

Optimal Power Flow Using Cuckoo Search Considering Voltage Stability

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Abstract: - Optimal power flow is the most complicated economical problem in power system operation. This is due to the nonlinearity of load flow equations and constraints on voltage, thermal limits and angle and voltage stability. The objective of this problem is to distribute system loads among generators with minimum cost while achieving all equality and inequality constraints. Nowadays, meta heuristic methods are used to solve this problem such as genetic algorithm, particle swarm and tabu search. In this paper Cuckoo search is investigated with additional constraints on voltage stability and output of reactive power compensators. Considering the initial load angles of generators will enhance the transient stability of power system. Results obtained by Cuckoo search are compared to that obtained by genetic algorithm, Tabu search and gradient descent methods. With the same loading conditions and constraints, Cuckoo search based optimal power flow gives less cost compared to these methods. The algorithm is investigated on IEEE 30 bus system.

Key-Words: - OPF, Cuckoo search, voltage stability, reactive power compensators.

1 Introduction

Optimal load flow (OPF) is one of the most important and complicated problem in power system operation, the objective of OPF is to minimize generation cost to meet power system load and to maintain many constraints. Reducing generation cost not only enhances the economic operation of power system but also reduces the amount of burned fuel, thus maintains the fuel and reduces its burning effect on environment. Equality constraints include the balance between both generated active and reactive powers with those of load and losses. Inequality constraints include voltage level of load buses, thermal limit on branches, and transient stability limits for generators. The problem of OPF is solved using traditional optimization methods such as interior point [1-2], linear programming [3-4], nonlinear programming [5-7] and quadratic programming [8-9]. Drawbacks in these methods are the difficulty to obtain the global minimum due to many local minimums that exist in OPF problem, also the simplification that made to convert the OPF problem into convex one, the fixed step size in solution and the initial starting point [10]. Heuristic optimization tools have been investigated such as evolutionary and genetic algorithm [11-13], particle swarm optimization [14-15], ant colony [16-17], Bees algorithm [18-19], gravitational search [20] and Tabu search [21-22] to solve OPF problem. In

this paper Cuckoo search [23] is investigated and applied to IEEE 30 bus system. Results obtained are compared with those in [11, 22, 24], Cuckoo search gave better results. Transient stability limit is considered by considering the reactive power limits especially for lead power factor operation. In this paper voltage stability is also considered by calculating smallest eigenvalue of the reduced Jacobian matrix [25] which is a measure for the sensitivity between bus voltages and reactive power. Participation factors are used to determine weak buses in the system. Reactive power compensators are used at these buses and Cuckoo search is used to tune the output of these compensators to enhance the generation cost.

List of Symbols

N_g	No. of generators.
a_i, b_i and c_i	Constants of the cost function.
P_{gi}	Real power generation at bus i .
$P_{gi \min}$	Minimum real power of generator at bus i .
$P_{gi \max}$	Maximum real power of generator at bus i .
Q_{gi}	Reactive power generation at bus i .
$Q_{gi \min}$	Minimum reactive power of generator at bus i .
$Q_{gi \max}$	Maximum reactive power of generator at bus i .
P_{Li}	Real power demand at bus i .

Q_{Li}	Reactive power demand at bus i.
$V_{gi \min}, V_{gi \max}$	Minimum and maximum output voltage of generation at bus i.
$V_{Li \min}, V_{Li \max}$	Minimum and maximum load voltage at bus i.
$I_{TLi \min}, I_{TLi \max}$	Minimum and Maximum current of transmission line no. i.
$Q_{comp \min, \max}$	Minimum and maximum reactive power output of compensator no. i.
$T_{i \min}, T_{i \max}$	Minimum and maximum tap setting of load tap changer no. i.
N	Total number of buses.
Y_{ij}	Magnitude of Ybus element i, j.
θ_{ij}	Angle of bus admittance element i, j.
δ_i	Voltage angle of bus i.
δ_j	Voltage angle of bus j.
V_{gi}	Voltage of generator no. i.
V_{Li}	Voltage of load bus no. i.
I_{TLi}	Current of transmission line no. i.
$Q_{comp i}$	Reactive power of compensator no. i.
T_i	Tap setting of transformer no. i.
N_L	No. of load buses.
N_{TL}	No. of transmission lines.
N_{comp}	No. of reactive power compensator.
N_{trans}	No. of load tap changing transformers.
$J_{11}, J_{12}, J_{21}, J_{22}$	Sub matrices of the Jacobian matrix.
λ	Reduced Jacobian matrix eigenvalues.

2 Optimal Load Flow

Optimal load flow problem is to find a steady state operating point for electrical power system, this point must satisfy power balance and all constraints related to generators capability, voltage, thermal loading, tap settings and reactive power compensators limits. All these equality and inequality constraints must be satisfied while minimizing the cost of generation. The problem can be formed mathematically as to minimize the following cost function:

$$\text{Min } J(x,u) \tag{1}$$

Where J is the total fuel cost:

$$J = \sum_{i=1}^{N_g} F_i \tag{2}$$

F_i is the fuel cost function of generator no. i.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad \$/\text{hr} \tag{3}$$

x : Is the vector of dependent variables that includes slack bus active and reactive power, load bus voltages, reactive power of generators and loading of transmission lines.

u : Is the vector of independent variables that can be varied within certain limits and affect the value of the objective function. These variables are active power output of all generators except slack bus, voltage of all voltage controlled buses in the system,

setting of load tap changers and setting of reactive power compensators.

The cost function in optimal load flow is subjects to both equality and inequality constraints:

$$g(x,u) = 0 \tag{4}$$

$$h(x,u) \leq 0 \tag{5}$$

g represents equality constraints which are balance equations for active and reactive power:

$$P_{gi} - P_{Li} = \sum_{j=1}^N |V_i Y_{ij} V_j| \cos(\theta_{ij} + \delta_j - \delta_i) \tag{6}$$

$$Q_{gi} - Q_{Li} = \sum_{j=1}^N |V_i Y_{ij} V_j| \sin(\theta_{ij} + \delta_j - \delta_i) \tag{7}$$

h represents inequality constraints which are:

$$P_{gi \min} \leq P_{gi} \leq P_{gi \max} \quad i = 1:N_g \tag{8}$$

$$Q_{gi \min} \leq Q_{gi} \leq Q_{gi \max} \quad i = 1:N_g \tag{9}$$

$$V_{gi \min} \leq V_{gi} \leq V_{gi \max} \quad i = 1:N_g \tag{10}$$

$$V_{Li \min} \leq V_{Li} \leq V_{Li \max} \quad i = 1:N_L \tag{11}$$

$$I_{TLi \min} \leq I_{TLi} \leq I_{TLi \max} \quad i = 1:N_{TL} \tag{12}$$

$$Q_{comp i \min} \leq Q_{comp i} \leq Q_{comp i \max} \quad i = 1:N_{comp} \tag{13}$$

$$T_{i \min} \leq T_i \leq T_{i \max} \quad i = 1:N_{trans} \tag{14}$$

3 Cuckoo Search Algorithm

Cuckoo search is one of nature-inspired metaheuristic algorithms, it is developed by Xin-She Yang and Suash Deb in 2009. Cuckoo search is based on the brood parasitism of some cuckoo species. Recent studies show that this algorithm is more efficient than genetic and particle swarm algorithms [22]. Cuckoo have aggressive reproduction strategy, some species engage the obligate brood parasitism by laying their eggs in nests of other host birds to increase the probability of hatching. Some species female parasitic cuckoos can mimic in color and pattern the eggs of chosen host species. Once first cuckoo chick is hatched, it will evict the host eggs by blindly propelling eggs out of nest, which increases the cuckoo chick's share of food, also it can mimic the call of host chicks to gain more feeding opportunity.

Flow chart in Figure (1) can summarize the steps of cuckoo search algorithm. A random n solution of control independent variables within their limits is generated. This solution is passed to a load flow and solved by Newton-Raphson method. Objective function is evaluated for each solution. A random set of solution is generated using Levy flight algorithm:

$$x_i^{t+1} = x_i^t + \alpha \oplus \text{Levy}(y) \tag{15}$$

Above equation is the stochastic equation of a random walk whose next step depends on current location (the first term) and the transition probability (the second term). α is the step size, the product \oplus means entry wise multiplications. Levy

flight provides a random walk whose random step length is drawn from Levy distribution:

$$Levy \sim = t^{-\lambda}, \quad (1 < \lambda < 3) \quad (16)$$

Steps form a random walk process with a power-law step length distribution with a heavy tail. Objective function with this new set is also evaluated. If new objective function is better than old one, a portion P_a of new set is replaces an equivalent random set of the initial solution. The process is repeated until the maximum number of epochs is reached. Initial set of nests (n nests) may vary from 15 to 40 and P_a of 0.25 are suitable values for most optimization problems [22].

4 Numerical Results

The proposed cuckoo search-based OPF algorithm has been applied to the IEEE 30-bus test system. The system SLD and line and bus data are given in the Appendix. The voltage magnitude limits are between 0.95 and 1.05 p.u. for all load buses and the slack bus (bus no. 5), while it is between 0.95 and 1.1 p.u. for all other generator buses. Tap setting of all transformer taps are between 0.9 and 1.1 p.u. In order to demonstrate the effectiveness of the technique, its results are compared with other results obtained by different techniques in the literature with the same power system and its operating conditions.

4.1 quadratic cost curve

The generator cost curves are represented by quadratic functions as given in (3). The values of the cost coefficients are given in Table 1.

Independent variables obtained by cuckoo search are given in Table (2). The total cost obtained by the proposed technique is 801.8796 \$/hr, while the best obtained after 100 runs by evolutionary programming was 802.62 \$/hr [11]. In addition, it was 802.29 \$/hr in [22] using Tabu search and with nonlinear programming solution was 802.40 \$/hr [24]. This confirms the superiority of the proposed technique over the evolutionary programming, Tabu search and nonlinear programming techniques.

Voltage profile with the obtained solution is shown in Figure (2). Minimum load voltage is 0.987 p.u. at bus no. 26 and maximum load voltage is 1.049 p.u. at bus no. 3. Total generated active power is 283.4 MW and reactive power 126.2 MVAR. Total system losses are 9.45 MW and 10.576 MVAR. No limit violation occurred with the solution. Table (3) shows the output power of the generators and the

internal load angles, maximum of load angles is that of generator no. 1 (35.3721°).

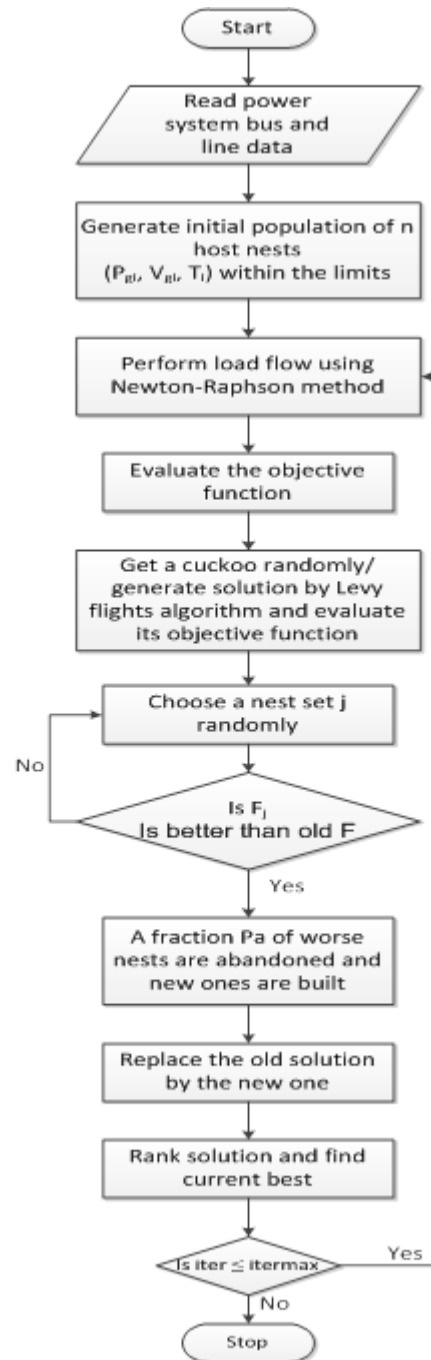


Fig. 1 Flow chart of cuckoo search based OPF

Table (1) Generator data and cost coefficients

Bus	$P_{g \min}$ p.u.	$P_{g \max}$ p.u.	$Q_{g \min}$ p.u.	$Q_{g \max}$ p.u.	a	b	c
1	0.50	2.00	-0.20	1.500	0.0	2.00	0.00375
2	0.20	0.80	-0.20	0.600	0.0	1.75	0.01750
5	0.15	0.50	-0.15	0.625	0.0	1.00	0.06250
8	0.10	0.35	-0.15	0.487	0.0	3.25	0.00834
11	0.10	0.30	-0.10	0.400	0.0	3.00	0.02500
13	0.12	0.40	-0.15	0.447	0.0	3.00	0.02500

Table (2) optimal values of independent variables

Independent variable	Proposed method	Ref. 11	Ref. 22	Ref. 24*
P_{g1} (MW)	178.0595	173.848	176.04	176.260
P_{g2} (MW)	48.1032	49.9980	48.760	48.8400
P_{g5} (MW)	21.332	21.3860	21.560	21.5100
P_{g8} (MW)	21.8178	22.6300	22.050	22.1500
P_{g11} (MW)	11.5379	12.9280	12.440	12.1400
P_{g13} (MW)	12.0000	12.0000	12.000	12.0000
V_1 (p.u.)	1.0792	1.05000	1.0500	1.05000
V_2 (p.u.)	1.0500	1.03600	1.0389	1.03820
V_5 (p.u.)	1.0293	1.0050	1.0110	1.01140
V_8 (p.u.)	1.0410	1.0160	1.0198	1.01940
V_{11} (p.u.)	1.0980	1.0690	1.0941	1.09120
V_{13} (p.u.)	1.0623	1.0550	1.0898	1.09130
T_{11} (p.u.)	1.0861	1.0200	1.0407	1.00275
T_{12} (p.u.)	0.9001	0.9000	0.9218	0.96020
T_{15} (p.u.)	1.0162	0.9500	1.0098	1.00474
T_{36} (p.u.)	0.9818	0.9400	0.9402	0.94163
Cost function (\$/hr)	801.8796	802.62	802.29	802.400

* There are many limits violated in this solution.

4.2 voltage stability consideration

In this part voltage stability of this power system is considered by evaluating the smallest eigenvalue of the reduced Jacobian matrix [25]. The change of active and reactive power is related by the change in voltage magnitude and angle by the relation:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \quad (17)$$

If we neglect the effect of ΔP on the voltage magnitude (decoupling phenomena), the reduced Jacobian J_R matrix is:

$$\Delta Q = (J_{22} - J_{21}J_{11}^{-1}J_{12})\Delta V = J_R\Delta V \quad (18)$$

Smallest eigenvalue of this matrix is a measure of steady state voltage stability limit since it is a sensitivity measure for the relation between reactive power and voltage. Table (4) shows the smallest six eigenvalues in this case. The smallest eigenvalue is 0.5096, as system loading increases or when contingences occurs, this value approaches zero, and the system is on the verge of voltage instability.

Table (4) shows the effect of outage of lines 24-25 and 6-28 on eigenvalues, smallest eigenvalue is reduced to 0.2608.

Using participation matrix [25], buses that contribute in the smallest eigenvalue are 26, 29 and 30. Cuckoo search is used to tune the output reactive power of three series compensators so that minimum generation cost is reduced. These series compensators are installed in lines 27-29, 27-30 and 29-30. Maximum compensation in each line 70% of its reactance, this is to avoid problems encountered by series compensation.

With lines 24-25 and 6-28 and by using series compensation, generation cost in this case is enhanced to 802.9186 \$/hr and the smallest eigenvalue is increased to 0.3499. Amount of compensation in line 27-29 is 50.4 %, in line 27-30 is 19.23 % and in line 29-30 is 30.13 %. In this case minimum load voltage is 0.989 at bus 24 and maximum load voltage is 1.056 at bus 27.

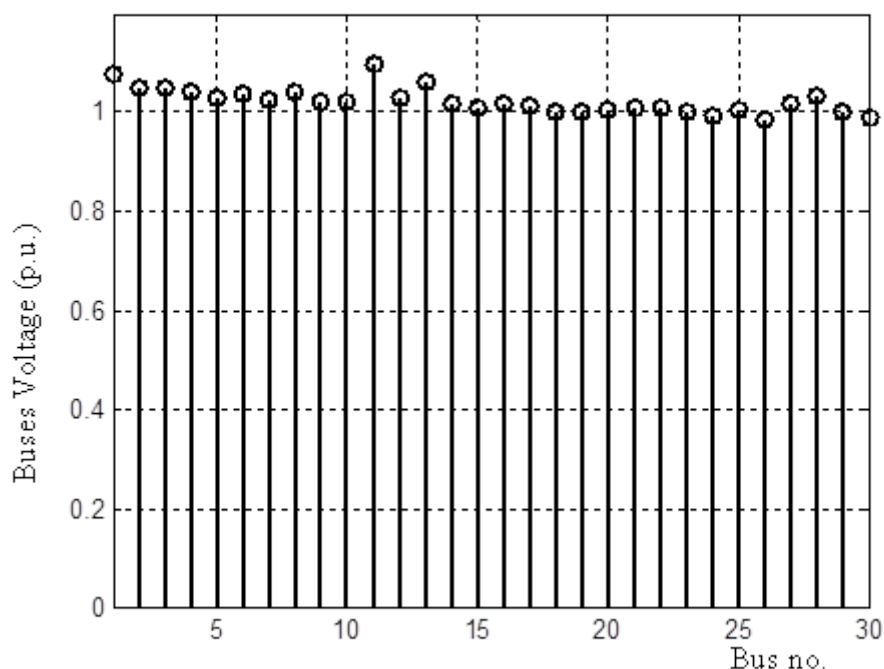


Fig. 2 Voltage profile using cuckoo search solution

Table (3) generators output power

Generator no.	P_g (MW)	Q_g (MVAR)	δ (Degree)
1	178.0595	17.878	35.3721
2	48.1032	-17.222	13.3002
3	21.332	30.069	5.0383
4	21.8178	42.431	4.8120
5	11.5379	39.666	2.3530
6	12.0000	23.952	2.7520

Table (4) smallest eigenvalues

No.	Base case	Outage of lines 24-25 and 6-28	Outage of lines 24-25 and 6-28 and with series compensators
1	0.5096	0.2608	0.3499
2	1.0287	0.7973	0.7907
3	1.7743	1.6254	1.9717
4	3.5335	3.3054	3.2554
5	3.9812	4.0327	3.9848
6	5.4206	5.3205	5.2548
Cost \$/hr	801.8796	803.9296	802.9186

5. Conclusion

In this paper Cuckoo search algorithm is used to solve the optimal load flow problem. Results obtained by Cuckoo search are compared with those obtained by evolutionary programming, Tabu search and interior point. It is found that cuckoo search algorithm enhanced the cost regarding that obtained by these algorithms for the same power system and

under the same conditions. Series compensators are used to accommodate voltage stability under outage conditions, tuning of this compensator resulted in enhancement of the cost function and also in voltage stability eigenvalue index.

References:

- [1] X. Yan and V. H. Quintana, "Improving an Interior Point based OPF by Dynamic Adjustments of Step Sizes and Tolerances," *IEEE Trans. on Power Systems*, Vol. 14, No. 2, 1999, pp. 709–717.
- [2] J. A. Momoh and J. Z. Zhu, 1999, "Improved Interior Point Method for OPF Problems," *IEEE Trans. on Power Systems*, Vol. 14, No. 3, 1999, pp. 1114–1120.
- [3] R. Mota-Palomino and V. H. Quintana, "Sparse Reactive Power Scheduling by a Penalty-Function Linear Programming Technique," *IEEE Trans. on Power Systems*, Vol. 1, No. 3, 1986, pp. 31–39.
- [4] A. A. Abou El-Ela and M. A. Abido, "Optimal Operation Strategy for Reactive Power Control," *Modelling, Simulation & Control, Part A*, AMSE Press, Vol. 41, No. 3, 1992, pp. 19–40.
- [4] R. Shoults and D. Sun, "Optimal Power Flow Based on P-Q Decomposition," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-101, No. 2, 1982, pp. 397–405.
- [6] K. R. C. Mamandur and R. D. Chenoweth, "Optimal Control of Reactive Power Flow for Improvements in Voltage Profiles and for Real Power Loss Minimization," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-100, No. 7, 1981, pp. 3185–3193.
- [7] R. C. Burchett, H. H. Happ, and K. A. Wirgau, "Large Scale Optimal Power Flow," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-101, 1982, pp. 3722–3732.
- [8] Zhijun Qin, and Yunhe Hou, "Application of Non-linear Programming for Large-Scale AC-DC Power Flow Analysis", *Power and Energy Society General Meeting, IEEE, San Diego, CA, 22-26 July 2012*, pp. 1-8.
- [9] K. Aoki, A. Nishikori, and R. T. Yokoyama, "Constrained Load Flow Using Recursive Quadratic Programming," *IEEE Trans. on Power Systems*, Vol. 2, No. 1, 1987, pp. 8–16.
- [10] A. A. Abou El-Ela and M. A. Abido, "Optimal Operation Strategy for Reactive Power Control," *Modelling, Simulation & Control, Part A*, AMSE Press, Vol. 41, No. 3, 1992, pp. 19–40.
- [11] Jason Yuryevich and Kit Po Wong, "Evolutionary Programming Based Optimal Power Flow Algorithm", *IEEE Transactions on Power Systems*, Vol. 14, No. 4, November 1999, pp. 1245-1250.
- [12] Narayana Prasad Padhy, "Wheeling Using Evolutionary Programming Based Optimal Power Flow Algorithm", *Proceedings of the 5th International Conference on Advances in Power System Control, Operation and Management, APSCOM 2000, Hong Kong, October 2000*, pp. 144-148.
- [13] C. H. Lo, C. Y. Chung, D. H. M. Nguyen and K. P. Wong, "A Parallel Evolutionary Programming Based Optimal Power flow Algorithm and Its Implementation", *Proceedings of the Third International Conference on Machine Learning and Cybernetics, Shanghai, 26-29 August 2004*, pp. 2543-2548.
- [14] M. A. Abido, "Multiobjective Particle Swarm Optimization for Optimal Power Flow Problem", *MEPCON 2008. 12th International Middle-East Power System Conference, Aswan, Egypt, 12-15 March 2008*, pp. 392-396.
- [15] Jing Zhang, Xiaoqing Zhang, Jingjing Sun, Qingyang Zou and Yuan Pan, "The Application of Improved Particle Swarm Optimization Algorithm Involtagage Stability Constrained Optimal Power Flow", *2nd International Conference on Measurement, Information and Control, Harbin, China, 2013*, pp. 1126-1130.
- [16] Soares J., Sousa T., Vale Z.A., Morais H. and Faria P. "Ant Colony Search algorithm for the optimal power flow problem", *Power and Energy Society General Meeting, IEEE, San Diego, CA, 24-29 July 2011* pp. 1-8.
- [17] Sreejith, S. ; Chandrasekaran, K. ; Simon, S.P. , " Application of Touring Ant colony Optimization technique for optimal power flow incorporating thyristor controlled series compensator", *TENCON 2009, IEEE Region 10 Conference, 23-26 Jan. 2009, Singapore*, pp. 1-6.
- [18] Umamom Kwannetr, Uthen Leeton and Thanatchai Kulworawanichpong, "Optimal Power Flow Using Artificial Bees Algorithm", *2010 International Conference on Advances in Energy Engineering, ICAEE 2010*, pp. 215-218.
- [19] Sumetha Anantasate and Pornrapeepat Bhasaputra, "A Multi-objective Bees Algorithm for Multi-objective Optimal Power Flow Problem", *The 8th Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTIT) Association of Thailand, Conference 2011*, pp. 852-856.
- [20] A. Bhattacharya P.K. Roy, "Solution of multi-objective optimal power flow using gravitational search algorithm", *IET Gener. Transm. Distrib.*, Vol. 6, No. 8, 2012, pp. 751–763.
- [21] Chunjie Li ,Huiru Zhao, Tao Chen, "The hybrid differential evolution algorithm for optimal power flow based on simulated annealing and tabu search", *Management and Service Science (MASS), 2010 International Conference, IEEE, 24-26 Aug. 2010, Wuhan*, pp. 1-7.

- [22] M. A. ABIDO, "Optimal Power Flow Using Tabu Search Algorithm", *Electric Power Components and Systems*, Taylor & Francis, 2002, pp. 469-483.
- [23] Xin She Yang, *Natural-Inspired Metaheuristic Algorithms*, second edition, Luniver Press, 2010.
- [24] O. Alsac and B. Stott, "Optimal Load Flow with Steady State Security," *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-93, 1974, pp. 745-751.
- [25] Gao, B. ; Morison, G.K. ; Kundur, P., "Voltage stability evaluation using modal analysis", *IEEE Transactions on Power Systems*, Vol. 7, No. 4 , , 1992, pp. 1529-1542.

Appendix

Fig. A.1 IEEE 30 bus power system

Table A.1 IEEE 30-bus test system line data

Line no.	From	To	R (p.u.)	X (p.u.)	B (p.u.)	Rating (p.u.)
1	1	2	0.0192	0.0575	0.0264	1.30
2	1	3	0.0452	0.1852	0.0204	1.30
3	2	4	0.0570	0.1737	0.0184	0.65
4	3	4	0.0132	0.0379	0.0042	1.30
5	2	5	0.0472	0.1983	0.0209	1.30
6	2	6	0.0581	0.1763	0.0187	0.65
7	4	6	0.0119	0.0414	0.0045	0.90
8	5	7	0.0460	0.116	0.0102	0.70
9	6	7	0.0267	0.0820	0.0085	1.30
10	6	8	0.0120	0.042	0.0045	0.32
11	6	9	0.0000	0.2080	0.0000	0.65
12	6	10	0.0000	0.5560	0.0000	0.32
13	9	11	0.0000	0.2080	0.0000	0.65
14	9	10	0.0000	0.1100	0.0000	0.65
15	4	12	0.0000	0.2560	0.0000	0.65
16	12	13	0.0000	0.1400	0.0000	0.65
17	12	14	0.1231	0.2559	0.0000	0.32
18	12	15	0.0662	0.1304	0.0000	0.32
19	12	16	0.0945	0.1987	0.0000	0.32
20	14	15	0.2210	0.1997	0.0000	0.16
21	16	17	0.0824	0.1923	0.0000	0.16
22	15	18	0.1070	0.2185	0.0000	0.16
23	18	19	0.0639	0.1292	0.0000	0.16
24	19	20	0.0340	0.0680	0.0000	0.32
25	10	20	0.0936	0.2090	0.0000	0.32
26	10	17	0.0348	0.0845	0.0000	0.32
27	10	21	0.0727	0.0749	0.0000	0.32
28	10	22	0.0116	0.1499	0.0000	0.32
29	21	22	0.0116	0.0236	0.0000	0.32
30	15	23	0.1000	0.2020	0.0000	0.16
31	22	24	0.1150	0.1790	0.0000	0.16
32	23	24	0.1320	0.2700	0.0000	0.16
33	24	25	0.1885	0.3292	0.0000	0.16
34	25	26	0.2544	0.3800	0.0000	0.16
35	25	27	0.1093	0.2087	0.0000	0.16
36	28	27	0.0000	0.3960	0.0000	0.65
37	27	29	0.2198	0.4153	0.0000	0.16
38	27	30	0.3202	0.6027	0.0000	0.16
39	29	30	0.2399	0.4533	0.0000	0.16
40	8	28	0.0636	0.2000	0.0214	0.32
41	6	28	0.0169	0.0599	0.0065	0.32

Table A.2 bus data of 30-bus system

Bus no.	P_L (MW)	Q_L (MVAR)	Bus no.	P_L (MW)	Q_L (MVAR)
1	0.0	0.0	16	3.5	1.8

2	21.7	12.7	17	9.0	5.8
3	2.4	1.2	18	3.2	0.9
4	7.6	1.6	19	9.5	3.4
5	94.2	19.0	20	2.2	0.7
6	0.0	0.0	21	17.5	11.2
7	22.8	10.9	22	0.0	0.0
8	30.0	30.0	23	3.2	1.6
9	0.0	0.0	24	8.7	6.7
10	5.8	2.0	25	0.0	0.0
11	0.0	0.0	26	3.5	2.3
12	11.2	7.5	27	0.0	0.0
13	0.0	0.0	28	0.0	0.0
14	6.2	1.6	29	2.4	0.9
15	8.2	2.5	30	10.6	1.9