A Cuckoo Search Algorithm Applied to the Electric Grid Interdiction Problem

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Abstract: - The Electric Grid Interdiction Problem (EGIP) considers the interaction of a disruptive or malicious agent and the system operator. The disruptive agent pretends to maximize damage to the network; for this he must decide a set of lines to attack in order to maximize load shedding. The independent system operator reacts to such attack by redispatching available generation aiming to minimize load shedding. The interaction of both agents is modeled as a Stackelberg leader-follower game and framed in a bilevel programming structure. Due to its non-convexity, the EGIP has been traditionally approached by means of linearized equivalents of the network. In this paper we used a nonlinear modeling of the network and expressed the EGIP as a mixed integer non-linear programming (MINLP) problem providing more accurate results. The model is solved by means of a cuckoo search algorithm which performance is compared with a hybridized genetic algorithm and a traditional mixed integer linear programming (MILP) approach. The proposed algorithm provides valuable information to the system operator and the system planner regarding the most critical lines. Results show the applicability and robustness of the proposed approach.

Key-Words: - Power systems vulnerability, terrorist threat problem, cuckoo search algorithm.

1 Introduction

Traditional power systems vulnerability studies are carried out by commonly known N-1 or N-2 criterion [1]. Nevertheless, transmission systems are vulnerable not only to natural occurring outages but also to deliberate attacks [2]. In this context, the assessment of power system security must bear in mind plausible deliberate outages. The electric grid interdiction problem (EGIP) takes into account such consideration. This problem consists on identifying a set of circuits which simultaneous outage would result in maximum load shedding. The EGIP, also kwon as the “terrorist threat problem”, was initially proposed in [3] and latter generalized in [4]. This problem considers the interaction of two agents. On one hand a disruptive agent (DA), with limited destructive resources, pretends to cause maximum damage to the network expressed as load shedding. For this, the DA must select a set of lines to attack and render out of service. On the other hand, the system operator (SO) must react to the attack by redispatching available generation resources in order to minimize load shedding. The interaction of both agents is represented as a Stackelberg leader-follower game and modeled as a bilevel programming (BP) problem. The DA is positioned in the upper-level optimization problem (leader) and the SO in the lower-level optimization problem (follower). Then, for every decision (or action) in the upper level there is another decision (reaction) on the lower level. Solving a BP problem is a challenging task since they are intrinsically non-convex [5].

Different approaches have been considered to solve the EGIP. In [6], a BP model is presented to identify maximally disruptive attack plans for terrorists who are assumed to have limited offensive resources. A mixed integer linear programming (MILP) model as well as a heuristic technique is presented, being the last one more suitable for large power systems. In [7], two BP models are proposed to approach the EGIP. The first one is a maximum vulnerability model which consists on identifying the greatest level of load shedding that can be attained with a fixed number of circuits that are simultaneously under attack. The second one consists on identifying the minimum number of simultaneous attacks to achieve a previously set goal of load shedding. In [8], a worst-case interdiction analysis is performed using Benders decomposition. The model is used to identify a set of lines and transformers which destruction maximizes economic losses to customers. In [9] and [10], the EGIP is solved including line switching as an alternative strategy in the lower level optimization problem. This means that the SO has also the chance of modifying the topology of the network after an attack. In [11], an interdiction analysis is performed considering cascading outages and medium-term impacts. The model is solved by means of a heuristic technique. Solving the EGIP is relevant to the SO and system planner since knowing the most critical elements of a power system can be used to device corrective or protective actions. Such information can also be used to drive transmission expansion planning. For example; in [12], the authors propose a model for transmission network expansion planning that considers the EGIP. In this way, the reinforcement and expansion of the transmission network is devised in such a way that mitigates the impact of plausible deliberate outages. In this context, the network planner selects the new lines to be built considering not only economic issues, as usually done, but also the vulnerability of the network against a set of credible intentional outages. A similar approach is proposed in [13]. In this case, the authors solve the investment planning problem of electric power systems under terrorist threat. The adopted technique is a tabu search hybridized with a greedy algorithm.

In [14], a trilevel programming model for transmission network expansion planning considering the EGIP is proposed. In this case, the authors optimize the allocation of defensive resources in an electric power grid to mitigate vulnerability against multiple contingencies. The system planner is located in the upper optimization level to identify the components to be defended, while the DA and the system planner are located in the middle and lower optimization levels, respectively.

In this paper, we present a cuckoo search (CS) algorithm as a new technique to solve the EGIP. Despite the common use of the CS technique to solve other problems related to power systems such as optimal power flow [15], optimal allocation of capacitors [16] and optimal power dispatch [17] among others; to the best of our knowledge, the CS technique has not been applied to the EGIP before. Also, unlike traditional approaches, we consider a nonlinear modeling of the network which leads to more realistic results than the common DC modeling. Several tests were performed using a 5 bus power system and the IEEE 24 bus reliability test system. The performance of the proposed technique was compared against traditional approaches reported in the specialized literature showing the effectiveness of the proposed approach.

2 Problem Formulation

The mathematical formulation of the EGIP is given by equations (1)-(14). Further details on the formulation are explained below.

2.1 Upper level optimization problem

The DA is located in the upper level optimization problem. In this level, the objective function consists on maximizing the total load shedding given by (1). In this case, N is the set of nodes, P_{DS_n} is the load shedding of bus n, and δ_l^{Lin} is a binary vector, named hereafter as interdiction vector that indicates the state of lines. If a given entry of such vector is 1 it means that the corresponding line is on service; conversely, if it is 0, it means that such line is out of service. Equation (2) indicates the limit of destructive resources given by M and the binary nature of the interdiction vector. Equation (3) represents the reaction of the SO which is further explained in the next section.

$$\max_{\delta_l^{Lin}} \sum_N P_{DS_n}; \quad \forall n \in N \tag{1}$$

$$\sum_l \left(1-\delta_l^{Lin}\right) \le M; \quad \delta_l^{Lin} \in \{0,1\}; \;\; \forall l \in L \tag{2}$$

$$Reaction\ of\ the\ SO \tag{3}$$

2.2 Lower level optimization problem

The lower level optimization problem represents the reaction of the SO. Given a set of lines out of service represented by the interdiction vector δ_l^{Lin}, the SO reacts by redispatching available generation resources. Equation (4) is the objective function of the SO which consists on minimizing the cost of load shedding (c_{DS_n}) and generation dispatch (c_g). Minimum and maximum limits on variables are expressed with the superscripts *min* and *max*, respectively. *Gen* and *Lin* represent the sets of generators and lines, respectively. Equations (5) and (6) represent limits on angles (θ_n) and voltage magnitudes (V_n) in all nodes, respectively. Equations (7) and (8) indicate limits on active (P_g^{Gen}) and reactive (Q_g^{Gen}) power provided by generators, respectively. Equation (9) indicates limits on power flows, where S_l^{Lin} is the apparent power flow in line l. Note that power flow limits are multiplied by the interdiction vector, which indicates that only lines on service are taken into account in equation (9). Equations (10) and (11) represent limits on active and reactive load shedding, respectively. In this case, the active load shedding in bus n (P_{DS_n}), and its reactive part (Q_{DS_n}) must be less than the total active and reactive load of the node, expressed as

P_{D_n} and Q_{D_n}, respectively. Equations (12) and (13) are the power balance equations for active and reactive power, respectively. In this case P_n and Q_n are the active and reactive power injections in bus n, respectively. Finally, equation (14) indicates that the reference angle must be zero.

$$\min_{x} \sum_{g} c_g P_g^{Gen} + \sum_{n} c_{DS_n} P_{DS_n} \tag{4}$$

$$\theta_n^{min} \leq \theta_n \leq \theta_n^{max}; \quad \forall n \in N \tag{5}$$

$$V_n^{min} \leq V_n \leq V_n^{max}; \quad \forall n \in N \tag{6}$$

$$P_g^{min} \leq P_g^{Gen} \leq P_g^{max}; \quad \forall g \in Gen \tag{7}$$

$$Q_g^{min} \leq Q_g^{Gen} \leq Q_g^{max}; \quad \forall g \in Gen \tag{8}$$

$$\delta_l^{Lin} S_l^{min} \leq S_l^{Lin} \leq \delta_l^{Lin} S_l^{max}; \quad \forall l \in Lin \tag{9}$$

$$0 \leq P_{DS_n} \leq P_{D_n}; \quad \forall n \in N \tag{10}$$

$$0 \leq Q_{DS_n} \leq Q_{D_n}; \quad \forall n \in N \tag{11}$$

$$P_g^{Gen} - P_{D_n} + P_{DS_n} = P_n; \quad \forall n \in N \tag{12}$$

$$Q_g^{Gen} - Q_{D_n} + Q_{DS_n} = Q_n; \quad \forall n \in N \tag{13}$$

$$\theta_{ref} = 0 \tag{14}$$

A BP problem is basically an optimization problem with optimization constraints. Due to their nature BP problems are intrinsically non-convex and difficult to solve, especially when the lower level optimization problem is nonlinear (such as the one presented above). Metaheuristic techniques are better suited to approach these problems than classical optimization techniques [18]. The proposed metaheuristic to solve the BP problem given by (1)-(4) is a Cuckoo Search algorithm which is explained in the next section.

3 Methodology

The Cuckoo Search (CS) is a metaheuristic proposed by Xin-She Yang and Suash Deb in 2009 [19]. It is inspired in the behavior of some cuckoo species which engage in brood parasitism by laying their eggs in the nest of other host birds. If the last ones discover the eggs are not their own, these might be thrown away or abandoned. Then, some species of female parasitic cuckoos are able to mimic in color and pattern the eggs of chosen host species. Once the cuckoos are hatched they throw away host eggs, increasing their share of food. Recent studies show the CS as a promising metaheuristic over performing some classical metaheuristics such as Genetic Algorithms and Particle Swarm Optimization in specific applications [20]. The CS algorithm follows three rules [19]: 1) cuckoos lay eggs in randomly chosen nests; 2) the best nests (high quality solution candidates) will carry over the next generation; 3) The number of available host nests (population of candidate solutions) is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability $P_a \in [0, 1]$. Once the egg is discovered the host bird throws the egg away, or abandons the nest and moves to a new one.

In the CS algorithm the generation of a new solution $x^{(t+1)}$ is done performing a Lévy flight as indicated in equation (15) where α>0 is the step size (typically $\alpha = 1$).

$$x_i^{(t+1)} = x_i^{(t)} + \alpha \oplus L\acute{e}vy(\lambda) \tag{15}$$

Equation (15) is a random walk. That is, a Markov chain in which the next status only depends on the current location $x_i^{(t)}$ and the transition probability (second term of equation (15)). The product $\oplus$ stands for entry wise multiplications. The traditional CS technique considers continuous variables. However, we adopted the binary CS version proposed in [21]. Fig 1 illustrates the pseudocode of the CS algorithm. A more detailed description of CS algorithm and its implementation can be consulted in [19-21].

```
begin
Generate initial population of n host nests
  while (t < MaxGenerators)
     Use Lévy flight to generate a new nest (say, i)
      Evaluate fitness function $F_i$
     Choose a nest (say, j) randomly
       if ($F_i > F_j$)
         Replace nest j by the new nest i
      end
     A fraction of worse nest are discovered
     New nests are built
     Keep and rank best solutions
   end while
 end
```

Fig.1 Pseudocode of the CS algorithm

3.1 Problem codification

A solution of the EGIP is an attack plan that indicates which lines outages maximize load shedding. Since the DA has limited resources, it is supposed that he can only attack a fixed number of lines (represented by *M* in equation (2)). Fig 2 illustrates a 5 bus power system and a codification example of an attack plan with *M*=3 in which lines 1-3, 2-3 and 4-5 are affected. In this case, the nest represents the interdiction vector δ_l^{Lin} that indicates the operative state of the lines. Every entry of such vector is a cuckoo egg. The fitness function of a given nest (interdiction vector) is computed by running an optimal dispatch considering the remaining elements. Such dispatch is given by equations (4)-(14) and is computed using the software Matpower [22].

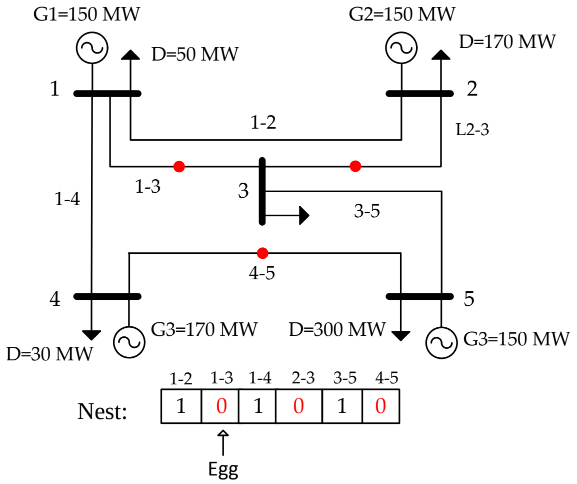

Fig.2 Codification example of the CS algorithm

4 Tests and Results

Several tests were carried out with three different power systems to show the effectiveness of the proposed approach.

4.1 Results with a 5 bus power system

The first tests were performed with the 5 bus power system illustrated in Fig 2. This system was chosen for didactic purposes and to compare results with [4] and [5]. Generation and demand data are provided in Fig 2 and line parameters are presented in Table 1.

Table 1. Line parameters of 5 bus power system

Line	X (p.u)	Max Power Flow (MW)
1-2	0.336	100
1-3	0.126	100
1-4	0.180	100
2-3	0.215	100
3-5	0.215	100
4-5	0.130	100

The CS algorithm was executed considering 100 nests and 1000 generations, α was set to 1 and the probability of discovering eggs was set to 10%.

Several tests were run increasing the DA destructive resources *M* which represent the number of lines that can be attacked at the same time. The results obtained considering up to 4 simultaneous outages are reported in Table 2. In this case L.S stands for load shedding. The results found with the C.S algorithm are the same as those reported in [4] and [5] in which an exact technique and a GA are used to solve the EGIP, respectively.

Note that for *M=1* the maximum load shedding obtained by the DA is 50MW. In this case, the single outage of either line 3-5 or 4-5 would result in such load shedding. This is because bus 5 has the greatest demand (300MW) and there is not enough local generation to supply such demand in case of a contingency of any of the lines connecting bus 5 with the rest of the system (see Fig. 2). For *M=2*, the best option of the DA is to attack lines 3-4 and 4-5 simultaneously which results in 150 MW of load shedding. Such result cannot be improved by attacking three lines. Finally, the maximum load shedding of 170MW takes place with *M=4*.

Table 1. Best solutions of the EGIP for the 5 bus power system

M	**Attacked lines**	**L.S (MW)**
1	3-5	50
	4-5	
2	3-5, 4-5	150
3	1-3, 3-5, 4-5	150
4	1-2, 2-3, 3-5, 4-5	170

4.2 Results with the IEEE 24 bus reliability test system

The IEEE 24 bus reliability test system comprises 38 branches, 11 generators and 17 loads. The data of this system can be consulted in [23]. All tests were performed using a winter day at 18:00 with a total demand of 2850MW. Minimum generation limits were considered to be 0 MW for all generators. Voltage magnitude limits were considered in the range [0.95, 1.05] in per unit.

Results for different destructive resources are presented in Table 2 and compared against values reported in [5] and [7]. In [5], the authors presented a hybridized GA to solve the EGIP while in [7] the authors used a classical optimization method. In both cases a linear modeling of the network is considered. Although the CS algorithm found the same solutions (attack plans), the results are slightly different in terms of load shedding. In general the load shedding found with the proposed algorithm is higher than that reported in [5] and [7]. The differences are attributable to the fact that we considered an AC modeling of the network. This leads to the conclusion that simplifications in the EGIP model result in conservative solutions. Differences might be greater in heavy loaded systems.

Fig. 3 illustrates the solutions (attack plans) found with the IEEE 24 bus reliability test system. Note that several lines appear in more than one solution. Such lines are identified as critical to the system in term of vulnerability. For example, transformers 10-12 and 9-12 and line 11-13 appear as common components of solutions for *M*=6 and *M*=8. With this information the system operator or system planner can make decisions regarding protective actions.

Table 2. Best solutions of the EGIP for the IEEE 24 bus reliability test system.

M	Attacked lines	LS Reported in [5] [7]	LS Cuckoo Search (MW)
4	3-24, 12-23, 13-23, 14-16	516	559.8
6	3-24, 7-8, 9-12, 10-12, 11-13, 14-16	1017	1022.9
8	9-12, 10-12, 11-13, 15-21, 15-21, 16-17, 20-23, 20-23	1198	1206.5

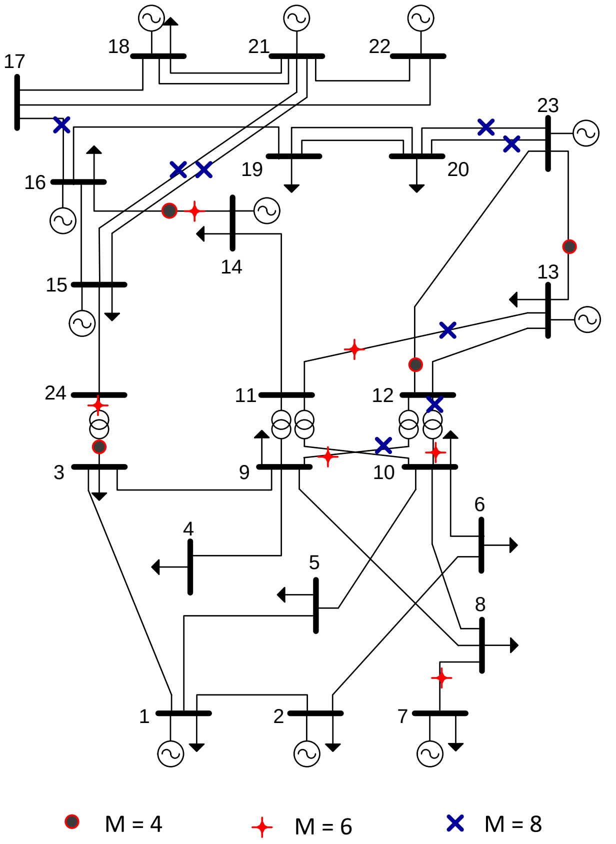

Fig. 3 Attack plans for the IEEE 24 bus reliability test system considering different values of *M*.

4.3 Results with the IEEE 30 bus power system

The IEEE 30 bus power system comprises 41 lines, 20 load buses, 6 generators with a total capacity of 335 MW and a total demand of 189.2 MW. Table 3 presents the results of the EGIP applied to this system considering 4, 6 and 8 simultaneous attacks. No results have been reported previously in this system regarding the EGIP. Thus, the results presented in this paper might serve for comparison purposes in future research. It can be observed that attack plans for *M*=4 and *M*=6 have several elements in common. These elements are identified as the most critical in terms of system vulnerability. Fig 4 illustrates the best attack plans identified by the CS algorithm.

Table 3. Best solutions of the EGIP for the IEEE 30 bus power system.

M	Attacked lines	LS Cuckoo Search (MW)
4	10-21, 10-22, 15-23, 28-27	45.9
6	6-9, 9-10, 12-14, 12-15, 12-16, 25-27	80.5
8	6-7, 6-9, 9-10, 12-14, 12-15, 12-16, 28-27, 27-29	93.5

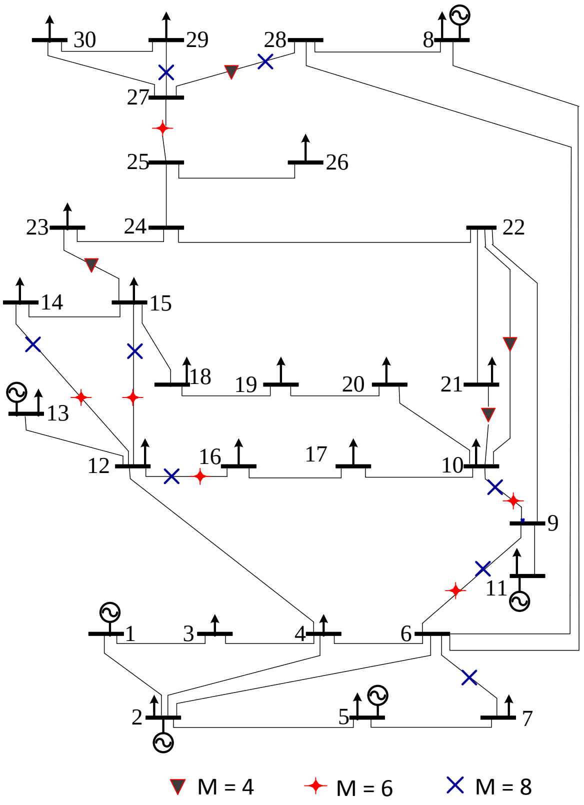

Fig. 4 Attack plans for the IEEE 30 bus test system considering different values of *M*.

5 Conclusion

A cuckoo search algorithm was presented to solve the EGIP. The tests carried out for three different power systems, as well as the comparison of results with other methodologies whenever it was possible, allows concluding that the proposed approach is robust and effective to tackle the EGIP.

The EGIP recreates an action-reaction game in which a malicious agent and the system operator are engaged. The solution of such game provides the most attractive interdiction plans from the point of view of a disruptive agent. Solving the EGIP gives the system operator (and system planer) valuable information about the most critical elements and provides signals for future reinforcements of the network or more strict surveillance.

The use of a metaheuristic such as the CS algorithm allows solving the EGIP with a more accurate modeling of the network providing more accurate results than those obtained with traditional optimization models.

Future work will explore the role of other protective actions such as line switching and demand response as strategies to mitigate the impacts of potential outages.

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References:

[1] J.M. López-Lezama, C.E. Murillo-Sanchez, L.J. Zuluaga and J.F. Gutierrez-Gomez, “A contingency-based security-constraint optimal power flow model for revealing the marginal cost of a blackout risk-equalizing policy in the Colombian electricity market,” in *Proc IEEE/PES Transmission & Distribution Conference and Exposition: Latin Amercia,* Caracas, 20016.

[2] P.H. Corredor, and M.E. Ruiz, “Against all odds: mitigating the impact of terrorist activity on Colombian´s power system,” *IEEE Power Energy Mag*. Vol.9, No. 2, 2011, pp. 59-66.

[3] J. Salmeron, K. Wood and R. Baldick, “Analysis of the electric grid security under terrorist threat,” *IEEE Transactions on Power Systems*, Vol.19, No. 2, 2004, pp. 905-912.

[4] J. Arroyo and F. Galiana, “On the solution of the bilevel programming formulation of the terrorist threat problem”. *IEEE Transactions on Power Systems.* Vol.20, No. 2, 2005, pp. 789-797.

[5] L. Agudelo, J.M. López-Lezama and N. Muñoz-Galeano, “Vulnerability assessment of power systems to intentional attacks using a specialized genetic algorithm,” *Revista DYNA Vol. 82, No. 192, 2015, pp. 78-84.*

[6] J. Salmeron, K. Wood and R. Baldick, “Analysis of electric grid security under terrorist threat” *Technical report NPSOR-04-001,* Naval Posgraduate School, Monterrey USA, 2004.

[7] J.M. Arroyo, “Bilevel programming applied to power system vulnerability analysis under multiple contingencies,” *IET Generation Transmission and Distribution, Vol. 4 No. 2, 2010, pp. 178-190.*

[8] J. Salmeron, K. Wood and R. Baldick, “Worst-case interdiction analysis of large-scale electric power grids,” *IEEE Transactions on Power Systems, Vol. 24, No. 1, 2009, pp. 96-104.*

[9] A. Delgadillo, J.M. Arroyo and N. Alguacil, “Analysis of electric grid interdiction with line switching,” *IEEE Transactions on Power Systems, Vol. 25, No. 2, 2010, pp. 633-641.*

[10] L. Zhao and B. Zeng, “Vulnerability analysis of power grids with line switching,” *IEEE Transactions on Power Systems, Vol. 28, No. 3, 2013, pp. 2727-2736.*

[11] Y. Wang, and R. Baldick, “Interdiction analysis of electric grids combining cascading outage and medium impacts,” *IEEE Transactions on Power Systems, Vol. 29, No. 5, 2014, pp. 2160-2168.*

[12] N. Alguacil, M. Carrión and J.M. Arroyo, “Transmission network expansion planning under deliberate outages,” *International Journal of Electrical Power and Energy Systems, Vol. 31, No. 9, 2009, pp 553-561.*

[13] R. Romero, X. Ningxiong, L.K. Nozick, I. Dobson and D. Jones, “Investment planning for electric power systems under terrorist threat,” *IEEE Transactions on Power Systems, Vol. 27, No. 1, 2012, pp. 108-116.*

[14] N. Alguacil, A. Delgadillo, Arroyo, J.M. “A trilevel programming approach for electric grid defence planning,” *Computers and Operations Research, Vol. 4 No. 1, 2014, pp. 282-290.*

[15] C. Mishar, S.P. Singh and J. Rokadia, “Optimal power flow in the presence of wind power using modified cuckoo search,” *IET Generation, Transmission and Distribution, Vol. 9 No. 7, 2015 pp. 615-626.*

[16] A.A. El-Fergany, and A. Abdelaziz, “Capacitor allocations in radial distribution networks using

cuckoo search algorithm," *IET Generation, Transmission and Distribution, Vol. 8 No. 2, 2014 pp. 223-232.*

[17] D.N. Vo, P. Schegner and W. Ongsakul, "Cuckoo search algorithm for non-convex economic dispatch," *IET Generation, Transmission and Distribution, Vol. 7 No. 6, 2013 pp. 645-654.*

[18] J.M. López-Lezama, L.F. Buitrago and F. Villada, "Location, sizing and optimal contracting price of distributed generation in distribution networks," *Información Tecnológica, Vol. 26, No. 6, 2015, pp. 109-120.*

[19] X.-S. Yang and S. Deb, "Cuckoo search via Lévy flights," in *Proc of World Congress on Nature & Biologically Inspired Computing,* India, 2009. IEEE Publications, USA, pp. 210-214.

[20] M.A. Elhameed and M.M. Elkholy, "Optimal power flow using cuckoo search considering voltage stability," *WSEAS Transactions on Power Systems, Vol. 11, 2016, pp. 18-26.*

[21] D. Rodrigues, L.A.M. Pereira, T.N.S. Almeida, J.P. Papa, A.N. Souza, C.C.O. Ramos and X.-S. Yang, "BCS: a binary cuckoo search algorithm for feature selection," in Proc of *IEEE International Symposium on Circuits and System, 2013.*

[22] R.D Zimmerman, C.E. Murillo-Sanchez, R.J. Thomas, "Matpower steady-state operations planning and analysis tools for power system research and education," *IEEE Transactions on Power Systems, Vol. 26, No. 1, 2011, pp. 12-19.*

[23] C. Grigg, P. Wong, P. Albrecht, R. Allan, M. Bhavaraju, R. Billinton, Q. Chen, C. Fong, S. Haddad, S. Kuruganty W. Li, R. Mukerji, D. Patton, N. Rau, D. Reppen, A. Schneider, M. Shahidehpour, C. Singh, "The IEEE reliability test system-1996. A report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Transactions on Power Systems, Vol. 14, No. 3, 1999, pp. 1010-1020.*