The Closest Stability Margin by Analyzing Full-dimensional Saddle-node Bifurcation Point in Power System

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Abstract: - The network equations of electric power system are formed in which the node voltages and branch currents are as s tate variables by π equivalent circuit simulation of power components. The explicit expressions of power system voltage equilibrium solution curves are obtained by solving the equations above, and then get the characteristic equation of the saddle-node bifurcation point, define the full-dimensional saddle-node bifurcation point. The full-dimensional saddle-node bifurcation point is the closest stability margin of the system after comparing the one-dimensional and full-dimensional saddle-node bifurcation point. In order to solve the system stability margin the dimensionality reduction algorithm of saddle-node bifurcation point is proposed. IEEE-14 nodes system simulation shows that the concepts and methods presented in this paper are correct.

Key-Words: - Full-dimensional, branch current, saddle-node bifurcation point, power flow equation, equilibrium solution curve, Stability Margin

1 Introduction

Now, the Bifurcation theory which studies the nonlinear problem has gained greater development and application in the study of voltage stability in [1-6]. A typical case that the power system loses its voltage stability is that the stable equilibrium point and the unstable equilibrium point overlap as the parameters change. The saddle-node bifurcation point (SNBP) appears because the Jacobian matrix of power network equations is singular.

Many methods are available to determine the SNBP. One of them is PU curve. The solutions of load voltages are often presented as a PU curve. But the PU curve can not be obtained near the SNBP because the Jacobian matrix tends to be singular, and the conventional power flow algorithm is failure. Therefore, the calculation of the SNBP often combines with morbid flow algorithm. Continuous power flow methods [7-12] track the trend of balanced solution by forecasting or correcting the power flow equations to improve the pathological phenomena and convergence. This method fails to give the accurate result if the step length is more. Though Interior Point method is efficient to solve the maximum loading problem[13-14], this method has the limitation of starting and terminating conditions [15]. The Sequential Quadratic Programming algorithm includes the differentiation of the constraints, and convert critical point conditions to optimized load, and solve it with Kuhn - Tucker optimality conditions[16]. This method is very slow as it involves many matrices during the iteration process. Fuzzy logic has been used to find the loadability limit in [17], this algorithm does not give the global optimal solution. Evolutionary algorithms have been applied to solve this problem. Particle swarm optimization is a computational intelligence-based technique that is not affected largely by the size and nonlinearity of the problem and can converge to the optimal solution in many problems [18-20].

As noted above, the current study of SNBP mainly focuses on the changes of single parameter (or two-dimensional parameter [21-22]) and the directions of node injection power are fixed. However, when the power injection changes in different directions the SNBP is likely to reach, which means that the SNBPs are different from each other. If the number and the location of zero eigenvalue appear different the SNBP should be different. The mechanism of SNBP has not been deeply identified.

The equilibrium solution curve intuitively reflects the bifurcation mechanism. It is crucial for localized states of bifurcation point as well as the global state if the exact equilibrium solution curve expression can be gotten. But, the explicit expression is often difficult to obtain due to the complexity of the non-linear problem, so commonly take the route of numerical calculation or simulation studies. Dimension is the basic properties of the bifurcation point. When the bifurcation occurs, if there are two smooth solution curve through the bifurcation point is called a simple bifurcation point, also known as one-dimensional saddle-node bifurcation point (ODSNBP), if there are more than two smooth solution curve through the bifurcation point is multi-dimensional saddle-node bifurcation point (MDSNBP). Compared to the one-dimensional, multi-dimensional bifurcation point needs more stringent generation conditions, but also contains deeper meanings, especially when all nodes are SNBPs (also called full-dimensional saddle-node bifurcation point, FDSNBP), MDSNBP is also more practical significance.

Saddle-node bifurcation problems in power system are also facing the same difficulties. Traditional power system analysis is based on node voltage equation, and the node voltage and power injection are as variables. This method is widely used because of its simple, practical, and intuitive physical meaning. But because of the interconnectedness among the nodes voltage the explicit expression of equilibrium solution curve is difficult to obtain, and numerical calculation or simulation can also not comprehensively and profoundly show the characteristics of the SNBP. Because of the problems as d escribed above the number of dimension, multi-dimensional, as well as FDSNBP are all not to carry out in-depth study, and can not further reveal the nature of the SNBP.

In this context, we proceed from the state variables of power network equation represented by the branch-current and node-voltage, form the explicit expression of equilibrium solution curves, and then describe the characteristics of SNBP. On this basis, define the one-dimensional, multi-dimensional and full-dimensional bifurcation point, analyze the feature of SNBP, and propose dimensionality reduction algorithm of SNBP.

2 The Explicit Expression of Equilibrium Solution Curve in Power System

The line (or transformer) of electric power system can be simulated by the π equivalent circuit model, as shown in Figure 1, called loop. Per loop is composed of three branches, an impedance branch and two grounded branch.

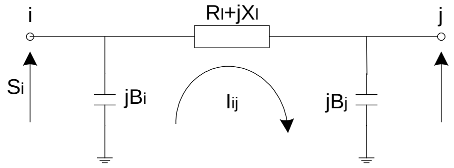

Fig.1 The π Equivalent Circuit

Wherein: i j are two nodes on both sides of branch l, $\dot{s}_i = p_i + jq_i$、 $\dot{s}_j = p_j + jq_j$ are node power injections of i and j, node voltages are as follows: $\dot{u}_i = e_i + jf_i$, $\dot{u}_j = e_j + jf_j$, branch l current is $\dot{i}_l = i_l^a + ji_l^r$, $R_l + jX_l$ is the impedance of branch l , jB_i and jB_j are the grounded susceptance of node i and j . The grounded conductivity is ignored for simple calculation in this paper. Figure 2 shows the grounded branch, node i is as an example.

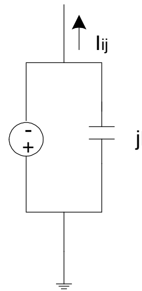

Fig. 2 Diagram of Grounded Branch

For a grounded branch, the current trend is in two ways including grounded capacitor branch and load branch. The current flows in the load branch are not only the current of the circuit itself but also the adjacent loop current. The analytical method of load branch is the same principle with node voltage. Node i , for example, the voltage of equivalent voltage source of the load branch is:

$$\overset{*}{u}_i = \frac{p_i - jq_i}{\sum_{l\in i} i_{li} - ju_i \sum_{l\in i} B_l} \tag{1}$$

Wherein: $l = 1,2,\cdots,L$ is branch collection, $i, j = 1,2,\cdots,N$ is node collection, $p_i - jq_i$ is the load of node i, $\sum_{l\in i} i_{li}$ is the sum of the injection current of node i, $\sum_{l\in i} B_l$ is the sum of the grounded susceptance of node i, $ju_i \sum_{l\in i} B_l$ is the sum of the susceptance current of node i, $\sum_{l\in i} i_{li} - ju_i \sum_{l\in i} B_l$ represents the load branch current of node i.

Derive from equation (1):

$$\begin{aligned} & e_i \sum_{l\in i} i_l^a + f_i \sum_{l\in i} i_l^r = p_i \\ & e_i \sum_{l\in i} i_l^r - f_i \sum_{l\in i} i_l^a + (e_i^2 + f_i^2) \sum_{l\in i} B_l = -q_i \end{aligned} \tag{2}$$

At the same time in the Cartesian coordinate system, the electric power network can be described as a mixed equation of branch currents and node voltage:

$$\dot{i}_l (R_{ij} + jX_{ij}) = \dot{u}_i - \dot{u}_j \tag{3}$$

Derived from equation (3):

$$\begin{aligned} & i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ & i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{aligned} \tag{4}$$

Node j has the equation like equation (4). Derived from equation (4):

$$\begin{aligned} & i_l^a = \frac{-X_{ij}(f_i - f_j) - (e_i - e_j)R_{ij}}{R_{ij}^2 + X_{ij}^2} = \\ & -G_{ij}(e_i - e_j) + B_{ij}(f_i - f_j) \\ & i_l^r = \frac{X_{ij}(e_i - e_j) - R_{ij}(f_i - f_j)}{R_{ij}^2 + X_{ij}^2} = \\ & -G_{ij}(f_i - f_j) - B_{ij}(e_i - e_j) \end{aligned} \tag{5}$$

Wherein: G_{ij}, B_{ij} is the admittance of branch l. Bring them into equation (2) can obtain the traditional node voltage equation:

$$\begin{cases} p_i - e_i \sum_{j\in i} (G_{ij} e_j - B_{ij} f_j) - \\ f_i \sum_{j\in i} (G_{ij} f_j + B_{ij} e_j) = 0 \\ q_i - f_i \sum_{j\in i} (G_{ij} e_j - B_{ij} f_j) + \\ e_i \sum_{j\in i} (G_{ij} f_j + B_{ij} e_j) + (e_i^2 + f_i^2) B_{i0} = 0 \end{cases} \tag{6}$$

The difference is that the grounded branch susceptance do not contain in the node self-susceptance B_{ii}. That is the nature of the equation has not been changed after the branch current introduced into the power network equation as state variables.

Suppose $x_i = \sum_{l\in i} i_{li}^a$, $y_i = \sum_{l\in i} i_{li}^r$ denote node injection current real part and imaginary part (excluding the grounded branch currents), and $B_{i0} = \sum_{l\in i} B_l$. Derived from equation (2):

$$\begin{aligned} e_i &= \frac{\left[2B_{i0} p_i x_i - y_i (x_i^2 + y_i^2)\right] \mp y_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2}}{2B_{i0}(x_i^2 + y_i^2)} \\ f_i &= \frac{\left[2B_{i0} p_i y_i + x_i (x_i^2 + y_i^2)\right] \pm x_i \sqrt{(x_i^2 + y_i^2)^2 - 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2}}{2B_{i0}(x_i^2 + y_i^2)} \end{aligned} \tag{7}$$

Obtain the explicit expressions of node voltage in which the branch current is as parameter. From equation (7):

$$(x_i^2 + y_i^2)^2 - 4B_{i0} q_i (x_i^2 + y_i^2) - 4B_{i0}^2 p_i^2 \geq 0 \tag{8}$$

Derived:

$$\left[(x_i^2 + y_i^2) - 2B_{i0} q_i\right]^2 \geq 4{B_{i0}}^2 {q_i}^2 + 4{B_{i0}}^2 {p_i}^2 \tag{9}$$

Only if meet '$\geq$' in the equation (9) the power network equation has solutions exist. The physical meaning of equation (9) is that the power network equation has solutions exist if the amplitude square of the node injection current distributes out of the circle of which the $2B_{i0} q_i$ is center and $2B_{i0}\sqrt{q_i^2 + p_i^2}$ is radius. If it distributes on t his circle (only '=' meet) two solutions coincide, the system is in the critical state of the SNBP. The conditions of solution exist has been found.

In the equation (7) '$\pm(\mp)$' symbol indicates that the electricity network equations exist two solution curves in per node, one is high voltage solution, the other is low.

When $B_{i0} = 0$ the node is called degraded node. If there is a degraded node the equation (2) becomes the following form:

$$\begin{cases} e_i \sum_{l\in i} i_{li}^a + f_i \sum_{l\in i} i_{li}^r = p_i \\ e_i \sum_{l\in i} i_{li}^r - f_i \sum_{l\in i} i_{li}^a = -q_i \end{cases} \tag{10}$$

And:

$$\begin{cases} e_i = \dfrac{p_i x_i - q_i y_i}{x_i^2 + y_i^2} \\ f_i = \dfrac{p_i y_i + q_i x_i}{x_i^2 + y_i^2} \end{cases} \tag{11}$$

In this case, the node voltage does not have two solutions, but only a single solution, so there is no issue of saddle-node bifurcation.

In addition, the generator node is typically a PU node, in the equation (2) reactive equation is replaced by the following:

$$e_i^2 + f_i^2 = U_i^2 \tag{12}$$

Wherein: U_i is node voltage amplitude. The node voltage analytical expression of PU is:

$$\begin{cases} e_i = \dfrac{p_i x_i \mp y_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \\ f_i = \dfrac{p_i y_i \pm x_i \sqrt{(x_i^2 + y_i^2)V_i^2 - p_i^2}}{x_i^2 + y_i^2} \end{cases} \tag{13}$$

PU node solution existing conditions:

$$(x_i^2 + y_i^2)U_i^2 - p_i^2 \geq 0 \tag{14}$$

Its physical meaning is the electricity network solutions exist if the amplitude square of the node injection current distributes out of the circle of which the 0 is center and p_i / U_i is radius.

3 The SNBP Characteristic Equation and the Definition of FDSNBP

If the equality of equation (9) or (14) is met the two solutions curves intersect, that is, a saddle-node bifurcation. Suppose the quantity of PQ node is N_L, PU node number is N_G, balance node number is N_S, then $N_L + N_G = N - N_S$, N is the sum of node. SNBP generating condition can be derived as:

$$(x_i^2 + y_i^2) = 2B_{i0}\gamma_i \quad i \in N_L \tag{15}$$

Or:

$$(x_i^2 + y_i^2) = \frac{p_i^2}{U_i^2} \quad i \in N_G \tag{16}$$

Suppose: $\gamma_i = q_i + \sqrt{p_i^2 + q_i^2}$. The node voltage changes to:

$$\begin{cases} e_i = \dfrac{p_i x_i - y_i \gamma_i}{2B_{i0}\gamma_i} \\ f_i = \dfrac{p_i y_i + x_i \gamma_i}{2B_{i0}\gamma_i} \end{cases} \quad i \in N_L \tag{17}$$

Or:

$$\begin{cases} e_i = U_i x_i \\ f_i = U_i y_i \end{cases} \quad i \in N_G \tag{18}$$

The equations (15) and (16) are called node characteristic equations of the SNBP. So can deduce that the establishment of equation (15) or (16) on any node will cause saddle-node bifurcation, that is the occurrence of SNBP is corresponding to the critical conditions of the power network equation solution existing.

If the characteristic equation (15) or (16) of saddle-node bifurcation only meets on one node, and only one pair solution curve intersects, the SNBP is called ODSNBP, it is called the MDSNBP if the characteristic equation (15) or (16) meets on multiple nodes, and FDSNBP if meets on all nodes. The dimension of SNBP equals to the number of node on which the equation (15) and (16) hold.

4 The SNBP Dimensionality Reduction Solution

4.1 Dimensionality reduction algorithm for ODSNBP

The power flow calculation based on Newton's method can not converge just because the Jacobian matrix is singular at SNBP. From the above analysis, the main reason that the eigenvalue of Jacobian matrix is zero is node voltage characteristic equation (15) or (16) holds. So the node voltage characteristic equation (15) or (16) can be used to replace the corresponding node voltage equation in (19) if calculate the ODSNBP, but the dimension of Jacobian matrix reduces one. The meaning is that the power network equations can be solved by the dimensionality reduction Jacobian matrix

$$\begin{cases} e_k \sum\limits_{l \in k} i_l^a + f_k \sum\limits_{l \in k} i_k^r = p_k \\ e_k \sum\limits_{l \in k} i_l^r - f_k \sum\limits_{l \in k} i_l^a + (e_k^2 + f_k^2) B_{k0} = -q_k \\ or \quad e_k^2 + f_k^2 = V_k^2 \end{cases}$$

$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases} \tag{19}$$

Wherein: $k \in (N_G + N_L)$ and $k \neq m$, m is the node at which characteristic equation (15) or (16) holds. To node m:

$$\begin{cases} e_m = \dfrac{p_m \sum_{l\in m} i_l^a - \gamma_m \sum_{l\in m} i_l^r}{2B_{m0}\gamma_m} \\ f_m = \dfrac{p_m \sum_{l\in m} i_l^r + \gamma_m \sum_{l\in m} i_l^a}{2B_{m0}\gamma_m} \\ (\sum_{l\in m} i_l^a)^2 + (\sum_{l\in m} i_l^r)^2 = 2B_{m0}\gamma_m \end{cases} \tag{20}$$

$$or \quad \begin{cases} e_m = V_m \sum_{l\in m} i_l^a \\ f_m = V_m \sum_{l\in m} i_l^r \\ (\sum_{l\in m} i_l^a)^2 + (\sum_{l\in m} i_l^r)^2 = \dfrac{p_m^2}{V_m^2} \end{cases}$$

Wherein: the equation (19) is the power network equation, (20) is the characteristic equation of SNBP. The voltage of ODSNBP can obtain if solve the two equations simultaneously. In the calculating process substitute equation (20) into the branch current formula of equation (19).

4.2 Dimensionality reduction algorithm for FDSNBP

Similarly, in all nodes, if the characteristic equation (15) and (16) are founded due to node injection power S_i, P_i, the power network equation has unique solution, it is the FDSNBP. The following shows:

$$\begin{cases} i_l^a R_{ij} - i_l^r X_{ij} - e_i + e_j = 0 \\ i_l^a X_{ij} + i_l^r R_{ij} - f_i + f_j = 0 \end{cases} \quad l \in L \tag{21}$$

$$\begin{cases} e_i = \dfrac{p_i \sum_{l\in i} i_l^a - \gamma_i \sum_{l\in i} i_l^r}{2B_{i0}\gamma_i} \\ f_i = \dfrac{p_i \sum_{l\in i} i_l^r + \gamma_i \sum_{l\in i} i_l^a}{2B_{i0}\gamma_i} \end{cases} \quad i \in N_L \tag{22}$$

$$\begin{cases} e_i = U_i \sum_{l\in i} i_l^a \\ f_i = U_i \sum_{l\in i} i_l^r \end{cases} \quad i \in N_G \tag{23}$$

While substitute (22) (23) into (21) there is only one solution because the formative equation is linear. But the calculation of FDSNBP needs the parameter conditions S_i, P_i , the following characteristic equations should be considered:

$$\begin{cases} (\sum_{l\in i} i_l^a)^2 + (\sum_{l\in i} i_l^r)^2 = 2B_{i0}\gamma_i & i \in N_L \\ (\sum_{l\in i} i_l^a)^2 + (\sum_{l\in i} i_l^r)^2 = \dfrac{p_i^2}{U_i^2} & i \in N_G \end{cases} \tag{24}$$

The FDSNBP can be calculated while solve equations (24), (21), (22) and (23) simultaneously. In the calculating process replace the voltages of all nodes in (21) with the voltage expression which meet the FDSNBP in (22) or (23).

5 The Closest Power System Static Voltage Stability Margin

SNBP represents the power system static voltage stability margin. From ODSNBP, MDSNBP to FDSNBP, different dimensional SNBPs represent static voltage stability margin are also different. ONSNBP is the power system static voltage stability margin of single node, MDSNBP is the margin of m nodes (area). Because ODSNBP and MDSNBP achieve the stability critical conditions on just part of nodes, so the SNBPs can not represent the static voltage stability margin of the whole system, and only FDSNBP represents the whole system static voltage stability margin.

Assume that the PQ node i, for example, in the same generator power increasing scheme, the node power injection are $S_i^{(1)}$ and $S_i^{(N_L)}$ while calculating the ODSNBP and FDSNBP respectively. It must be:

$$S_i^{(N_L)} \le S_i^{(1)} \tag{25}$$

The PU node is also similarly. In conclusion: if regard the space distance of node injection power as a measure, the static voltage stability margin of FDSNBP is the most adjacent in the whole system, the ODSNBP is the farthest, and MDSNBP is between them.

Get γ_i, p_i according to the calculation above, then:

$$S_i = \frac{\gamma_i}{\sin\phi_i + 1} \quad or \quad P_i = p_i \tag{26}$$

Wherein: $S_i = \sqrt{p_i^2 + q_i^2}$, $\sin\phi_i = q_i / S_i$, ϕ_i is power factor Angle, then S_i or P_i represents the establishing conditions of SNBP, and also is the static voltage stability margin of node i.

6 Case Study

In this paper, take IEEE-14 node system for an example to carry on the calculation. The number of node 1 and node 14 exchanges, node 14 is balance node, power factor is 0.9. Calculate the SNBPs in two cases: 1) ODSNBP calculation (calculate node each to each); 2) FDSNBP calculation (assume all nodes achieve SNBP at the same time). In the process of computation, suppose that the reactive power adjustment ability of PU nodes is limited, and all of them reach the limit value before arriving at the SNBP. Table 1 shows the calculation results by use of the method proposed.

Define:

P_l : Load active power;

Q_l : Load reactive power;

U_s : Voltage amplitude initial value;

U_g : ODSNBP voltage amplitude calculated value. That is when a node is a SNBP, calculate the node voltage amplitude;

ΔU_g : ODSNBP voltage amplitude variation rate. It is the rate of U_g divided by U_s ;

U_a : FDSNBP voltage amplitude calculated value. That is when a node is a FDSNBP, calculate the node voltage amplitude;

θ_a : FDSNBP voltage phase Angle calculated value. That is when a node is a FDSNBP, calculate the node voltage phase Angle;

ΔU_a : FDSNBP voltage amplitude variation rate. It is the rate of U_a divided by U_s ;

P_s : Node load active power initial value;

P_g : ODSNBP node load active power calculation value. When a node is SNBP, calculate the node load active power;

ΔP_g : ODSNBP node load active power critical closer degree. It is the degree of P_s divided by P_g , it can represent the stability redundance;

P_a : FDSNBP node load active power calculation value. When a node is FDSNBP, calculate the node load active power;

ΔP_a : FDSNBP node load active power critical closer degree. It is the degree of P_s divided by P_a ;

Table 1 Calculation results of FDSNBP

No.	U_a	θ_a	P_l	Q_l
1	0.7072	-34.464	0.2373	0.1103
2	0.8200	-17.836	0.5834	0.2718
3	0.7703	-21.196	1.1104	0.5174
4	0.7881	-17.607	0.6093	0.2835
5	0.8191	-14.647	0.2212	0.1031
6	0.7921	-28.920	0.1884	0.0877
7	0.7762	-28.389	0.1254	0.0582
8	0.8014	-30.378	0.1211	0.0571
9	0.8244	-31.748	0.4012	0.1864
10	0.7551	-32.034	0.1184	0.0551
11	0.7664	-30.835	0.0511	0.0237
12	0.7623	-30.315	0.0887	0.0411
13	0.7511	-31.074	0.1574	0.0732

Table 2 The comparison of voltage amplitude variation rate between ODSNBP and FDSNBP

No.	U_s	U_g	ΔU_g (%)	U_a	ΔU_a (%)
1	1.03	0.6403	37.83	0.7072	31.34
2	1.04	0.7133	31.41	0.8200	21.15
3	1.01	0.6891	31.77	0.7703	23.73
4	1.01	0.7374	26.99	0.7881	21.97
5	1.02	0.7222	29.20	0.8191	19.70
6	1.07	0.6931	35.22	0.7921	25.97
7	1.06	0.6833	35.54	0.7762	26.77
8	1.05	0.6652	36.64	0.8014	23.68
9	1.05	0.7627	27.36	0.8244	21.48
10	1.05	0.7187	31.55	0.7551	28.09
11	1.05	0.6532	37.79	0.7664	27.01
12	1.05	0.6651	36.66	0.7623	27.40
13	1.05	0.6881	34.47	0.7511	28.47

Table 2 s hows the comparison of voltage amplitude variation rate between ODSNBP and FDSNBP. Voltage amplitude of ODSNBP is lower than that of FDSNBP from table 2. This is because when one node arrives at ODSNBP the other nodes are still in a normal load level, the system can be maintained at a lower voltage level and does not collapse. But when the FDSNBP happens all nodes are in high load level, the system voltage collapses before ODSNBP. In all nodes, the voltage reduction of nodes 1 is the greatest, especially in FDSNBP, this condition is the same with the fact that node 1 is far away from the power supply in IEEE-14 system.

Table 3 The comparison of node load active power critical closer degree between ODSNBP and FDSNBP

No.	P_s	P_g	ΔP_g (%)	P_a	ΔP_a (%)
1	0.1490	1.1881	87.46	0.2373	37.21
2	0.2170	4.1172	94.73	0.5834	62.80
3	0.9240	1.9021	51.42	1.1104	16.79
4	0.4780	1.9984	76.08	0.6093	21.55
5	0.0760	2.1133	96.40	0.2212	65.64
6	0.1120	1.0427	89.26	0.1884	40.55
7	0	1.0626	100	0.1254	100
8	0	0.7732	100	0.1211	100
9	0.2950	1.1850	75.11	0.4012	26.47
10	0.0900	0.8425	89.32	0.1184	23.99
11	0.0350	0.7553	95.37	0.0511	31.51
12	0.0610	0.6552	90.69	0.0887	31.23
13	0.1350	0.7877	82.86	0.1574	14.23

In table 3 ΔP_g , ΔP_a represent the distance between the node load active power initial value and ODSNBP, FDSNBP respectively, illustrate the stability margin size of each node. The initial load active power of node 7 and 8 are zero, so the load power margin is 100%.

From the table 3, the power critical closer degree of ODSNBP is much larger than that of FDSNBP, also proves that FDSNBP is the closest stability margin. This is mainly because the load of ODSNBP has larger growth, other nodes load are constant, but all nodes load are increasing at FDSNBP, so each node load stability margin becomes low.

It is shown that the voltage amplitude drop and load stability margin is not directly proportional by contrasting them. The voltage amplitude drop of node 1 is the greatest, but its load stability margin is not the least. This is because the load stability margin major decision factor is the difference value between current node load level and the SNBP power. The higher the node current load level is, the lower the difference value is, and its stability margin is smaller. It also explains the reason from another aspect that the heavy load nodes have the possibility of voltage collapse.

The FDSNBP can observe the stability margin of each node from the global point of view, and can analyze the stability of all nodes comparatively in the same level. The ODSNBP can represent the local situation, but sometimes can not truly reflect the whole stability of the system. For example, the stability margin of node 13 is the smallest in FDSNBP, but not in ODSNBP. If adjust the node power to improve the system stability, should be based on the calculation results of FDSNBP. In this system, for example, should first adjust the node like 3 and 13.

7 Conclusion

In this paper, power system network equations have been established by introducing branch current as a variable based on the traditional node voltage equations. The existing conditions of solution have been found by the analysis of voltage high and low solution curve, and propose the characteristic equations of SNBP. Define the ODSNBP, MDSNBP and FDSNBP, and prove FDSNBP is the closest stable margin to the system. The conclusions of simulation are:

1) The method proposed can be applied to calculate SNBP and analyze static voltage stability;

2) Comparison of ODSNBP, MDSNBP and FDSNBP, the FDSNBP calculating results reflect the system stability information more rich, and can embody more approaching to the reality;

3) The calculation results of FDSNBP can obtain system closest stability margin, observe system stability overall, and provide the basis for the stable adjustment.

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