Study of Non-Linear Systems: PI and Fuzzy Controllers Performances

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Abstract: This work aims to present the development and study of a nonlinear system structure capable of moving in one degree of freedom, performed by a brushless motor coupled to a propeller. For this, the mathematical modeling of the developed structure was based on resembling a simple physical pendulum. Regarding the system control, two techniques were considered: PID and Fuzzy, implemented in Matlab. Simulation tests were carried out to validate and compare the control technique results with different input signals. Both controller topologies tracked the respective setpoints, but Fuzzy controller had the best system performance. The structure developed can serve as a base platform for future improvements, studies, and experiment tests, such as the analysis of motion inthehorizontalplane.

Key–Words: PID controller, Fuzzy Controller, 1 DoF prototype.

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1 Introduction

Automation and robotics are closely related, since automation deals with the control of mechanical, electronic and computer-based systems and robotics present in a large area of industrial automation, as robotic machines to improve the efficiency of industrial automation [1].

In this context, measuring the linear or angular displacement of a robotic arm and detecting objects are essential requirements in many industrial applications [2]. Also, its positioning systems are widely used in different automation processes, such as detecting the position of tools, other objects or security systems for supervision. They can be controlled in various forms with classic controllers, such as through the PID (Proportional-Integral-Derivative Controller) or through advanced controllers [3].

In work developed by [4] was presented a genetic algorithm adjusted for Fuzzy PID controllers applied to robotic arms, capable of moving in three Degree of Freedom (DoF). From the tests, the authors concluded that the strong nonlinear characteristics and parameter variations in real environments led the arm tracking control system with Fuzzy PID controller to get better performance when compared to the other ones.

The authors of the paper [3] described an angular positioning system to perform camera rotation. Regarding the controllers, the authors did not compare the best performance of them. However, they stated that the classic PI controller requires the knowledge of the system transfer function and the controller parameters. Besides, the adaptive controller does not need these settings since they are determined during the execution.

In paper [5], another interesting work was developed, presenting the modeling and positioning control of one DoF robotic arm. Different strategies and classic PD control structures were applied to obtain data for system performance analysis. The comparison of the Fuzzy PD with the traditional PD showed that the Fuzzy PD had superior performance compared to the conventional one. The authors also emphasized that Fuzzy PD is better adequate if there is a dynamic variation in the system. Another three interesting works can be highlighted: [6], [7] and [8].

Taking these aspects into consideration, this work aims to analyze an application case of a nonlinear system controlled by two techniques through computational simulations: Fuzzy and PID control. Firstly, for better get its real measurements, a prototype was built to allow the experimentation of the control techniques in nonlinear systems. It consists of a wooden structure representing a physical pendulum, equipped with sensors and actuators. Its pendulum movement is performed by the actuation of a brushless motor coupled to a propeller, where a potentiometer acts on the system position measuring. Concerning the system control, a Fuzzy controller was simulated through MATLAB software to perform a comparison with the traditional PID controller.

This paper is organized as follows: Section 2 presents the prototype developed, considered as a base platform; Section 3 shows the system modelling used to implement the proposed controllers; Section 4 presents the controllers: PID and Fuzzy; Section 5 shows the simulation results; and finally, Section 6 concludes this work.

2 Prototype Developed

The prototype structure for angular control with one DOF was developed in wood, composed of brushless motor, potentiometer, Electronic Speed Controller (ESC), and Arduino MEGA.

For the prototype assembly, a 68 cm long X wood base was used, coupled with a vertical fixed stem, also of wood, 72 cm length. The fixed rod is coupled to a rotating upper base with a bearing, which allows the prototype horizontal movement. The top rotating base is interconnected through another bearing to a rod of 52x2x1.5 cm length capable of performing the vertical action, studied in this work.

In the prototype, the joint movement is performed by the brushless motor actuation response. This movement will occur by driving the brushless motor, where the manipulated variable was monitored through a potentiometer, fixed close to the rotation axis of the vertical movement rod. Also, a three-cell LiPo rechargeable battery has been attached to the rotary base, as well as a circuit board designed to connect the Arduino to the brushless motor and the potentiometer. Figure 1 shows it.

The 2 boards are the main one (where the Arduino is embedded) and a board to support and connect the ESC. On the main board there are two connections to the Arduino PWM (Pulse Width Modulation) and 3 ports, which can be used to drive several devices. In this work, one of these connections was used to drive the brushless motor, through the ESC. The main board also has two ports for connection to the digital sensors of the communication standard I2C, connected to the SCL and SDA pins of the Arduino, and two ports for connection of analog sensors, such as the potentiometer previous mentioned. All connections are made through six RJ11 connectors.

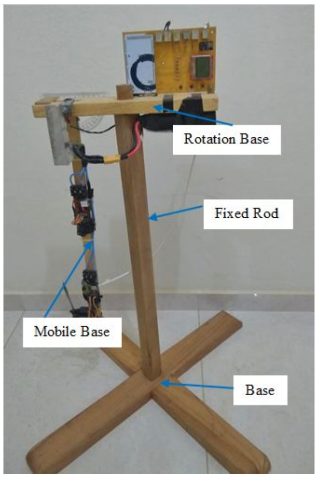


Figure 1: Illustration of the prototype developed for future implementation, modeled for the simulation simulations.

3 System Modelling

According to the prototype physical characteristics, it can be approximated to a physical or compound pendulum. The forces acting on the pendulum can be written through (1).

$$I\ddot{\theta} + D\dot{\theta} + mg\frac{L}{2}\sin\theta = \tau \tag{1}$$

where the term $I\ddot{\theta}$ corresponds to the torque generated by the circular motion reaction, $D\dot{\theta}$ is related to the torque due to the viscous friction, and the term $mg\frac{L}{2}\sin\theta$ refers to the torque around the suspension point, due to the gravitational force action. Moreover, I corresponds to the system moment of inertia, m indicates the mass, g is the gravity force, and τ is the input signal.

Replacing the system moment of inertia in (1):

$$\begin{aligned} \frac{mL^2}{3}\ddot{\theta} + D\dot{\theta} + mg\frac{L}{2}\sin\theta &= \tau \qquad (2) \\ k_1\ddot{\theta} + k_2\dot{\theta} + k_3\sin\theta &= \tau \qquad (3) \end{aligned}$$

where $k_1 = \frac{mL^2}{3} = 0.005769Kgm^2$, $k_2 = D = 0.0005769Kgm^2/s^2$ and $k_3 = mg\frac{L}{2} = 0.163m^2/s$.

4 Design of Controllers

4.1 PID Controller

According to [9], the PID controller is widely used in industry due to its simple topology and easy adaptation. Work [10] completes that the PID controller has a simple control structure, reduced power consumption, and satisfactory efficiency. Also, approximately 90 % of industrial controllers still are currently PIDs [11].

The author from work [12] stated that the use of the PID controller causes the insertion of a pole into the system where this additional pole usually assists on limiting the action or gain, of the controller at high frequency.

Also, according to [12], another analytical method can be used to design the PID controller. Basing on it, a pole in closed-loop (generally complex) must be located in s_1. The location of this pole is chosen assuming that the system can be approximated to a second-order system since it is possible to set a relationship between the pole positions, the settling time (T_s) and the maximum overshoot (M_p (%)), as shown in Fig. 2.

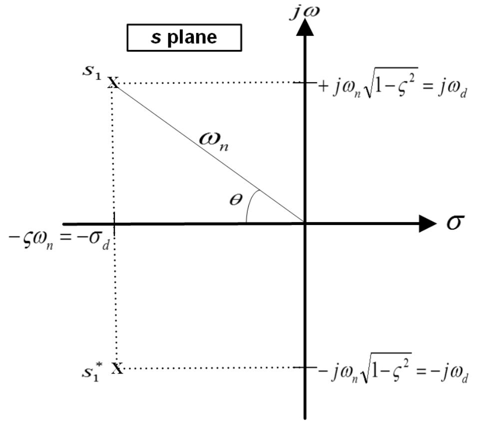


Figure 2: General *Root-Locus* for a second order system.

Assuming that the dominant conjugate poles given by s_1 and s_1^* are desired, the design procedure described in [12] allows to calculate the gains k_P, k_I and k_D, by means of:

$$k_P = \frac{-\sin(\beta+\psi)}{|G_p(s_1)H(s_1)|\sin\beta} - \frac{2k_I\cos\beta}{|s_1|} \tag{4}$$

$$k_P = \frac{\sin(\psi)}{|s_1||G_p(s_1)H(s_1)|\sin\beta} + \frac{2k_I}{|s_1|^2} \tag{5}$$

The angles β and ψ are obtained from (6) and (7):

$$S_1 = |s_1|e^{j\beta} \tag{6}$$

$$G_p(s_1)H(s_1) = |G_p(s_1)H(s_1)|e^{j\psi} \tag{7}$$

Since there are two equations and three variables, one of the controller gains must be adopted to obtain the others. It is worth mentioning that the equations above can also be used for the design of PI and PD controllers, by defining the corresponding gains as zero.

Only for the tuning procedure, the system represented by (3) was linearized around the angle of 45 degrees, in order to satisfy tests around the system angular range mid-point. The linearized equation representing the system is given by (8):

$$0.005769\Delta\ddot{\theta} + 0.00005769\Delta\dot{\theta} + 0.11526\sin\Delta\theta = \tau - 0.11526 \tag{8}$$

where $\Delta\theta$ is the angular rate around 45 degrees.

Based on the procedure described above and considering the project criteria maximum overshoot $M_p = 5\%$, settling time $T_s = 10$ sec, the controllers gains $k_I = 0.2$, $k_P = 0.36293$ and $k_D = 0.59987$ were obtained.

Although the closed-loop zeros influenced the response, and produced the observed characteristic between 1 and 6 seconds, the controller constants were adopted for the tests.

4.2 Fuzzy Controller

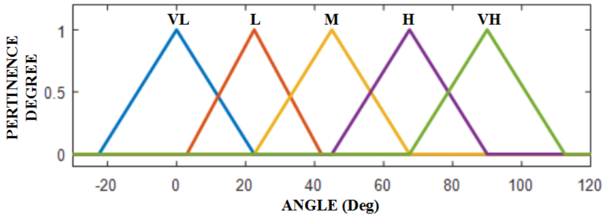
As an alternative to the PID controllers, the Fuzzy controller is appropriate considering it allows the use of operator intelligence for automatic control. Also, the Fuzzy control logic is a mathematical tool that was introduced by Lofti Zadeh in 1965 to deal with problems with uncertainties. This tool is used to develop intelligent control and system information, being applied in the control of highly non-linear processes or with unknown characteristics. Fuzzy logic allows an expert to implement control strategies used by human operators [13, 14].

The developed Fuzzy controller was implemented in MATLAB software, where it takes into account three inputs: angle, angle error, and angle error derivative. It is verified in the literature the frequent use of error and derivative error variables in the project of Fuzzy controllers. For the input variable angle, the following Fuzzy sets were defined: Very Low Angle (VL), Low Angle (L), Medium Angle (M), High Angle (H) and Very High Angle (VH).

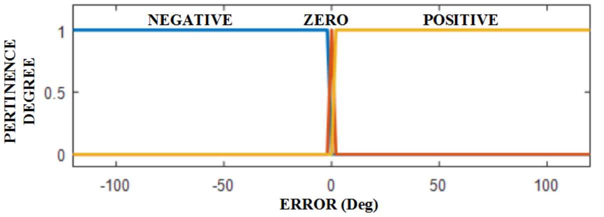
For the angle error input variable, the Fuzzy sets created were: Negative Error (N), Zero Error (Z), Positive Error (P). Finally, the last Fuzzy input sets were defined for angle error derivative input variable: Negative Error Derivative (N), Zero Error Derivative (Z), Positive Error Derivative (P).

Considering the output Fuzzy variable, the torque follows: Zero Torque (Z), Low Torque (L), Medium Torque (M), High Torque (H), Light High Torque (LH), and Very High Torque (VH).

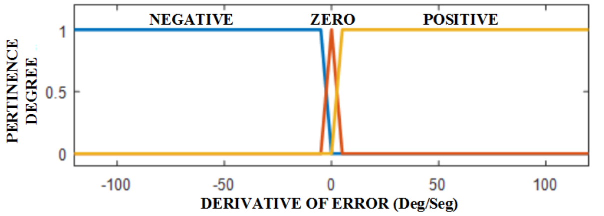
The Fuzzy pertinence functions for the input variables are presented in Figs. 3(a), 3(b) and 3(c). Figure 3(d) shows the respective output Fuzzy pertinence sets. It is worth mentioning that the distance between the peaks of the output relevance functions Z, L, M, H, LH, and VH incorporate the nonlinear system aspect, as explained below. In fact, it is observed the asymmetric distribution of sets and the difference in their widths before the discourse universe.



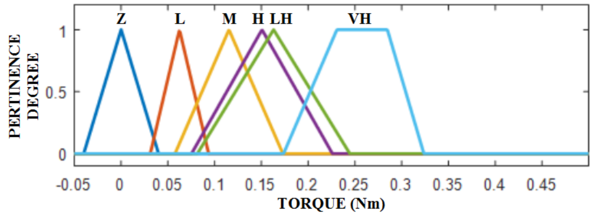
(a) Pertinence functions of angular position input variable.



(b) Pertinence functions of angular error input variable.



(c) Pertinence functions of angular error derivative input variable.



(d) Pertinence functions of torque output variable.

Figure 3: Pertinence functions of input and output variables.

It should be noted that for each input variable, a corresponding discourse universe was stated. The rod position to be controlled may range from 0 to 90 degrees. However, for the angle input variable, the discourse universe was -30 to 120 degrees. This difference of 30 degrees was chosen, assuming that the system may oscillate during the performance. The same criteria were adopted for the error and error derivative input variables.

Also, the input signal variation of the Fuzzy set cannot exceed the discourse universe established for the inputs. Thus, the saturation is applied to the signal, which is inserted before each information in the simulation step.

The low angle set is centered at 0 degrees, just as the very low angle set is 22.5 degrees. The medium angle set is centered at 45 degrees, and the high angle set at 67.5 degrees. Finally, the very high angle set is centered at 90 degrees. These characteristics are essential in the definition of the associated torque sets.

In fact, the torque associated with each angle was determined from (3), where in steady-state regime, it is desired the terms $\ddot{\theta}$ and $\dot{\theta}$, reducing to $\tau = 0.163\sin(\theta)$. For better illustration, some data are shown in Table1.

Table 1: Associated torque to the angle set.

Angle (deg)	**Torque (Nm)**
0	0
22.5	0.06238
45	0.1153
67.5	0.1506
90	0.163

After determining the controller inputs and outputs, the inference rules were established, which relates the pertinence functions of the inputs to the output. The rules are presented in Tables 2, 3, 4, 5 and 6.

Table 2: Rules used for Very Low Angles (VL).

		Very Low Angle (VL)		
		Error		
		N	Z	L
de/dt	N	Z	Z	Z
	Z	Z	Z	L
	L	Z	L	L

Table 3: Rules used for Low Angles (L).

		Low Angle (L)		
		Error		
		N	Z	L
de/dt	N	Z	Z	L
	Z	Z	L	M
	L	L	M	LH

Table 4: Rules used for Median Angles (M).

		Median Angle (M)		
		Error		
		N	Z	L
de/dt	N	Z	L	M
	Z	L	M	LH
	L	M	LH	H

Table 5: Rules used for Light High Angle (LH).

		Light High Angle (LH)		
		Error		
		N	Z	L
de/dt	N	L	M	LH
	Z	M	LH	H
	L	LH	H	VH

Table 6: Rules used for Very High Angle (VH).

		Very High Angle (VH)		
		Error		
		N	Z	L
de/dt	N	L	M	H
	Z	M	H	VH
	L	LH	VH	VH

5 Simulation Results

This section will present the simulation results performed using the designed Fuzzy and PID controllers. It will be presented in two subsections, divided into single and multiple setpoints.

5.1 Unique Setpoint Results

Here will be shown the simulation tests where only 1 setpoint requested during each experiment. The results considered 40 seconds of simulation with step application in 0 seconds for reference angles of 15, 53 and 80 degrees, respectively shown in Figs. 4, 5 and 6.

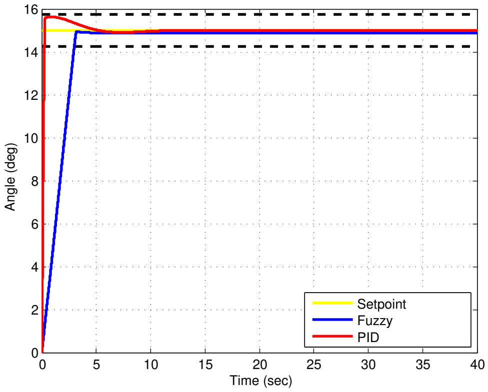


Figure 4: Controlled responses with angular setpoint of 15 degrees.

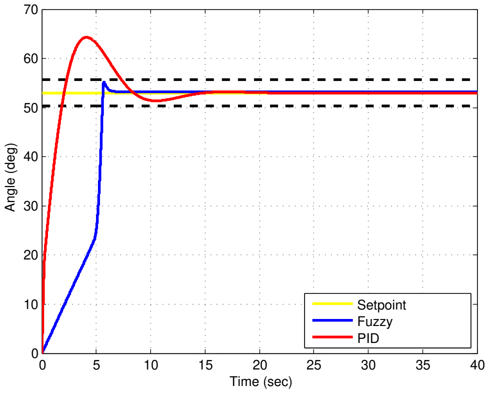


Figure 5: Controlled responses with angular setpoint of 53 degrees.

In all cases, the output tracked the reference value for Fuzzy and PID controllers. However, the Fuzzy controller made the system to stabilize faster. It is also possible to note that for a reference angle of 15 degrees (Fig. 4) only the PID controller showed overshoot signals, but within the established range of 5% (black dashed lines), where both controlled performed system stabilization before 10 seconds.

Increasing the reference angle, for 53 and 80 degrees (Figs. 5 and 6, respectively), the controlled responses generated by the PID controllers presented overshoot peaks more than 5%, whereas the Fuzzy controller remained within the desired range. For Fig. 5, the overshoot values were 21.7% and 4.6%, respectively for PID and Fuzzy controller. For Fig. 6, they were 25% and 4.4% also for PID and Fuzzy controllers, respectively.

Regarding the system stabilization, the Fuzzy controller made the system to reach the reference value close to 10 seconds for both cases, where the

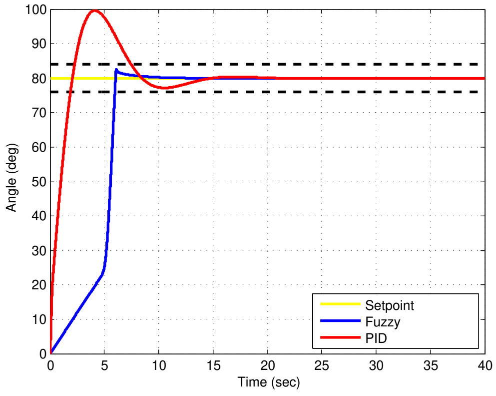


Figure 6: Controlled responses with angular setpoint of 80 degrees.

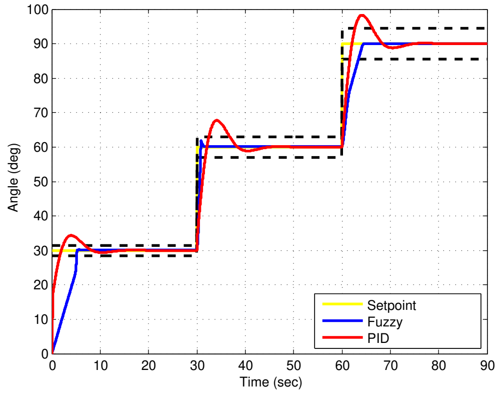


Figure 7: Controlled responses considering crescent setpoints.

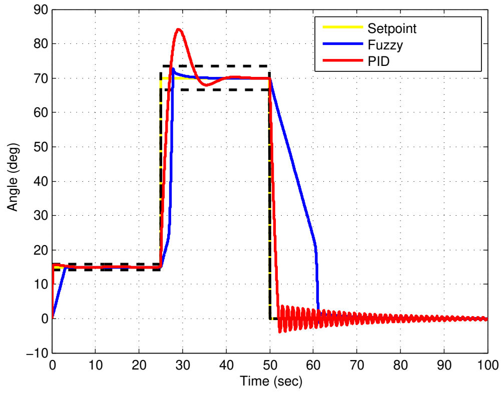


Figure 8: Controlled responses considering crescent and descendent setpoints.

PID controller obtained this value around 15 seconds.

5.2 Multiple Setpoint Results

This section will show the controller performances considering setpoint changes during the simulation.

The results were performed considering 90 seconds of simulation, applying a step inputs at 0, 30, and 60 seconds, for reference angles of 30, 60, and 90 degrees, respectively, shown in Fig. 7. Figure 8 shows the controlled responses for 100 seconds of simulation considering the application of a step in 0, 25 and 50 seconds for reference angles of 15, 70 and 0 degrees, respectively. Figure 9 displays the curves obtained considering 90 seconds of simulation and the application of a step of 18 degrees in 0 seconds. After, a ramp reference signal was requested at 30 seconds until it reaches 88 degrees in 60 seconds, remaining in that value until the end of the simulation. Finally, the results obtained for 20 seconds simulation considered a reference input with sinusoidal oscillation, shown in Fig. 10.

As can be seen in Fig. 7, both controllers were able to track the reference setpoints. However, the Fuzzy controller was able to control the system faster, also presenting lower overshoot values for all reference setpoints compared to the PID. This behavior is mainly due to the interaction of the integral action with the torque saturation procedure.

In the second result, it is possible to observe that for the reference value of 15 degrees, both controllers remained within the overshoot established range. Besides, it is remarked that the PID controller caused the output to be tracked faster than the Fuzzy controller. For the reference of 70 degrees, the PID controller presented a maximum overshoot of 20%, also performing slower settling time than the Fuzzy controller. Finally, for a reference of 0 degrees, the Fuzzy controller took more time to track the input. Also, it made the system stabilize faster than the PID controller, which in turn presented oscillatory behavior.

In the third result (Fig. 9) can be seen that both controllers reached the control requirements, tracking the reference signal. Besides, the PID controller was able to control the system faster than the Fuzzy controller, even with higher overshoot peaks.

In the last result (Fig. 10) is possible to observe that soon in the first half sinusoidal cycle, the Fuzzy controller was able to track the input and remained practically without overshoots throughout the simulation. The PID controller, in turn, took around the entire first cycle to follow the reference signal, as well as out of the $\pm 5\%$ error range set for performance evaluation.

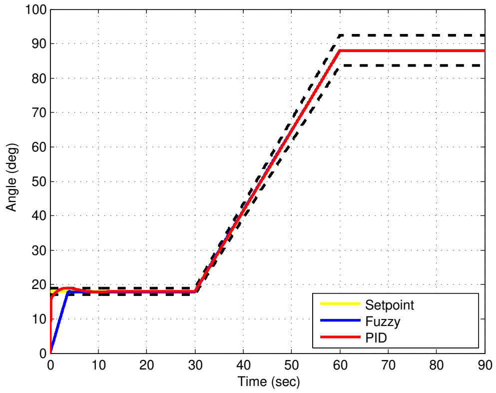


Figure 9: Controlled responses considering step and ramp setpoints.

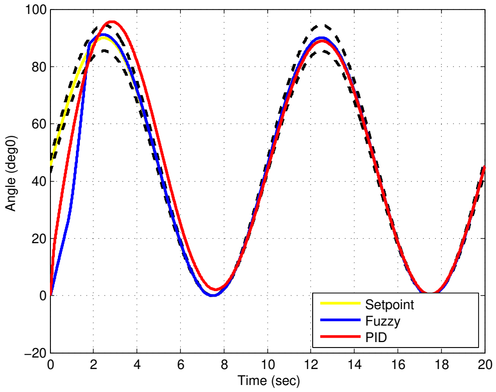


Figure 10: Controlled responses considering sinusoidal setpoint.

6 Conclusions

This work proposed the study and development of a structure that represents a nonlinear system capable of performing one DOF movement. The objective of this work was to control the position of a rod in the vertical plane, where two control techniques were implemented: PID and Fuzzy.

Initially, researches were carried out involving the subjects to understand the proposed work, as well as a bibliographical review highlighting the importance of the study and understanding of the students about nonlinear systems.

After analyzing the simulation results, it was verified that both simulated controllers (PID and Fuzzy) reached the control requirements. Among the simulated cases, most of the observed conditions, the Fuzzy controller obtained the best performance.

6.1 Future Works

The main next step for this work is to perform the experimental tests, implementing the controllers in the respective control board, embed in the prototype shown in Fig. 1.

Acknowledgment

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References:

[1] K. Jahnavi and P. Sivraj, "Teaching and learning robotic arm model," in *International Conference on Intelligent Computing, Instrumentation and Control Technologies (ICICICT)*, pp. 1570–1575, IEEE, 2017.

[2] F. C. Ferreira, M. F. Santos, R. Z. Nogueira, and A. F. Santos Neto, "Modeling and control of one DOF arm controlled by brushless motor: A first class experimental case," in *21st International Conference on System Theory, Control and Computing (ICSTCC)*, pp. 352–357, IEEE, 2017.

[3] K. Agoston and C. Nagy, "Data acquisition for angular measuring and positioning system," in *International Conference on Automation, Quality and Testing, Robotics*, pp. 175–178, IEEE, 2012.

[4] S. Alavandar and M. J. Nigam, "Genetic fuzzy based tracking control of 3 DOF robot arm," in *International Conference on Emerging Trends in Engineering and Technology*, pp. 547–552, IEEE, 2008.

[5] A. A. Aly and F. A. Salem, "Design of intelligent position control for single axis robot arm," *1st International Journal of Scientific & Engineering Research*, vol. 5, no. 6, pp. 790–795, 2014.

[6] R. Manikandan and R. Arulmozhiyal, "Modeling and simulation of fuzzy based BLDC fed vertically rotating one DOF robot arm position control system," in *10th International Conference on Intelligent Systems and Control (ISCO)*, pp. 1–7, IEEE, 2016.

[7] A. Rojas-Moreno and B. Apaza-Palomino, "FO position control of a dc motor an experimental comparative study," in *URUCON*, pp. 1–4, IEEE, 2017.

[8] C. C. Mouli, P. Jyothi, K. N. Raju, and C. Nagaraja, "Design and implementation of robot arm control using labview and arm controller," *Journal of Electrical and Electronics Engineering (IOSR)*, vol. 6, no. 5, pp. 80–84, 2013.

[9] Y. Wang, H. Zou, J. Tao, and R. Zhang, "Predictive fuzzy PID control for temperature model of a heating furnace," in *36th Chinese Control Conference (CCC)*, pp. 4523–4527, IEEE, 2017.

[10] V. Priya and V. Kavitha, "Design of efficient multiply-accumulate block for PID controllers," in *2nd International Conference on Electronics and Communication Systems (ICECS)*, pp. 322–325, IEEE, 2015.

[11] F. A. Castro, N. D. Bernardes, M. A. Cuadros, and G. M. de Almeida, "Comparison of fractional and integer PID controllers tuned by genetic algorithm," in *12th International Conference on Industry Applications (INDUSCON)*, pp. 1–7, IEEE, 2016.

[12] C. L. Phillips and J. M. Parr, *Feedback control systems*. Pearson, 2011.

[13] A. K. Gaurav and A. Kaur, "Comparison between conventional PID and fuzzy logic controller for liquid flow control: Performance evaluation of fuzzy logic and PID controller by using matlab/simulink," *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, vol. 1, no. 1, pp. 84–88, 2012.

[14] X. Han, W. Deng, Q. Sun, and Y. Zhao, "Direct adaptive fuzzy H∞ tracking control for a class of uncertain nonlinear systems based on LMI technique," in *Chinese Control and Decision Conference*, pp. 3019–3023, IEEE, 2010.