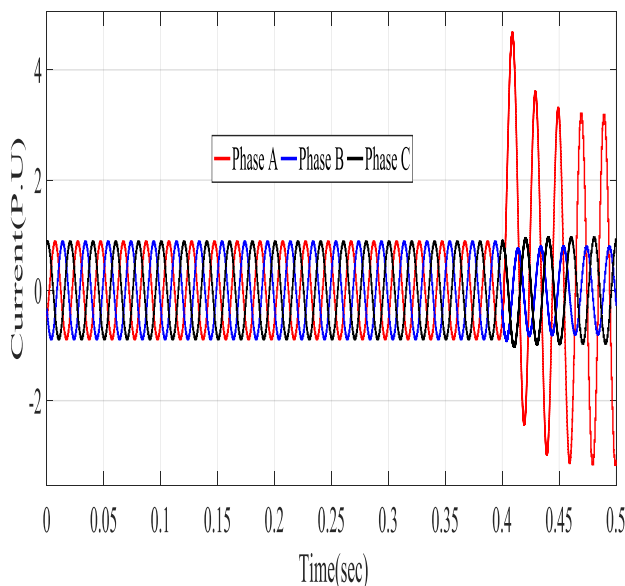


Fig. 12: 3-phase current during L-G fault with R-L load.

b. L-L Fault

A L-L (unsymmetrical) fault is simulated with the fault between phase A and Phase B. The two conductors are short circuited and the result is shown in Figure 13. The value of the L-L voltage

magnitude between phase A and phase B reduces during the fault conditions.

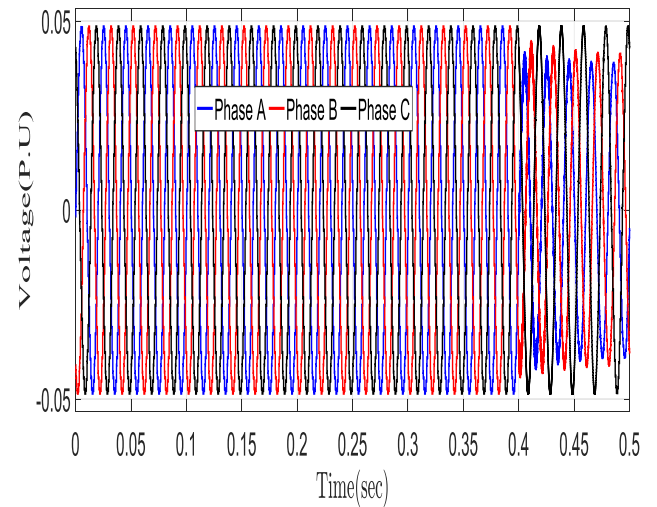


Fig. 13: 3-phase voltages during L-L fault with R-L load.

During the L-L fault, the current drawn by the load during the fault with R-L load is depicted in Figure 14 and it observed from this Figure 14 the value of current is exceeds above the normal conditions.

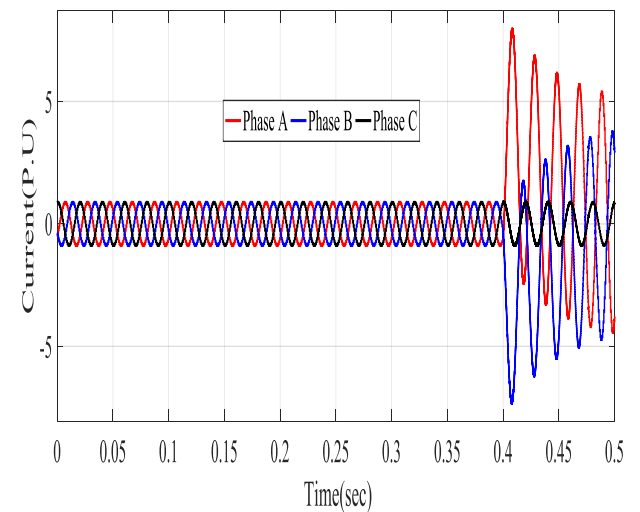


Fig. 14: 3-phase current during L-L fault with R-L load.

C. 3-Phase Fault (LLL-G)

The 3-phase fault to ground involving earth has been simulated by connecting all 3-phases to earth simultaneously. The resulting waveform of the voltage is shown in Figure 15. The results show that all the 3-phases are reduced below the normal value (0.05 or -0.05pu).

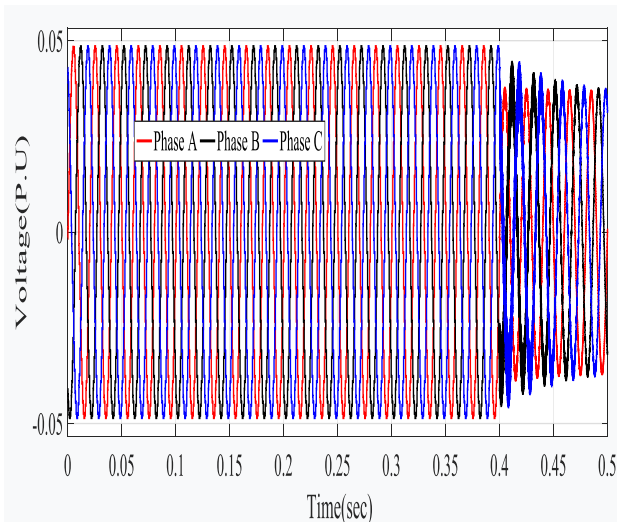


Fig. 15: 3-phase voltages during LLL-G fault with R-L load.

In Figure 16, it is observed that the 3-phase current drawn by the load during the LLL-G fault rises above the normal value (5 or -5 pu) at different magnitudes and amplitudes.

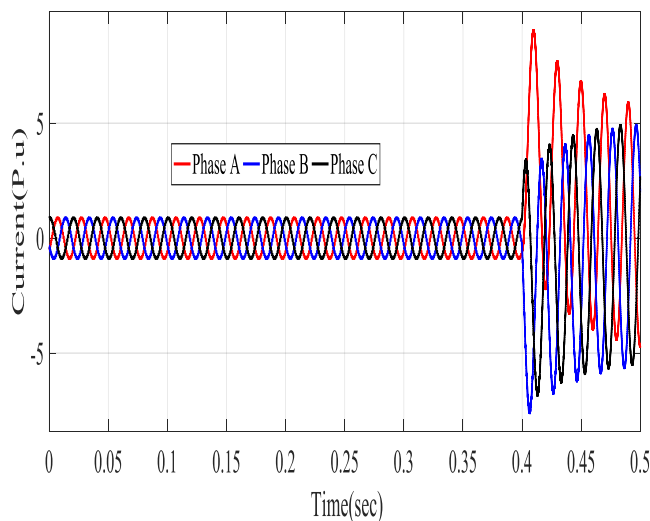


Fig. 16: 3-phase current during LLL-G fault with R-L load.

4 Conclusions

This paper presented the analysis of faults in a 3-phase transmission line with resistive and inductive loads. A significant increase in current above the normal conditions was obtained for asymmetrical faults. Consequently, the current and voltage wave forms are reduced. The effects of the voltage wave forms on 3-phase faults was seen to affect both the symmetrical and unsymmetrical 3-phase faults.

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