Optimal power dispatch of DGs in DC power grids: a hybrid Gauss-Seidel-Genetic-Algorithm methodology for solving the OPF problem

Oscar Danilo Montoya
Universidad Tecnológica de Bolívar
Km 1 vía Turbaco
Cartagena, COLOMBIA
omontoya@utb.edu.co

Walter Gil-González
Universidad Tecnológica de Pereira
Carrera 27 # 10-02 Barrio Alamos
Pereira, COLOMBIA
wjgil@utp.edu.co

Luis F. Grisales-Noreña
Instituto Tecnológico Metropolitano
Calle 73 No. 76A - 354, Vía al Volador
Medellin, COLOMBIA
luisgrisales@itm.edu.co

Abstract: This paper addresses the optimal power flow (OPF) problem in direct current (DC) power grids via a hybrid Gauss-Seidel-Genetic-Algorithm methodology through a master-slave optimization strategy. In the master stage, a genetic algorithm is employed to select the power dispatch for any distributed generator while the slave stage, Gauss-Seidel method is used for solving the resulting power flow equations without recurring to matrix inversions. This approach is important since it can be easily implementable over any simple programming toolbox finding the optimal solution of the OPF problem. Genetic-Algorithm proposed in this paper corresponds to a continuous variant of the conventional binary approaches. Computational results show the efficiency and accuracy of the proposed optimization method when is compared to GAMS/CONOPT nonlinear solver.

Key–Words: Direct current power grids, distributed generation, Gauss-Seidel method, genetic algorithm, hybrid master-slave optimization strategy, optimal power flow problem.

1 Introduction

1.1 General context

Electrical power grids are an indispensable part of the human development including all technological advances, which have allowed improving people quality life [1, 2]; nevertheless, conventional electrical power systems have also produced harmful effects around the world mainly evidenced as global warming, which is caused by the consumption of fossil fuels (transportation system and thermo-electric plants) producing a lot of tons of greenhouse effect gases [3].

To deal with these problems new paradigms in electrical systems have been developed in recent decades as are the cases of smart grids and microgrids, based on a combination of renewable energy resources and energy storage technologies as can be seen in Fig. 1[1]. These combinations allow replacing gradually the dependence of fossil fuels for electricity generation [3]. To supply all power consumption for the constantly increasing demand two main electrical distribution technologies based on alternating and direct current (AC and DC) have been developed as well as their hybrid combinations [5].

Power grids operating under AC reference frame

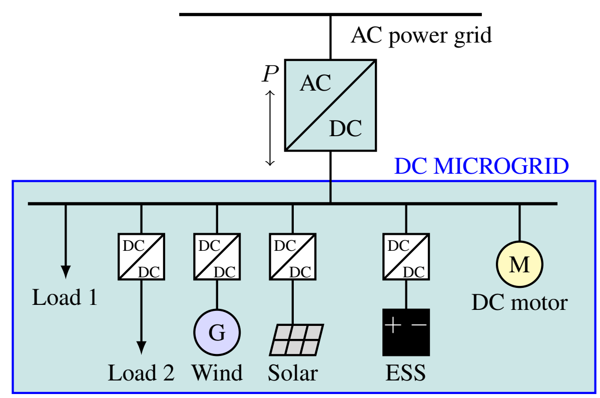

Figure 1: Typical interconnection of distributed energy resources, which conform a DC power microgrid

have been widely explored in specialized literature from the point of view of dynamical and static analysis [6], i.e., under transient and steady-state conditions. In case of dynamical analysis, differential equation methods are required, while in case of static analysis nonlinear equations appear in the numerical reasoning [7, 8]. In this sense, power flow analyses in case of static approaches correspond to one of the most studied problems in AC power grids by using linear and nonlinear techniques [9, 10, 11].

Electrical DC power grids are not the exception

[1]This figure was transformed from the AC configuration presented in [4] into an equivalent DC grid

of these studies, mainly when power electronics have allowed real DC power grid implementations [5], for improving the conventional distribution system performance [12]; following this line, DC power flow analysis corresponds to the essential technique for planning and operation [13, 14, 15], which becomes this research topic to an excellent opportunity to propose novel, efficient and easily implementable solving methodologies.

1.2 Motivation

When DC power grids are analyzed via power flow methodologies, it is necessary to know that in specialized literature the expression **DC optimal power flow** corresponds to an AC simplified power flow formulation and it is not related with power flow analysis in direct current power networks [14, 16]; nevertheless, the usage of the same expression for two different problems could cause confusion between non-familiarized readers. For this reason, we prefer to use the complete name of the problem (i.e., optimal power flow analysis in DC power grids) to make reference to this research area.

This work is motivated by two main reasons. The first corresponds to the needed of implementing an optimal power flow in DC microgrids for obtaining the power values of the distributed generators that allow reducing the active power losses and satisfying the technical restrictions of the system. The second reason is the importance of having easily implementable tools for solving important and recurrent problems in electrical engineering as is the case of optimal power flow analysis avoiding to recur to sophisticated software or optimization packages for solving these problems.

1.3 Brief state-of-art

In specialized literature exist multiple research papers which analyses the OPF problem in electrical power grids. These investigations can be divided into an AC and DC power grids, respectively [9]. Nevertheless, it is important to point out that both OPF models share the same characteristics in terms of mathematical complexity, i.e., nonlinear, non-convex problem [14]. We focus this review of state of the art on DC power grids from high-voltage to low-voltage DC power grids.

For solving the OPF problem in DC power grids have been proposed equivalent convex formulations of the problem as presented in [17] and [18]. The first case proposes a convex reformulation of the OPF equations via semidefinite programming by relaxing the non-convex constraint associated to rank one of the matrix of variables, then, after solving the OPF problem, the voltage profiles are recovering via eigenvalues and eigenvectors decomposition [19, 20]. In the second case, a second-order cone programming model is proposed, the authors apply the same relaxing concept to solve and recover the solution variables. Both approximations are compared to the exact solution of the problem obtained through a GAMS optimization package with a high grade of fidelity in terms of objective function.

A port-Hamiltonian approach for solving OPF problems in DC power grids is proposed in [21]. This formulation guarantees stability properties in the sense of Lyapunov for passive DC circuits; nevertheless, not constant power loads are taking into account in the formulation, which reduces its applicability to linear circuits [22].

On the other hand, the existence of the power flow solutions for DC power grids have also studied in [13, 14, 23]. They mainly focus on the convergence properties of the power flow equations and their solving region; nevertheless, they do not analyze the OPF for power grids directly, since their objective is to analyze the structural and geometrical properties of the power flow equations.

Notice that the microgrids' control theory solves the OPF problem when analyses hierarchical controllers [24] or consensus algorithms [25]; notwithstanding, they concentrate their analysis on the control design from the differential equations point of view, which relegates the OPF for DC problem to a second plan since constant power loads are included into the DC grid via power electronic converters, which facilitate their manipulation in terms of stability properties [26, 27, 28].

In terms of optimization, some approximations of the OPF problem for DC power grids have been presented, and their corresponding OPF equations are solved via optimizing packages and optimization techniques [17, 29, 30] considering the possible interconnection of distributed energy resources, including wind and photovoltaic generation as well as battery energy storage systems [31, 32, 33, 34].

It is important to stand out that in the revision of state-of-the-art made in this paper was found that the integration of DGs in DC grids is a research topic in progress, for this reason in the specialized literature exists low documentation and investigations about of optimal sizing of distributed generation in DC electrical distribution system. The aforementioned situation highlights the importance of exploring this problem and proposing new methodologies in this research line. Additionally, to the best knowledge of the authors, in the specialized literature, the OPF problem for DC power grids have not addressed from hybrid metaheuristic optimization techniques (i.e., Genetic-Algorithm) and conventional numerical methods (i.e., Gauss-Seidel), which is a clear gap that this research tries to fill.

1.4 Contribution and scope

This paper presents as main contribution the hybridization of a conventional numerical method known as Gauss-Seidel [10] and optimization technique named genetic-algorithms [35] for solving the optimal power flow problem in DC power grids. Additionally, we present the possibility to adapt the classical binary-integer genetic-algorithm for solving continuous optimization problems. Another important fact, it is that we propose the solution of the optimal power flow problem in DC power grids from the point of view 100% algorithmic avoiding the needed of using any specialized software to carry out this task.

The optimal power flow analysis for DC power grids presented in this paper assumes that electrical DC network has been designed to support all power consumption guaranteeing voltage stability conditions [36], which implies that we assume that the conventional power flow equations exhibit solution for any power load and distributed generation value in the range of analysis [23].

1.5 Document organization

The remain of this document is organized as follows: Section 2 presents of mathematical formulation for the optimal power flow problem in DC power grids by using its nonlinear non-convex representation. Section 3 shows the hybrid Gauss-Seidel–Genetic-Algorithm methodology, focusing on the main aspects associated to the evolution strategies in the genetic algorithm as well as the recursive equations for solving power flow equations via Gauss-Seidel numerical method. Section 4 presents the main characteristics of the test system and the proposed simulation scenarios. Section 5 shows computational results via MATLAB software and their comparison to GAMS conventional optimization package. Finally, some concluding remarks are provided in Section 6.

2 Mathematical formulation

For obtaining the general formulation of the optimal power flow problem in a DC grid, let us consider a DC grid as a set of nodes represented by $\mathcal{N} = \{1, 2, ..., n\}$, a set of generator terminals $\mathcal{G} \subseteq \mathcal{N}$ and a set of constant power loads $\mathcal{L} \subseteq \mathcal{N}$. The DC grid lines are represented by a set $\mathcal{E} = \{(i, j)\} \subseteq \mathcal{N} \times \mathcal{N}$ and the DC nodal conductance matrix is defined as $\mathcal{G}_{bus} \in \mathbb{R}^{n\times n}$ which is a symmetric and positive semidefinite matrix such that $[\mathcal{G}]_{i,j} = \mathcal{G}_{ij}$. Notice that, any consumption modeled as a constant resistance value is included into the conductance matrix since its mathematical model is defined by a straightforward linear relation (i.e., Ohm's law). Besides, it is important to highlight that there is only two continuous variables per node, this is, the voltage profile v_i and the net power injected p_i, for the i^{th} node, respectively [37, 13].

Optimal power flow problem for DC power grids can be formulated as a nonlinear-non-convex optimization problem [13, 14], as follows:

Objective function:

$$\min z = \sum_{i\in\mathcal{N}} \left[\left(\sum_{j\in\mathcal{N}} \mathcal{G}_{ij} v_i v_j \right) - \mathcal{G}_{i0} {v_i}^2 \right] \quad (1)$$

Set of constraints:

$$p_i^g - p_i^d = \sum_{j\in\mathcal{N}} \mathcal{G}_{ij} v_i v_j \ \{\forall i \in \mathcal{N}\}, \quad (2)$$

$$v_i^{\min} \leq v_i \leq v_i^{\max} \ \{\forall i \in \mathcal{N}\}, \quad (3)$$

$$p_i^{g,\min} \leq p_i^g \leq p_i^{g,\max} \ \{\forall i \in \mathcal{G}\}, \quad (4)$$

where p_i^g is the active power generated in the node i, $p_i^{g,\min}$ and $p_i^{g,\max}$ correspond to the minimum and maximum power generation limits for each generator located in the node i, respectively; $v_i^{\min}$ and $v_i^{\max}$ are the minimum and maximum allowed voltage profiles at node i, while z is the total active power losses in the DC network. Notice, that $\mathcal{G}_{i0}$ represents the constant impedance load connected at i^{th}, which corresponds to linear consumption (resistive load in the DC network), and it can not be considered part of the active power losses as presented in the first part of 1, (remember that the conductance matrix contains all resistive effects in the network including the constant resistive loads).

The mathematical optimization model given from (1) to (4) has the next interpretation. Equation (1) determines the total active power losses in the grid caused by the resistive effects in all distribution DC lines, these active power losses are calculated as function of the voltage profiles in the entire network; expression (2) determines the power balance per node, i.e., this equation corresponds to a set of nonlinear non-convex equations widely well-known in specialized literature as power flow equations [13, 36, 37]. On the other hand, expressions (3) and (4) are bounded by constraints associated with voltage regulation policies and power capabilities in all power generators.

Notice that this paper focuses on the possibility to decouple the optimization problem above presented in two subproblems, which allows solving it via numerical methods without approximating its mathematical model; The general OPF problem is composed first by the optimal selection of the total power generated by any distributed generator, and second, by the calculation of the voltage profiles in the entire power grid, for this reason, we propose a hybrid Gauss-Seidel–Genetic-Algorithm (GS-GA) for decoupling this optimization problem into a generation problem named *master problem* and classical power flow problem named *slave problem*. The main advantage of this approach lies that for solving the optimal power flow problem in DC power grids any specialized optimization package or specialized software is required to enhance its optimal solution, since it corresponds only to an algorithmic solution (evolution optimization process), as will be evidenced in next sections.

3 Proposed optimization methodology

The optimal power flow problem for DC power grids is addressed in this paper from the point of view of master-slave solution strategy where the master problem defines the power generation for each distributed generator through a genetic-algorithm optimization approach that guarantees minimum power losses in the grid; while the slave problem solves the conventional DC power flow equations via Gauss-Seidel numerical method [14].

3.1 Master problem

In general terms the master problem consists to determine the power generation in each power controlled node, i.e., in all distributed generators without the capability to control voltage profile. In this sense, we consider:

Assumption 1 *The DC power grid contains at least one constant voltage node.*

A constant voltage node is completely necessary to avoid trivial solution to the power flow equations for electrical AC or DC power grids, in this sense, this node has the capacity to generate (absorb) the missing (excessing) power in the entire electrical network, in other words, this node is widely used in specialized literature as oscillating node or slack node [8, 17].

Considering that, the constraint (2) can be rewritten as follows:

$$p_i^g - p_i^d = \sum_{j \in \mathcal{N}} \mathcal{G}_{ij} v_i v_j \ \{\forall i \in \mathcal{N} - \mathcal{S}\}, \quad (5)$$

$$v_k = v_k^c \ \{\forall k \in \mathcal{S}\}, \quad (6)$$

$$p_k^g \in \mathbb{R} \ \{\forall k \in \mathcal{S}\}, \quad (7)$$

where $\mathcal{S}$ represents the set of constant voltage nodes with well-know output voltages v_k^c.

Now, it is important to point out that to solve (5), i.e., to find all voltage profiles in the remains of set of nodes, we need to know the power generation or consumption in all these nodes, which does not happen yet, since p_i^g is an unknown variable for each generation node. Based on the aforementioned requirement, a genetic algorithm is proposed to determine the power generation in all distributed generators.

3.2 Genetic algorithm

We propose a continuous genetic algorithm metaheuristic technique to solve the optimal power flow problem considered that in each step the slave problem has been solved satisfactorily as will be presented in next section. Now, we are going to explain the main aspects of the genetic algorithm implementation. Following this line, a genetic algorithm corresponds to a classical well-known optimization technique to solve mainly binary-integer optimization problems, i.e., multi-stage transmission planning [38] or optimal placement and sizing distributed generators in distribution networks [39], among others; nevertheless, multiple authors have previously adapted this optimization technique for continuous optimization, such as, optimization of nonlinear continuous functions [40], second-order boundary differential equations [41] or optimal AC power dispatch [42], among others; with satisfactory results.

The genetic algorithm has five main characteristics to know:

i. Generation of the initial population.

ii. Fitness function calculation.

iii. Genetic operators for generating the descending population.

iv. New population calculation.

v. Stopping criteria.

All of them are extremely important to solve satisfactorily any optimization problem via genetic algorithms, for this reason, each one of them is going to be explained as follows:

3.2.1 Initial population

This is the first step for any optimization technique, in this sense, we propose a population with a size of a rows and s columns, i.e., an $a \times s$ matrix, where a corresponds to the number of potential solutions named set of individuals and s is the number of distributed generators to be dispatched ($s = |\mathcal{S}|$). In the case of the DC power flow problem, this initial population has the following structure:

$$\begin{bmatrix} p^g_{11} & p^g_{12} & \cdots & p^g_{1k} & \cdots & p^g_{1s} \\ p^g_{21} & p^g_{22} & \cdots & p^g_{2k} & \cdots & p^g_{2s} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ p^g_{l1} & p^g_{l2} & \cdots & p^g_{lk} & \cdots & p^g_{ls} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ p^g_{a1} & p^g_{a2} & \cdots & p^g_{ak} & \cdots & p^g_{as} \end{bmatrix}_{a \times s}$$

where p_{lk} represents the active power generated by the generator k at the l solution individual. This value is calculated as random number contained between $p^{g,\min}_k$ and $p^{g,\max}_k$, i.e., $p^g_{lk} = p^{g,\min}_k + \left(p^{g,\max}_k - p^{g,\min}_k\right) rand$, where $rand \in (0,1)$. Notice that, this initial population fulfills generation capabilities defined by (4), which implies that all row in the initial population is feasible in terms of power generation.

3.2.2 Fitness function

The fitness function in metaheuristics theory corresponds to the performance function assigned to any individual contains in the population, in other words, it determines what is the quality of an arbitrarily solution l. It is important to mention that genetic algorithms solve optimization problems by becoming a constraint optimization problem into a conditional problem. For this reason, we propose the following fitness function.

$$\tilde{z} = z + f_p \sum_{i \in \mathcal{N}} \left(f_i\left(v_i, v^{\min}_i, v^{\max}_i\right)\right), \tag{8}$$

where $\tilde{z}$ represents the fitness function, f_p corresponds to the penalty factor ($f_p \gg 0$) and $f_i(\cdot)$ represents a binary function which is calculated as presented below:

$$f_i\left(v_i, v^{\min}_i, v^{\max}_i\right) = \begin{cases} 1, & v_i < v^{\min}_i \\ 1, & v_i > v^{\max}_i \\ 0, & \text{otherwise} \end{cases}, \; \{\forall i \in \mathcal{N}\}$$

Notice that to calculate the fitness function is required to know the voltage profiles in all nodes of the system, which will be solved via slave problem. Additionally, this penalty strategy tries to eliminate any individual such that presents bad voltage performance by assigning to its fitness function a higher losses value; on the other hand, if the voltage profile is fulfilled in all nodes of the system, then, the fitness function corresponds to the real active power losses of the DC power grid, i.e., $\tilde{z} = z$. It is important to highlight that penalty factors are commonly employed for evolutive algorithms, since they allow exploring infeasible regions, that would be closed to promissory solutions.

3.2.3 Descending population

As a genetic algorithm corresponds to an iterative optimization process it is necessary to generate new potential solutions to the studied problem, to replace the bad solutions contained in the current population. To generate this set of solving individuals a classical selection, recombination and mutation operators are adapted to solve continuous optimization.

Selection: The descending population starts selecting an arbitrary subset of individuals contained in the current population, in this selection a random number r between 1 to a is chosen, i.e., $r = 1 + (a-1)rand$. If $r < a$, an additional $(a - r) \times s$ matrix with potential solutions are generated by using the same strategy employed for the initial population. The total set of selected individuals are conformed by the combination of the both aforementioned strategies.

Recombination: This process alters the descending population though the following principle. If the recombination probability r_p is grater than 50% (this value has been arbitrary selected), then, two arbitrary individuals (randomly selected) are recombined in an arbitrary position selected via random number between 1 to $s - 1$. If r_p is lower than 50%, then two arbitrary individuals (randomly chosen) are averaged to generate a new potential individual; notice that, this operation always generates feasible individuals, since the initial population as well as random solutions are generate inside of the admissibility region of the distributed generators. This process continues to obtain descending population with a potential solutions.

Mutation: In this point the mutation probability m_p is explored, i.e., if m_p is grater than 50% (this value has been arbitrary selected), an arbitrary position of the potential solution l is modified by an arbitrary power generation value guaranteeing that (4) be satisfied. If m_p is lower than 50% the potential solution l is not modified. This process continues until all descending individuals are analyzed.

Once the descending population has been generated its fitness function are calculated as given in (8).

3.2.4 New population

In the new population will be saved the set of best solutions found by the genetic algorithm, until the current iteration t. To generate the new population, we proceed as follows: A new population is generated by combining the current and descending set of individuals, which produces a population with $2a$ potential solutions; then, two potential solutions are identical, then, one of them is eliminated to this list. This procedure is repeated until guaranteeing that all potential solutions are different.

Now, with the resulting potential solution list, we ordered in ascendant form all individuals as a function of their fitness function, and the first a potential solutions are selected as a new population to pass to the next iteration cycle $t + 1$.

3.2.5 Stopping criteria

The proposed continuous genetic algorithm finishes its optimization process, when one of the following stopping conditions are achieved:

i. The total iteration cycles has been reached.

ii. The best potential solution does not been improved after m consecutive iterative cycles.

Otherwise, the genetic algorithm back to the descending population step.

3.3 Slave problem

The solution of the slave problem is indispensable to carry out to determine the fitness function of each potential solution contained in the population of the genetic algorithm. The slave algorithm resolves the conventional power problem given from (5) to (7) via Gauss-Seidel numerical method as we will be presented in next section.

3.3.1 Power flow solution via Gauss-Seidel method

Gauss-Seidel power flow method corresponds to one of the first numerical techniques reported in specialized literature to solve power flow equations in power grids. This solution methodology has been mainly used for AC power flow problems; nevertheless, this methodology is easily applicable on DC power grids since its structure preserves the same structure of AC grids, both formulations only differ in their solution space, i.e., AC power flows are analyzed inside of the complex number set, while DC power grids are analyzed inside of the real numbers set.

In the case of the DC power flow problem the Gauss-Seidel method solve iteratively (5) considering that the following assumptions are fulfilled:

Assumption 2 *The graph that describe the DC grid is connected (radial or mesh grids), i.e., there are not islanded nodes on the DC power grid.*

Assumption 3 *The DC power grid is operating under steady state conditions, i.e., there are not external perturbations.*

Assumption 4 *All possible generation-load scenarios are inside of the admissible power flow solution region, i.e., the DC grid is stable in terms of voltage.* and,

$$v_k^{b+1} = \frac{1}{\mathcal{G}_{kk}} \left(\frac{p_k^g - p_k^d}{v_k^b} + \sum_{j<k} |\mathcal{G}_{kj}|\, v_j^{b+1} \right) + \frac{1}{\mathcal{G}_{kk}} \sum_{j>k} |\mathcal{G}_{kj}|\, v_j^b,\ \{\forall k \in \{\mathcal{N} - \mathcal{S}\}\} \quad (9)$$

where b is the current iteration of the Gauss-Seidel method and the voltage profile and generation in the slack node(s) is(are) given by (6) and (7).

The accuracy of any power flow solution method (Newton-Raphson, Gauss-Seidel, linear methods) is highly dependent of the starting point, in other words, of the assigned values for the first iteration calculation. A common practice in specialized literature is to start all voltage in the grid as 1 p.u; nevertheless, we employ the open voltage circuit of the network calculated as the voltage profile in all nodes of the network when constant power loads and distributed generators are disconnecting, since this practice allows improving the convergence rates of the numerical methods for power flow analysis, in terms of number of iterations and processing times [43].

3.3.2 Advantages of the Gauss-Seidel method

Gauss-Seidel numerical method for solving power flow equations in DC power grids has the following advantages [10]:

i. its convergence can be guaranteed through point fixed theorems as presented in [14].

ii. it is not required to make inverse of the matrices to obtain the solution vector which contains all voltage profiles of the entire DC network.

iii. it can be applied over mesh or radial DC power grids indistinctly.

It is important to point out that Gauss-Seidel method was selected over other power flow solution methods since it is easily implementable of any programming language, which implies that not specialized software or optimization package is needed.

3.4 Pseudo-code for the proposed methodology

Algorithm 1 (pseudo-code version) shows the main characteristics for solving optimal DC power flow problem via hybrid GS–GA by using a master-slave strategy.

Data: DC power grid, genetic algorithm, Gauss-Seidel parameters.
for $t = 1 : t_{\max}$ **do**
  $m = 0$;
  if $t == 1$ **then**
    Generate the initial population;
    for $i = 1 : a$ **do**
      Solve the power flow problem;
      Evaluate the fitness function;
    end
  else
    Generate the descending popupulation;
    for $i = 1 : a$ **do**
      Solve the power flow problem;
      Evaluate the fitness function;
    end
    Determine the new population;
    if $(m > m_{\max} \mid\mid t == t_{\max})$ **then**
      Result: Impress results
      Break;
    end
  end
end

Algorithm 1: Proposed pseudo-code for the hybrid Gauss-Seidel–Genetic-Algorithm for solving the optimal DC power flow problem

4 Test system and simulation scenarios

As test system we employ a DC power grid reported in [14] which has 10 nodes operating under radial topology. This DC power grid has constant resistive load as well as constant power loads as presented in Table 1. Notice that we assume that all values are showed in per-unit, considering 1 kV and 100 kW as voltage and power bases. Besides, the power capabilities in all distributed generators are contained from 0 p.u to 3.0 p.u, while the maximum and minimum voltages in the grid are assigned from 0.9 p.u to 1.1 pu.

Table 1: Electrical parameters of the test system

From	To	R [pu]	Type of node	P [pu] - R [pu]
1	2	0.0050	Step-node	—
2	3	0.0015	P	-0.8
2	4	0.0020	P	-1.3
4	5	0.0018	P	0.5
2	6	0.0023	R	2.0
6	7	0.0017	Step-node	—
7	8	0.0021	P	0.3
7	9	0.0013	P	-0.7
3	10	0.0015	R	1.25

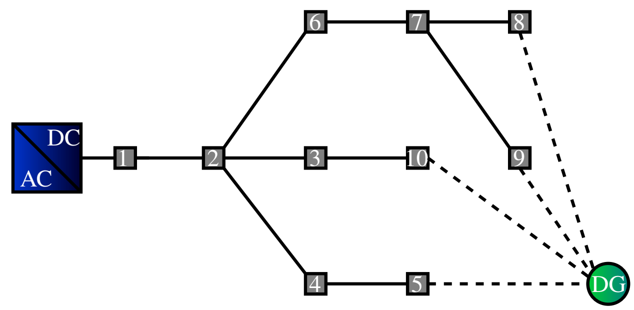

Figure 2: Electrical configuration of the low-voltage dc power grid

The proposed Gauss-Seidel–Genetic-Algorithm is validated by considering two simulation scenarios as described below:

Esc. 1: The OPF problem is solved considering the possibility to allocate a distributed generator (one at time) at the ending nodes of the test system as presented in Fig. 2.

Esc. 2: The OPF problem is solved considering the possibility to allocate two distributed generators (two at time) at the ending nodes of the test system.

It is important to mention that the location of the DGs in the test system is made in arbitrary form, since the approach of this article it is to analyze the optimal sizing of the DGs (OPF in DC grids) and it does not corresponds the optimal location of these. Additionally, all simulation results are compared in terms of objective function (see (8)) with GAMS optimization package.

5 Computational results

The computational implementation was carried-out through MATLAB 2017a software in a desk computer with 8 Gb RAM, 3.6 GHz, windows 10 Home Single Language, 64 bits.

For comparison purposes, the active power losses to the base case corresponds to 6.447 kW, which has been calculated by solving the power flow problem via GS method considering null the power injection at all distributed generators. Additionally, for the GA, the population size, number of iteration and convergence's error are selected in 10, 2000 and 1×10^{-9}, respectively; while the GS numerical method allows 1000 iterations and the convergence's error is fixed at 1×10^{-6}.

5.1 First simulation scenario

Table 2 presents the power losses after applying the optimization process (see **Esc. 1.**). It is important to highlight that the proposed GS–GA and the CONOPT solver find the same objective function, which implies that the proposed method converges 100% to the global optima; nevertheless, the power generation per node suffers small variations, which may be attributed to the numerical precision of GAMS and MATLAB. On the other hand, Fig. 4 shows the percentage of power losses reduction for each possible location of a distributed generator analyzed in the **Esc. 1.**

Table 2: Power generation and active power losses for **Esc. 1.** [kW]

Node	GAMS/CONOPT		GS–GA	
	p^g	z	p^g	z
5	196.323	3.037	196.474	3.037
8	168.603	3.283	168.067	3.283
9	195.994	2.455	195.558	2.455
10	252.285	1.353	252.469	1.353

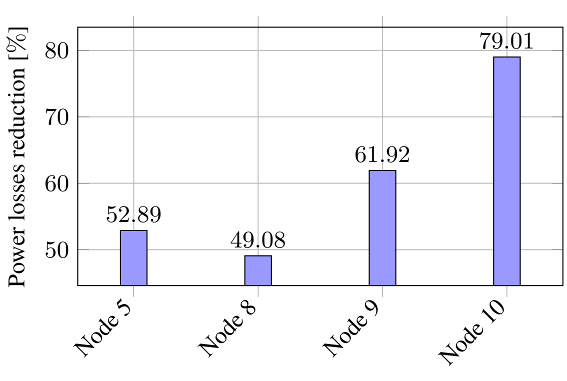

Figure 3: Total reduction of power losses for for **Esc. 1.**

Notice that the location of the distributed generator affects significantly the objective function performance, in this sense, when one distributed generator is considered, the node 8 is less attractive for power losses reduction (49.08%), while the node 10 is the most attractive node with 79.01% of power losses reduction. Nonetheless, their difference (29.93%) implies around of 84.402 kW additional of power injection, which may be not efficient from the economical point of view.

5.2 Second simulation scenario

In this simulation scenario, we present the possibility to solve the OPF problem for a combination of two distributed generators in four nodes, which produces six different alternatives. Table 3 presents the results obtained when GS-GA as well as CONOPT solver are employed to solve this problem.

Notice that the GS–GA and GAMS optimizing package find exactly the same optimal solution, which guarantees the 100% of convergence of the proposed GS–GA when is compared to a commercial widely-known solver.

In Fig. 4 are presented the percentage of power losses reduction as function of the distributed generators allocation. In this since, it is possible to observe that the combination between nodes 9 and 10 represent 91.59% of power losses reduction, which is the most important reduction obtained in the **Esc. 2.**, while the combination between 8 and 9 reduces the active power losses around 62.40%, which corresponds to the lower reduction in this scenario. Nevertheless, the combination between 9 and 10 nodes requires 291.538 kW, while 8 and 9 nodes requires only 199.986 kW, which implies that for improving the power losses reduction from 62.40% to 91.59% are needed 91.552 kW additional, which may be non-economical sustainable by the grid operator.

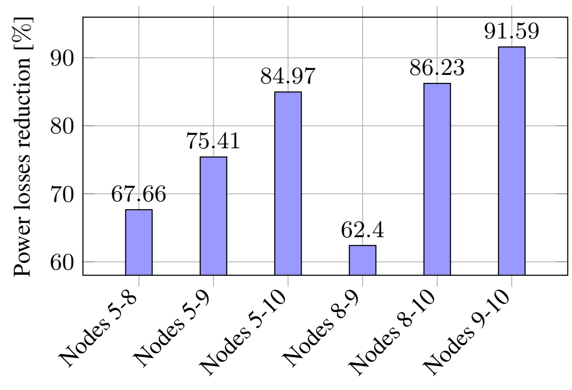

Figure 4: Total reduction of power losses for **Esc. 2.**

Table 3: Power generation and active power losses for **Esc. 2.** [kW]

Nodes	GAMS/CONOPT			GS–GA		
	p^g	p^g	z	p^g	p^g	z
5 - 8	135.121	107.367	2.085	135.941	106.607	2.085
5 - 9	116.762	139.144	1.585	115.063	140.601	1.585
5 - 10	82.237	200.568	0.969	83.160	198.985	0.969
8 - 9	31.171	168.815	2.424	32.117	169.578	2.424
8 - 10	76.349	204.355	0.888	75.616	205.247	0.888
9 - 10	106.012	185.526	0.542	105.657	185.740	0.542

5.3 Additional analysis and commentaries

For the sake of completing, in Fig. 5 is presented the voltage profile performance for the best power reduction impact, i.e., node 10 for **Esc. 1.** and nodes 9 and 10 for **Esc. 2.** as well as the base case. Recall that the performance of the voltage profile is highly dependent of the total power injection, nevertheless, it is not possible to affirm that the voltage profile increases linearly with the power injection, since the power flow equations have nonlinear intrinsic relations between both variables that complicates their analysis. On the other hand, it is important to mention that, when the **Esc. 2.** is observed, the voltage profile evidences a constant tendency for all nodes, this behavior occurs because the injection of active power in different nodes reduce significantly the current through the distribution lines, which reduces the voltages drops between neighborhood nodes which tend to equilibrate their voltage profiles. Nevertheless, in the **Esc. 2.** this situation is less evident since the total power injection is concentrated in a unique point, which has local and not global consequences in the voltage profile.

We consider that the results presented in this research are important for optimization as well as control issues since the GS–GA presents a straightforward form to solve complex nonlinear problems with classical and well-known optimization and numerical techniques, without recurring to specialized software, which is mainly attractive for free-software developers and researchers.

Finally, it is important to mention that for future comparison purposes the averaged time employed for the Gauss-Seidel method for each power flow evaluation is 2.8 ms; with 439 iterations.

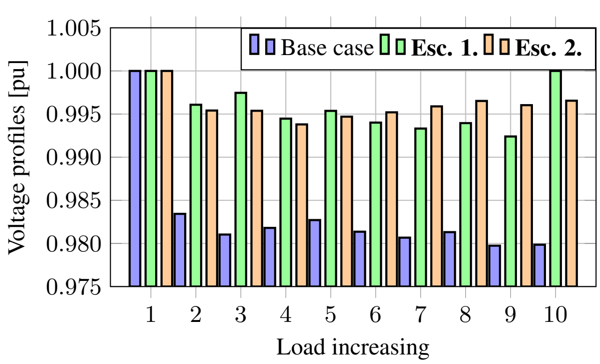

Figure 5: Voltage profile at load nodes when for different increments of the capacity of generation and consumption

6 Conclusions and future works

A hybrid GS–GA for solving the optimal power flow problem in DC power grids was presented. The main advantage of the proposed methodology was that it did not require any specialized software or optimization packages to the determine the optimal power generation in each distributed generator for minimizing the total power losses. Besides, the proposed methodology avoided making inverse matrices since GS worked directly on the power flow equations by recursively solving the convergence under normal operating conditions of the network.

A modification of the conventional binary GA was proposed to solve nonlinear continuous optimization problems through transforming the constrained optimization problem into an equivalent non-constrained problem; in addition, most of the constraints were directly fulfilled by the GA codification making easier to resolve the optimizing problem under analysis.

Simulation results allowed validating the applicability and the quality of the results regarding the values of the objective function since these were the same found by the GAMS commercial optimization package. Additionally, the numerical results presented in this paper showed that for minimizing power losses on DC power grids, not only the optimal power flow problem was important, since the location of the distributed generators, as well as the quantity of them, affects the total power losses of the grid significantly.

As future investigation works, the optimal location and dimensioning of distributed energy resources such a renewable generation and energy storage systems can be explored by using hybrid algorithms, such as binary-continuous genetic algorithms as well as conventional power flow solutions like Gauss-Seidel or Gauss-Jacobi or linear approximations. In addition, the hybrid GS–GA proposed in this paper can be used for microgrid control application to determine the set point of the controllers under any possible operating condition (combination of loads and distributed energy resources.)

Financial support

This work was partially supported by the National Scholarship Program Doctorates of the Administrative Department of Science, Technology and Innovation of Colombia (COLCIENCIAS), by calling contest 727-2015.

References:

[1] T. Slough, J. Urpelainen, and J. Yang, "Light for all? Evaluating Brazils rural electrification progress 2000 2010," *Energy Policy*, vol. 86, pp. 315 – 327, 2015. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0301421515300124

[2] D. R. Thomas and J. Urpelainen, "Early electrification and the quality of service: Evidence from rural india," *Energy for Sustainable Development*, vol. 44, pp. 11 – 20, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0973082617308700

[3] O. D. Montoya, A. Grajales, A. Garces, and C. A. Castro, "Distribution systems operation considering energy storage devices and distributed generation," *IEEE Latin America Transactions*, vol. 15, no. 5, pp. 890–900, May 2017.

[4] O. D. Montoya, A. Garcs, and G. Espinosa-Prez, "A generalized passivity-based control approach for power compensation in distribution systems using electrical energy storage systems," *Journal of Energy Storage*, vol. 16, pp. 259 – 268, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2352152X17304644

[5] S. Parhizi, H. Lotfi, A. Khodaei, and S. Bahramirad, "State of the art in research on microgrids: A review," *IEEE Access*, vol. 3, pp. 890–925, 2015.

[6] "Chapter 2 - uhv ac grid and system stability," in *UHV Transmission Technology*. Oxford: Academic Press, 2018, pp. 21 – 50. [Online]. Available: http://www.sciencedirect.com/science/article/pii/B9780128051931000021

[7] A.-F. Attia, R. A. E. Sehiemy, and H. M. Hasanien, "Optimal power flow solution in power systems using a novel Sine-Cosine algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 99, pp. 331 – 343, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0142061517330272

[8] A. Garces, "A linear three-phase load flow for power distribution systems," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 827–828, Jan 2016.

[9] H. Abdi, S. D. Beigvand, and M. L. Scala, "A review of optimal power flow studies applied to smart grids and microgrids," *Renewable and Sustainable Energy Reviews*, vol. 71, pp. 742 – 766, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1364032116311583

[10] P. Murty, "Chapter 10 - power flow studies," in *Power Systems Analysis (Second Edition)*, second edition ed., P. Murty, Ed. Boston: Butterworth-Heinemann, 2017, pp. 205 – 276. [Online]. Available: http://www.sciencedirect.com/science/article/pii/B9780081011119000100

[11] O. D. Montoya-Giraldo, W. J. Gil-González, and A. Garcés-Ruíz, "Optimal power flow for radial and mesh grids using semidefinite programming," *Tecno Lógicas*, vol. 20, no. 40, pp. 29–42, 2017.

[12] D. Gandini and A. T. de Almeida, "Direct current microgrids based on solar power systems and storage optimization, as a tool for cost-effective rural electrification," *Renewable Energy*, vol. 111, no. Supplement C, pp. 275 – 283, 2017.

[13] A. Garces, "On Convergence of Newtons Method in Power Flow Study for DC Microgrids," *IEEE Transactions on Power Systems*, pp. 1–1, 2018.

[14] ——, "Uniqueness of the power flow solutions in low voltage direct current grids," *Electric Power Systems Research*, vol. 151, no. Supplement C, pp. 149 – 153, 2017.

[15] O. D. Montoya, L. F. Grisales-Noreña, D. González-Montoya, C. Ramos-Paja, and A. Garces, "Linear power flow formulation for low-voltage DC power grids," *Electr. Power Syst. Res.*, vol. 163, pp. 375 – 381, 2018.

[16] K. S. H. Beagam, R. Jayashree, and M. A. Khan, "A new dc power flow model for q flow analysis for use in reactive power market," *Engineering Science and Technology, an International Journal*, vol. 20, no. 2, pp. 721 – 729, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2215098616306152

[17] A. Garces, D. Montoya, and R. Torres, "Optimal power flow in multiterminal hvdc systems considering dc/dc converters," in *2016 IEEE 25th International Symposium on Industrial Electronics (ISIE)*, June 2016, pp. 1212–1217.

[18] J. Li, F. Liu, Z. Wang, S. Low, and S. Mei, "Optimal Power Flow in Stand-alone DC Microgrids," *IEEE Transactions on Power Systems*, pp. 1–1, 2018.

[19] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[20] Y. Nesterov, *Lectures on Convex Optimization*, ser. Springer Optimization and Its Applications. Springer International Publishing, 2018. [Online]. Available: https://books.google.com.co/books?id=JSyNtQEACAAJ

[21] E. Benedito, D. del Puerto-Flores, A. Dria-Cerezo, and J. M. Scherpen, "Optimal Power Flow for resistive DC Networks: a Port-Hamiltonian approach," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 25 – 30, 2017, 20th IFAC World Congress. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2405896317300162

[22] C. Alexander and M. Sadiku, *Fundamentals of Electric Circuits*. McGraw-Hill Higher Education, 2006. [Online]. Available: https://books.google.com.co/books?id=dm6VPwAACAAJ

[23] N. Barabanov, R. Ortega, R. Gri, and B. Polyak, "On existence and stability of equilibria of linear time-invariant systems with constant power loads," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 1, pp. 114–121, Jan 2016.

[24] Z. Shuai, J. Fang, F. Ning, and Z. J. Shen, "Hierarchical structure and bus voltage control of dc microgrid," *Renewable and Sustainable Energy Reviews*, vol. 82, pp. 3670 – 3682, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1364032117314788

[25] C. DePersis, E. R. Weitenberg, and F. Drfler, "A power consensus algorithm for dc microgrids," *Automatica*, vol. 89, pp. 364 – 375, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109817306131

[26] Z. Liu, M. Su, Y. Sun, H. Han, X. Hou, and J. M. Guerrero, "Stability analysis of dc microgrids with constant power load under distributed control methods," *Automatica*, vol. 90, pp. 62 – 72, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0005109817306386

[27] Shivam and R. Dahiya, "Stability analysis of islanded DC microgrid for the proposed distributed control strategy with constant power loads," *Computers & Electrical Engineering*, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0045790617309606

[28] P. G. Khorasani, M. Joorabian, and S. G. Seifossadat, "Smart grid realization with introducing unified power quality conditioner integrated with dc microgrid," *Electric Power Systems Research*, vol. 151, pp. 68 – 85, 2017. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0378779617302122

[29] H. Wu, X. Liu, and M. Ding, "Dynamic economic dispatch of a microgrid: Mathematical models and solution algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 63, pp. 336 – 346, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0142061514003482

[30] C. Phurailatpam, B. S. Rajpurohit, and L. Wang, "Planning and optimization of autonomous DC microgrids for rural and urban applications in India," *Renewable and Sustainable Energy Reviews*, vol. 82, pp. 194 – 204, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1364032117312509

[31] A. A. Hamad and E. F. El-Saadany, "Multi-agent supervisory control for optimal economic dispatch in DC microgrids," *Sustainable Cities and Society*, vol. 27, pp. 129 – 136, 2016. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2210670716300294

[32] V. Bhattacharjee and I. Khan, "A non-linear convex cost model for economic dispatch in microgrids," *Applied Energy*, vol. 222, pp. 637 – 648, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0306261918305439

[33] O. D. Montoya, A. Grajales, L. F. Grisales, and C. A. Castro., "Optimal Location and Operation of Energy Storage Devices in Microgrids in Presence of Distributed Generation (in Spanish)," *Revista CINTEX*, vol. 22, no. 1, pp. 97–117, jun 2017.

[34] J. Yue, Z. Hu, C. Li, J. C. Vasquez, and J. M. Guerrero, "Economic Power Schedule and Transactive Energy through an Intelligent Centralized Energy Management System for a DC Residential Distribution System," *Energies*, vol. 10, no. 7, 2017. [Online]. Available: http://www.mdpi.com/1996-1073/10/7/916

[35] M. T. Bhoskar, M. O. K. Kulkarni, M. N. K. Kulkarni, M. S. L. Patekar, G. Kakandikar, and V. Nandedkar, "Genetic algorithm and its applications to mechanical engineering: A review," *Materials Today: Proceedings*, vol. 2, no. 4, pp. 2624 – 2630, 2015, 4th International Conference on Materials Processing and Characterzation. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S2214785315004642

[36] O. D. Montoya, "Numerical approximation of the maximum power consumption in dc-mgs with cpls via an sdp model," *IEEE Transactions on Circuits and Systems II: Express Briefs*, pp. 1–1, 2018.

[37] J. E. Machado, R. Griñó, N. Barabanov, R. Ortega, and B. Polyak, "On existence of equilibria of multi-port linear ac networks with constant-power loads," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 64, no. 10, pp. 2772–2782, Oct 2017.

[38] R. A. Gallego, A. Monticelli, and R. Romero, "Transmission system expansion planning by an extended genetic algorithm," *IEE Proceedings - Generation, Transmission and Distribution*, vol. 145, no. 3, pp. 329–335, May 1998.

[39] W. Sheng, K. Y. Liu, Y. Liu, X. Meng, and Y. Li, "Optimal placement and sizing of distributed generation via an improved nondominated sorting genetic algorithm ii," *IEEE Transactions on Power Delivery*, vol. 30, no. 2, pp. 569–578, April 2015.

[40] M. Esmaelian, M. Tavana, F. J. Santos-Arteaga, and M. Vali, "A novel genetic algorithm based method for solving continuous nonlinear optimization problems through subdividing and labeling," *Measurement*, vol. 115, pp. 27 – 38, 2018. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0263224117306061

[41] O. A. Arqub and Z. Abo-Hammour, "Numerical solution of systems of second-order boundary value problems using continuous genetic algorithm," *Information Sciences*, vol. 279, pp. 396 – 415, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0020025514004253

[42] M. Todorovski and D. Rajicic, "An initialization procedure in solving optimal power flow by genetic algorithm," *IEEE Transactions on Power Systems*, vol. 21, no. 2, pp. 480–487, May 2006.

[43] M. Todescato, "DC power flow feasibility: Positive vs. negative loads," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Dec 2017, pp. 3258–3263.