A Nonlinear Thermal Process Modeling and Identification using a fourth-order S-PARAFAC Volterra Model

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Abstract: - This paper proposes a new reduced complexity Volterra model called S-PARAFAC-Volterra. The proposed model is yielded by using the symmetry property of the Volterra kernels and their tensor decomposition using the PARAFAC technique. It takes advantage from previous results where an algorithm for the estimation of the memory and the order of the Volterra model has been presented. The proposed model has been tested to yield a suitable modeling for the nonlinear thermal process Trainer PT326 and the validation results are satisfactory.

Key-Words: - PT326 Trainer, Nonlinear systems, Volterra model, PARAFAC.

1 Introduction

The identification of nonlinear dynamical systems from a given input/output data set is of fundamental importance for practical applications since many physical systems exhibit nonlinear characteristics. The Volterra model can be used to represent a broad class of nonlinearities. The use of Volterra models for the analysis of nonlinear systems was, for the first time, conducted by N. Wiener [1] in order to characterize the response of a n onlinear system. Several other studies in this research (Billings [2] and Schetzen [5]) have followed. The study of discrete nonlinear systems using Volterra series began with the work of Alper [6].

The main problem with the Volterra models is the determination of its kernels which provide an adequate representation of the system to be modeled. A very nice property of the Volterra model is its linearity with respect to its parameters, so standard parameter estimation techniques like Least Squares (LS) can be applied. However, the large number of parameters associated with the Volterra models limits their practical use to problems involving only small values for the memory and the truncation order. This limitation arises since the estimation of a large number of parameters is problematic, but also design procedures based upon such models may be cumbersome.

To eliminate this drawback, two ways were followed: By expanding the kernels on a n orthonormal basis such as the Laguerre or Kautz functions basis ([7-10]) or Generalized Orthonormal Bases (GOB) ([11-13]) or by considering Volterra kernels as high order tensors and proceeding to tensor decomposition (PARAFAC, TUCKER, PARATUCK...) to arrange the kernels coefficients in decomposition matrices [14-16].

The purpose of this paper is to propose a n ew reduced complexity Volterra model to represent a thermal process: Trainer PT326. Our motivation is emphasized by the fact that the complexity of a Volterra model can be further reduced and especially with estimated structure parameters.

The proposed model is based on t he PARAFAC tensor decomposition of the Volterra kernels which pride with a symmetry property. In fact, each k-order kernel is represented by k matrices which can be equal using the symmetry property [17]. The scalar representation of the kernel decomposition was introduced to write the input/output relation of the reduced complexity Volterra model [15-16-17]. The new Model is then defined and called S-PARAFAC Volterra model. The Trainer PT326 was represented in [18] using a classical Volterra model and its structural parameters (order and memory) were estimated using the non-singularity property of correlation matrices between the inputs and the outputs of the nonlinear process. The Trainer PT326 structural parameters estimated in latter work were used to construct the S-PARAFAC Volterra model.

This paper is organized as follows. In section 2, we present the new 4th order reduced complexity Volterra model called S-PARAFAC Volterra model. In section 3, we present the thermal process: Trainer PT 326 and we represent it using a 4th order S-PARAFAC Volterra model highlighting the complexity reduction achieved. Finally the fourth section is devoted to practical simulations where the memory and the order are set and the coefficients of the S-PARAFAC Volterra model are estimated using input/output observations collected from the Process Trainer PT 326. The resulting model behavior is compared to that of the PT 326 for other input/output measurements.

2 New Reduced Complexity PARAFAC Volterra Model

2.1 Volterra model

In practice, the input/output relation of a discrete-time, time invariant, truncated Volterra model is given by:

$$\hat{y}(n)=\sum_{m=1}^{L}\sum_{n_1=1}^{M_1}\cdots\sum_{n_m=1}^{M_m}h_m(n_1,\ldots,n_m)\prod_{j=1}^{m}u(n-n_j) \quad (1)$$

where u, y and h_m are the input signal, the output signal and the parameters of the mth-order kernel, respectively, $\{M_m\}_{m=1}^{L}$ are the memories of the Volterra kernels and L is the model order. This model constitutes a nonlinear generalization of the impulse response.

2.2 PARAFAC Decomposition

The PARAFAC (PARAllel FACtor) decomposition also called CANDECOMP (CANonical DECOMPosition) was introduced by Harshman [19] and by Caroll and Chang [20] in order to reduce the complexity of an Nth order tensor. This decomposition entirely preserves the information contained in the original tensor.

The PARAFAC decomposition of a kth-order $(N_1,...,N_K)$ tensor H_k is written using k matrices of respective dimensions (N_i,R_k). Its scalar representation is written as:

$$h\,(n_1,n_2,...,n_k)=\sum_{r=1}^{R_K}v_{n_1,r}^{(1)}\,v_{n_2,r}^{(2)}\ldots v_{n_k,r}^{(k)} \quad (2)$$

where $h_{n_1,...,n_k}$ is the $(n_1,...,n_k)$ element of the tensor Hk, $v_{n_i,r}^{(i)}$ constitutes the (n_i,r) element of the matrix $V^{(i)}$ of dimensions (N_i,R_k) and R_k is the number of the PARAFAC model factors and the rank of the tensor H_k defined by Kruskal [21]. For example, the scalar representation of the PARAFAC decomposition of a 3^{rd}-order (N_1,N_2,N_3) tensor H_3 is written as:

$$h_3\,(n_1,n_2,n_3)=\sum_{r=1}^{R_3}b_{n_1r}^{(1)}.b_{n_2r}^{(2)}.b_{n_3r}^{(3)} \quad (3)$$

2.3 Fourth-order PARAFAC Volterra model

The input/output relation of a discrete-time, time invariant 4th-order Volterra model is written as:

$$\begin{aligned}\hat{y}(n)=&\sum_{n_1=1}^{M}h_1(n_1)u(n-n_1)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}h_2(n_1,n_2)u(n-n_1)u(n-n_2)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}\sum_{n_3=1}^{M}h_3(n_1,n_2,n_3)\prod_{i=1}^{3}u(n-n_i)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}\sum_{n_3=1}^{M}\sum_{n_4=1}^{M}h_4(n_1,n_2,n_3,n_4)\prod_{i=1}^{4}u(n-n_i)\end{aligned} \quad (4)$$

where $u(n)$ and $\hat{y}(n)$ are the model input and output respectively. $\{h_k(n_1,...,n_k)\}$; $k=1,...,4$ are the Volterra kernels, respectively described by an M-dimensional tensor H_k.

By using the scalar representation (2) of PARAFAC, equation (4) becomes:

$$\begin{aligned}\hat{y}(n)=&\sum_{n_1=1}^{M}h_1(n_1)u(n-n_1)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}\left(\sum_{r=1}^{R_2}a_{n_1r}^{(1)}\,a_{n_2r}^{(2)}\right)u(n-n_1)u(n-n_2)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}\sum_{n_3=1}^{M}\left(\sum_{r=1}^{R_3}b_{n_1r}^{(1)}.b_{n_2r}^{(2)}.b_{n_3r}^{(3)}\right)\prod_{i=1}^{3}u(n-n_i)\\&+\sum_{n_1=1}^{M}\sum_{n_2=1}^{M}\sum_{n_3=1}^{M}\sum_{n_4=1}^{M}\left(\sum_{r=1}^{R_4}c_{n_1r}^{(1)}.c_{n_2r}^{(2)}.c_{n_3r}^{(3)}.c_{n_4r}^{(4)}\right)\prod_{i=1}^{4}u(n-n_i)\end{aligned} \quad (5)$$

$\hat{y}(n)$ can also be written as :

$$\begin{aligned}\hat{y}(n) &= \sum_{n_1=1}^{M} h_1(n_1)u(n-n_1) \\ &+ \sum_{r=1}^{R_2}\left(\sum_{n_1=1}^{M} a_{n_1 r}^{(1)} u(n-n_1)\right)\left(\sum_{n_2=1}^{M} a_{n_2 r}^{(2)}\ u(n-n_2)\right) \\ &+ \sum_{r=1}^{R_3}\prod_{i=1}^{3}\left(\sum_{n_i=1}^{M} b_{n_i r}^{(i)} u(n-n_i)\right) \\ &+ \sum_{r=1}^{R_4}\prod_{j=1}^{4}\left(\sum_{n_j=1}^{M} c_{n_j r}^{(j)} u(n-n_j)\right)\end{aligned} \tag{6}$$

where

- h_1 represents the linear Volterra kernel,
- $a_{n_1 r}^{(1)}$ and $a_{n_2 r}^{(2)}$ represent respectively the elements (n_1, r) and (n_2, r) of the (M, R_2)-dimensional matrices $A^{(1)}$ and $A^{(2)}$ components of the PARAFAC decomposition of quadratic kernel H_2.
- $b_{n_1 r}^{(1)}, b_{n_2 r}^{(2)}$ and $b_{n_3 r}^{(3)}$ represent respectively the elements (n_1, r), (n_2, r) and (n_3, r) of the (M, R_3)-dimensional matrices $B^{(1)}$, $B^{(2)}$ and $B^{(3)}$, components of the PARAFAC decomposition of the cubic kernel H_3.
- $c_{n_1 r}^{(1)}$, $c_{n_2 r}^{(2)}$, $c_{n_3 r}^{(3)}$ and $c_{n_3 r}^{(4)}$ represent the elements (n_1, r), (n_2, r), (n_3, r) and (n_4, r) of the (M, R_4)-dimensional matrices $C^{(1)}, C^{(2)}$, $C^{(3)}$ and $C^{(4)}$ component of the PARAFAC decomposition of the 4^{th} order kernel H_4.

2.4 Fourth-order S-PARAFAC Volterra model

In the Volterra kernel, there always exists an associated symmetrized kernels $\left\{\tilde{H}_k\right\}_{k=1\ldots4}$ [22] computed according to:

$$\tilde{h}_k(n_1, ..., n_k) = \frac{\alpha}{k!}\sum_{\pi\in P} h_k\left(n_{\pi(1)}, ..., n_{\pi(k)}\right);\ k = 1, ..., 4 \tag{7}$$

where ! stands for the factorial notation and P is the permutation set respectively of cardinal $k!/\alpha$ with $\alpha = n_1!\cdots n_r!$ and r is the number of distinct values respectively in the set $\{n_1, ..., n_k\}$.

With this symmetry property, it is proved that for each k^{th} order symmetric Volterra kernel $\tilde{H}_k\ (k = 1, \cdots, 4)$, the PARAFAC decomposition matrices $\{V^{(k)}\}_{k=1,\ldots,4}$ are all identical and equal to $\tilde{V}$ according to the following theorem.

Theorem [17] : Any symmetric k^{th}-order tensor H_k can always be decomposed as the sum of symmetric outer products of vectors:

$$H_k = \sum_{r=1}^{R^{(s)}} \underbrace{\tilde{V}_{.r} \circ \cdots \circ \tilde{V}_{.r}}_{k\ times} \tag{8}$$

or, in scalar form:

$$h_{i_1,\ldots,i_k} = \sum_{r=1}^{R^{(s)}}\prod_{n=1}^{k} \tilde{v}_{i_n r}\ ;\quad i_n = 1, \ldots, k \tag{9}$$

where $R^{(s)}$ is the number of symmetric outer products needed to generate H_k using (8) or (9) ■

Unlike (2), such symmetric decomposition called a symmetric PARAFAC model needs a unique $(M, R^{(s)})$ matrix factor $\tilde{V}$. The symmetric rank of H_k, is defined by:

$$rank_s(H_k) = \min(R^{(s)}) \tag{10}$$

It is also defined using the following inequality between its rank and its symmetric rank [17]:

$$rank(H_k) \le rank_s(H_k) \tag{11}$$

So the output of the 4th-order S-PARAFAC Volterra model described in (6) can be written as:

$$\begin{aligned}\hat{y}(n) &= \sum_{k=1}^{M} h_1(k)u(n-k) + \sum_{r=1}^{R_2}\left(\sum_{k=1}^{M} \tilde{a}_{kr} u(n-k)\right)^2 \\ &+ \sum_{r=1}^{R_3}\left(\sum_{k=1}^{M} \tilde{b}_{kr} u(n-k)\right)^3 + \sum_{r=1}^{R_4}\left(\sum_{k=1}^{M} \tilde{c}_{kr} u(n-k)\right)^4\end{aligned} \tag{12}$$

The Volterra kernels $\{H_k\}_{k=1\ldots4}$ can be viewed as a k^{th}-order $(M, ..., M)$ tensors with a p arameter complexity $M^k; k = 1, ..., 4$ in terms of its coefficients number. So, the complexity of a standard 4th order Volterra model is $C = \sum_{k=1}^{4} M^k$.

The k^{th} -order S-PARAFAC kernel complexity is $M.R_k$. So, the complexity of the new 4^{th} order S-PARAFAC Volterra model is $C_S = M.\sum_{k=1}^{4} R_k$ with $R_1 = 1$ for the linear Volterra kernel.

The Ratio of Complexity Reduction (*RCR*) with respect to the standard 4^{th}-order Volterra model is:

$$RCR(\%) = 100\left(1 - \sum_{k=1}^{4} R_k \Big/ \sum_{k=1}^{4} M^{k-1}\right) \quad (13)$$

When $R_k << M$, a significant complexity reduction can be achieved.

2.5 Model parameter estimation

The input/output relation (12) can be implemented using a parallel-cascade structure, as shown in Fig.1, where:

- $U(n) = [u(n-1) \cdots u(n-M)]^T$,
- h_1 is the vector containing the coefficients of the linear kernel.
- $\tilde{A}_{.i}$ represents the i^{th} column of the matrix $\tilde{A}$ related to the quadratic Volterra kernel and the boxes of stage *r* correspond to the convolution operation $U^T(n)\tilde{A}_{.r}$.
- $\tilde{B}_{.j}$ represents the j^{th} column of the matrix $\tilde{B}$ related to the cubic Volterra kernel and the boxes of stage *r* correspond to the convolution operation $U^T(n)\tilde{B}_{.r}$.
- $\tilde{C}_{.k}$ represents the k^{th} column of the matrix $\tilde{C}$ related to the fourth-order Volterra kernel and the boxes of stage *r* correspond to the convolution operation $U^T(n)\tilde{C}_{.r}$.

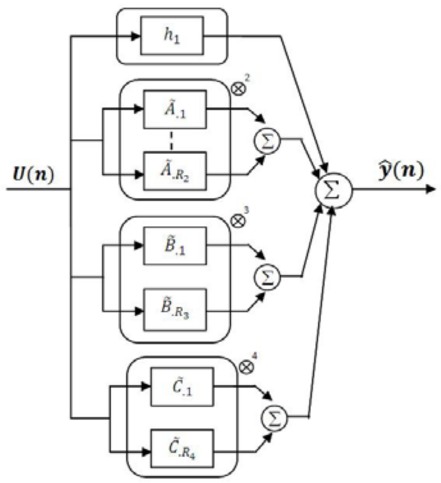

Fig.1. Parallel-cascade realization of the 4th-order S-PARAFAC Volterra model

The output writing of the 4^{th} order S-PARAFAC Volterra model given by (12) can be written in a linear regression way. Let us define:

$$\phi_r^A(n) = \sum_{k=1}^{M} \tilde{a}_{k\,r}\, u(n-k) \quad (14)$$

$$\phi_r^B(n) = \left(\sum_{k=1}^{M} \tilde{b}_{k\,r}\, u(n-k)\right)^2 \quad (15)$$

$$\phi_r^C(n) = \left(\sum_{k=1}^{M} \tilde{c}_{k\,r}\, u(n-k)\right)^3 \quad (16)$$

Equation (12) can be written as:

$$\hat{y}(n) = \sum_{k=1}^{M} \left(\begin{array}{l} h_1(k) + \sum_{r=1}^{R_2} \tilde{a}_{k\,r}.\phi_r^A(n) \\ + \sum_{r=1}^{R_2} \tilde{b}_{k\,r}.\phi_r^B(n) + \sum_{r=1}^{R_3} \tilde{c}_{k\,r}.\phi_r^C(n) \end{array} \right) u(n-k) \quad (17)$$

$$= \underbrace{\begin{bmatrix} h_1 \\ vec(\tilde{A}) \\ vec(\tilde{B}) \\ vec(\tilde{C}) \end{bmatrix}^T}_{\Theta^T} \underbrace{\begin{bmatrix} \begin{pmatrix} 1 \\ \Phi_A(n) \\ \Phi_B(n) \\ \Phi_C(n) \end{pmatrix} \otimes U(n) \end{bmatrix}}_{\Lambda(n)} \quad (18)$$

$$= \Theta^T.\Lambda(n) \quad (19)$$

where $vec(X) = \left[x_{.i}^T\right]^T$ and $x_{.i}$ is the i^{th} column of a given matrix *X*,

$$\Phi_A(n) = \left[\phi_1^A(n) \quad \phi_2^A(n) \quad \cdots \quad \phi_{R_2}^A(n)\right]^T \quad (20)$$

$$\Phi_B(n) = \left[\phi_1^B(n) \quad \phi_2^B(n) \quad \cdots \quad \phi_{R_3}^B(n)\right]^T \quad (21)$$

$$\Phi_C(n) = \left[\phi_1^C(n) \quad \phi_2^C(n) \quad \cdots \quad \phi_{R_4}^C(n)\right]^T \quad (22)$$

and $\otimes$ denotes the Kronecker product.

Therefore, the estimation of vector Θ implies the estimation of the components of the vector h_1 and the matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ characterizing the proposed model. To do, we apply the Recursive Least Square algorithm that updates from the model outputs (18-19-20), the linear kernel h_1, the matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ by minimizing the following least squares cost function $\eta(L)$.

$$\eta(L) = \sum_{n=1}^{L} \left(y(n) - \hat{y}(n)\right)^2 \quad (23)$$

$$= \sum_{n=1}^{L} \left(y(n) - \Theta^T \Lambda(n)\right)^2 \quad (24)$$

where y denotes the output of the system to be modeled, $\hat{y}$ denotes the output of the proposed model given by (18-19-20) and L the length of the input sequence. The estimation is achieved by the four following algorithm.

a. Initialization

- $\Theta(0)$
- $Q(0) = I_{M(1+R_2+R_3+R_4)}$

b. Update of the S-PARAFAC Volterra model components.

- $U(n) = \left[u(n-1)\cdots u(n-M)\right]^T$
- $$\begin{cases} \text{Calculate } \phi_r^A(n), \phi_r^B(n) \text{ and } \phi_r^C(n) \\ \text{Construct } \Phi_A(n), \Phi_B(n) \text{ and } \Phi_C(n) \\ \Lambda(n) = \left[1 \;\; \Phi_A^T(n) \;\; \Phi_B^T(n) \;\; \Phi_C^T(n)\right]^T \otimes U(n) \\ \varepsilon(n) = y(n) - \Lambda^T(n).\hat{\Theta}(n-1) \\ K(n) = \dfrac{Q(n-1)\Lambda(n)}{1-\Lambda^T(n)Q(n-1)\Lambda(n)} \\ Q(n) = [1-K(n)\Lambda^T(n)]Q(n-1) \\ \hat{\Theta}(n) = \hat{\Theta}(n-1) + K(n)\varepsilon(n) \\ \hat{\Theta}(n) = \left[[h_1^T(n) \;\; vec^T\hat{A}(n) \;\; vec^T\hat{B}(n) \;\; vec^T\hat{C}(n)\right]^T \end{cases}$$

c. Reconstruction of the Volterra model kernels

- Linear kernel : h_1
- Quadratic kernel : $h_2(n_1,n_2) = \sum_{r=1}^{R_2} \tilde{a}_{n_1 r}\,\tilde{a}_{n_2 r}$
- Cubic kernel : $h_3(n_1,n_2,n_3) = \sum_{r=1}^{R_3} \tilde{b}_{n_1 r}\,\tilde{b}_{n_2 r}\,\tilde{b}_{n_3 r}$
- 4^{th} order kernel :

$$h_4(n_1,n_2,n_3,n_4) = \sum_{r=1}^{R_3} \tilde{c}_{n_1 r}\,\tilde{c}_{n_2 r}\,\tilde{c}_{n_3 r}\,\tilde{c}_{n_4 r}$$

d. Go to step (b) till convergence

3 Modeling and identification of a nonlinear thermal process PT326

3.1 Process description

The application involves the identification of a bench-scale hot air-flow device, the PT 326 Process Trainer as shown in Fig.2, from Feedback Ltd. This laboratory thermal process exhibits many linear range, output drift and inherent process noise. It has been used many times to illustrate the performances of other identification methods, as in [23]. Fig.3 shows a schematic description of the heat transfer process. Air is pulled by a fan into a 30cm length tube through a valve and heated by a mesh of resistor wires at the inlet.

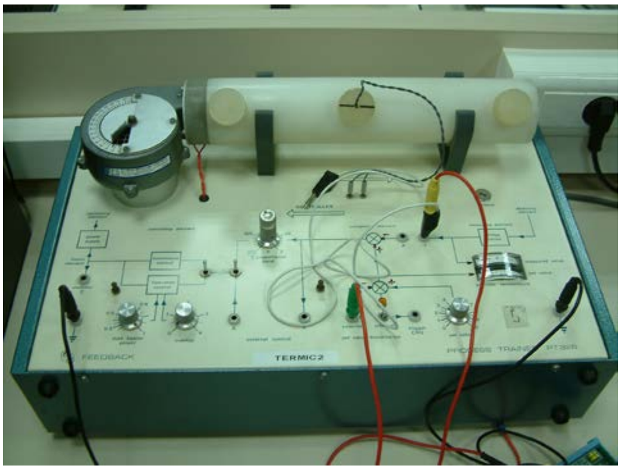

Fig.2. Process Trainer PT 326

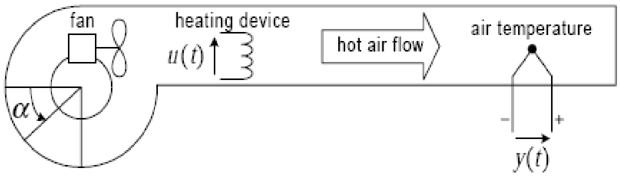

Fig.3. Schematic description of the heat transfer process

The process input u_n is the voltage over of resistor wires to heat the incoming air and the output y_n is air temperature measured by a thermocouple at the outlet. The process perturbation can be realized by adding the ambient air the quantity of which is tuned by the angle α .

3.2 Process modeling

The algorithm proposed by Chouchane et al [18], estimates, from input/output observations, the minimal values of the nonlinearity degree (order) K and the system memory M to represent the Process Trainer PT326. The results found give a V olterra model memory $M = 4$ and an order $K = 4$. Let us rewrite the output of the 4^{th} order S-PARAFAC Volterra model previously defined in equation (12) adapted to the heat transfer process.

$$\hat{y}(n) = \sum_{k=1}^{M} h_1(k)u(n-k) + \sum_{r=1}^{R_2}\left(\sum_{k=1}^{M}\tilde{a}_{kr}u(n-k)\right)^2 + \sum_{r=1}^{R_3}\left(\sum_{k=1}^{M}\tilde{b}_{kr}u(n-k)\right)^3 + \sum_{r=1}^{R_4}\left(\sum_{k=1}^{M}\tilde{c}_{kr}u(n-k)\right)^4 \quad (25)$$

3.3 Complexity reduction

Tab.1 presents the Ratio of Complexity Reduction (*RCR*) given by (13) for different values of the Symmetric rank $rank_s(H)$ defined in (11). We note that $rank_s(H) \leq M$

Tab.1. RCR for different symmetric rank of the Volterra kernels with memory *M=4*.

$R^{(s)}$	1	2	3	4
RCR	95,29 %	90,59 %	85,88 %	81,18 %

4 Numerical Simulation

A set of 1000 input/output observations were collected from the process at a sampling time of 0.08s. The process input was chosen to be a binary random signal shifting between 3.41V and 6.41V as in Fig.4 plots.

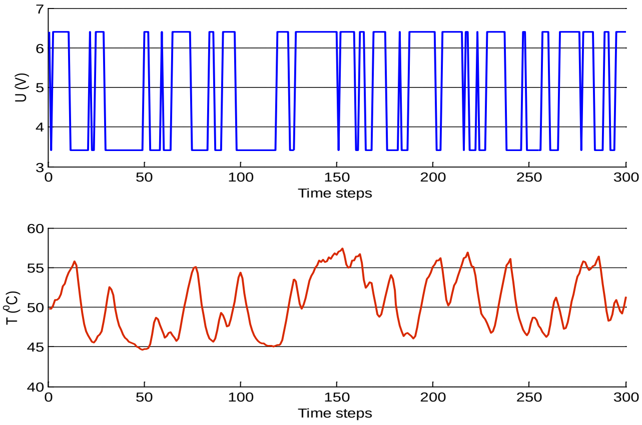

Fig.4. Input and output sequences of the thermal process

The simulation results were obtained using the Monte Carlo method with 20 different additive Gaussian white noise sequences and with a signal to Noise Ratio $SNR = 20dB$.

$$SNR_{dB} = 10\log\left[\sum_{n=1}^{1000}\left(y(n)-\bar{y}\right)^2 \Big/ \sum_{n=1}^{1000}\left(\gamma(n)-\bar{\gamma}\right)^2\right] \quad (25)$$

$\bar{y}$ and $\bar{\gamma}$ are the mean values of the system output and the disturbance respectively.

The accuracy of the proposed model and the estimation algorithm is evaluated by means of the Normalized Mean square Error (*NMSE*) given by:

$$NMSE = \sum_{n=1}^{1000}\left(y(n)-\hat{y}(n)\right)^2 \Big/ \sum_{n=1}^{1000}\hat{y}^2(n) \quad (26)$$

where y and $\hat{y}$ are the outputs of the thermal process and the estimated model respectively. These outputs are simultaneously plotted in Fig.5.

Tab.2 presents the estimated components of the linear Volterra kernel h_1 and the matrices *A*, *B* and *C* of the S-PARAFAC Volterra model. To highlight the efficiency of the estimated model, Fig.6 shows the convergence of some of its components.

Tab.2. Estimated components of the linear Volterra kernel h_1 and the matrices *A*, *B* and *C* of the S-PARAFAC Volterra model

$\boldsymbol{h}_1$	0.0219	0.1394	0.0805	0.1412
A	-0.0169	0.0871	-0.0476	-0.0278
B	0.0516	0.1295	-0.0339	-0.0007
C	-0.0019	0.1424	-0.0801	0.0379

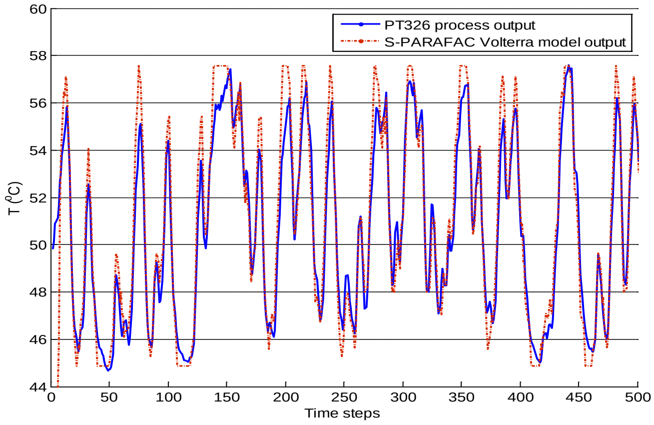

Fig.5. Outputs of the thermal process and the S-PARAFAC Volterra model

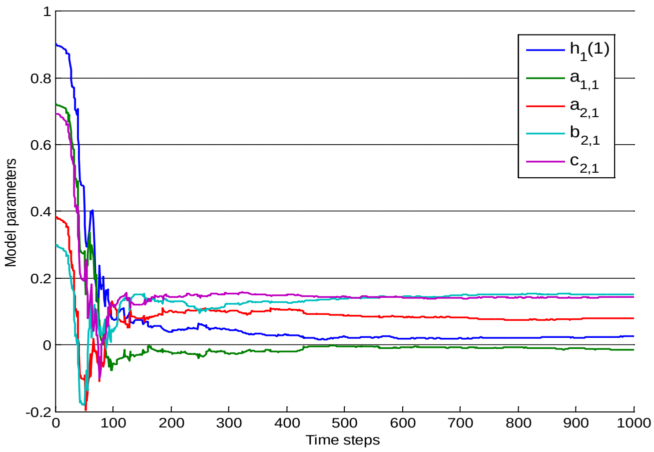

Fig.6. Convergence of some parameters of the estimated S-PARAFAC Volterra model

The Normalized Mean square Error is plotted in Fig.7 for different SNR values. For *SNR=20dB* and after 1000 time steps, it reaches $4{,}9.10^{-3}$.

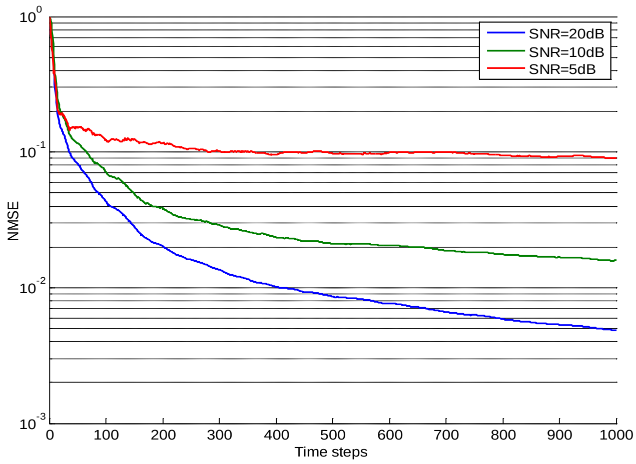

Fig.7. *NMSE* between the thermal process and the estimated model outputs for different *SNR* values

5 Conclusion

In this paper, we have proposed a new approach to design a reduced Volterra model using tensorial decompositions and the symmetry property of the Volterra kernels so that the parameter number of the resulting model is significantly reduced.

To illustrate the efficiency of the proposed S-PARAFAC Volterra model, it was tested to describe the behaviour of a n onlinear thermal process; the PT326. It resorts that the proposed model handles the process perfectly despite its reduced parameter number.

References:

[1] Wiener N. *Nonlinear problems in random theory*. New York : Wiley, 1958.

[2] Billings S.A, Identification of nonlinear systems - a survey. *IEE Proceedings Part D*, Vol.127, No.6, 1980, pp.272-285.

[3] Schetzen M, *The Volterra and Wiener theories of nonlinear systems*, New York, Wiley, 1980.

[4] Alper A.A, Consideration of the discrete Volterra series, *IEEE Trans. Automatic Control*, Vol.10, No.3, 1965, pp.322-327.

[5] Boyd S, Chua L.O, Fading memory and the problem of approximating nonlinear operators with Volterra series, *IEEE Tr. Circuits and Systems*, Vol.32, No.11, 1985, pp.1150-1171.

[6] Campello R.J.G.B, Amaral W, Favier G, Optimal expansions of discrete time Volterra models using Laguerre functions, *Automatica*, Vol.40, 2004, pp.815–822.

[7] Dumont G.A, Fu Y, Nonlinear adaptive control via Laguerre expansion of Volterra kernels. *Int. J. Adaptive Control and Signal Processing*, Vol.7, No.5, 1993, pp.367-382.

[8] da Rosa A, Campello R.J.G.B, Amaral W.C, An optimal expansion of Volterra models using independent Kautz bases for each kernel dimension, *International Journal of Control*, Vol.81, No.6, 2008, pp.962-975.

[9] Garna T, Bouzrara K, Ragot J, Messaoud H, Nonlinear system modeling based on bilinear Laguerre orthonormal bases, *ISA Transactions*, Vol.52, No.3, 2013, pp.301-317.

[10] Kibangou A, Favier G, Hassani M M. A, growing approach for selecting generalized orthonormal basis functions in the context of system modeling, *In Proceding of IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing, NSIP'03*, 2003.

[11] Favier G, Kibangou A, Khouaja A, Nonlinear System Modelling by means of Volterra Models. Approaches for the Parameter Complexity Reduction, *Invited paper in Symposium Techniques Avancées et Stratégies Innovantes en Modélisation et Commandes Robustes des processus Industriels, Martigues, France*, 2004.

[12] Mbarek A, Garna T, Bouzrara K, Messaoud H, Reduced complexity Volterra model of non-linear MISO system. *Int. J. of Modelling, Identification and Control*, Vol.6, No.2, 2012, pp.134-148.

[13] Khouaja A, Favier G, Identification of PARAFAC-Volterra Cubic Models using an Alternating Recursive Least Squares Algorithm. *In Proceedings of European Signal Processing Conference (EUSIPCO)*, 2004, pp.1903-1906.

[14] Khouaja A, Favier G, Identification of SVD-PARAFAC based Third-order Volterra Models using an ARLS Algorithm. *In Proceedings of IFAC World Congress*, 2005.

[15] Kibangou A, Favier G, Tensor-based Identification of the Structure of Block-Oriented Nonlinear Systems. *In Proceedings of IFAC Symposium on System Identification*, 2009.

[16] Comon P, Golub G, Lim LH, Mourrain B, Symmetric tensors and symmetric tensor rank. *SIAM Journal on Matrix Analysis and Applications*, Vol.30, No.3, 2008, pp.1254-1279.

[17] Harshman R A, Foundation of the PARAFAC procedure : Models and conditions for an .explanatory. multimodal factor analysis, *UCLA working papers in phonetics*, Vol.16, 1970, pp.1-84.

[18] Chouchane S, Bouzrara K and Messaoud H, Structural identification of a thermal process using Volterra model, *Heat Transfer Research*, Vol.45, No.5, 2014, pp.471-484
[19] Carroll J.D, Chang J.J, Analysis of individual differences in multidimensional scaling via an N-way generalization of Eckart-Young decomposition, *Psychometrika*, Vol.35, 1970, pp.283-319.
[20] Kruskal JB, Three-way arrays : Rank and uniqueness of trilinear decompositions, with application to arithmetic complexity and statistics, *Linear algebra and its applications*, Vol.18, 1977, pp.95-138.
[21] Favier G, Bouilloc T, Parametric Complexity Reduction of Volterra Models using Tensor Decompositions. *In Proceedings of European Signal Processing Conference (EUSIPCO)*, 2009, pp.2288-2292.
[22] Ljung L. *System identification. Theory for the user*. Prentice Hall, 1987.
[23] S. M. Abd-Elazim, and E. S. Ali, Synergy of Particle Swarm Optimization and Bacterial Foraging for TCSC Damping Controller Design. *Int. J. of WSEAS Transactions on Power Systems*, Vol. 8, No. 2, April 2013, pp. 74-84.
[24] S. M. Abd-Elazim, and E. S. Ali, Optimal Power System Stabilizers Design for Multimachine Power System Using Hybrid BFOA-PSO Approach, *International Journal of WSEAS Transactions on Power Systems*, Vol. 8, No. 2, April 2013, pp. 85-94.
[25] E. S. Ali, S. M. Abd-Elazim, Optimal PSS Design in a Multimachine Power System via Bacteria Foraging Optimization Algorithm, *Int. J. of WSEAS Transactions on Power Systems*, Vol. 8, No. 4, October 2013, pp. 186-196.
[26] E. S. Ali, and S. M. Abd-Elazim, Power System Stability Enhancement via Bacteria Foraging Optimization Algorithm, *Int. Arabian J. for Science and Engineering (AJSE)*, Vol. 38, No. 3, March 2013, pp. 599-611.
[27] A. S. Oshaba, and E. S. Ali, Bacteria Foraging: A New Technique for Speed Control of DC Series Motor Supplied by Photovoltaic System, *International Journal of WSEAS Transactions on Power Systems*, Vol. 9, 2014, pp. 185-195.