New Algorithms for Removal of DC Offset and Subsynchronous Resonance terms in the Current and Voltage Signals under Fault Conditions

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Abstract: - Two filtering algorithms for removal of DC offset and subsynchronous resonance terms in the current/voltage signals in a transmission line under fault condition are presented. One of the algorithms utilizes the samples of the signal over a time interval slightly greater than one cycle time of the fundamental component and is computationally less demanding than the existing Fourier filters. The second algorithm is based on sampling the signal at arbitrary locations and hence, the signal samples can be acquired during a time interval much less than the full-cycle period of the fundamental component. This results in the calculation of the fundamental component in a time less than the full-cycle time of the signal, and hence is more attractive. It is shown that the Fourier filters used for the removal of DC offset etc. can also be interpreted as the weighted moving-average filters. Moreover, the Fourier filters, although having similarity with the discrete Fourier transform (DFT) computation of a signal, do not precisely involve the computation of the DFT coefficients of the sliding windowed sampled signal. Lastly, simulation results are presented to validate the approach of both the proposed algorithms under noiseless as well as noisy conditions.

Key-Words: -Discrete Fourier transform, Power system protection, Power system transients, Power transmission lines

1 Introduction

The current and voltage signals in a power transmission line under fault conditions is essentially a non-periodic non-sinusoidal signal, which contain DC offset, subsynchronous resonance and several harmonic terms of the fundamental frequency [1]-[5]. Filtering of the DC offset, subsynchronous resonance term and the harmonic terms form the current and voltage signals under fault conditions and estimating the fundamental frequency component is of prime importance in the operation of digital distance relays used for the protection of the power transmission lines [1]-[5].

The accuracy and time required for the estimation of the fundamental frequency component are also very critical parameters for the proper functioning of various protective devices (such as digital distance relays) and consequent protection of several power system components. For instance, the power companies around the world demand that the time elapsed between the occurrence of a fault and the operation of the digital distance relays should not exceed the time corresponding to one-and-a-half cycle of the power line fundamental frequency [1].

Many elegant algorithms have been proposed in the literature for filtering the DC offset, subsynchronous resonance term and harmonics of the fundamental frequency from the total current and voltage signals under fault conditions [1]-[2].

These algorithms, popularly termed as Fourier filters, are based on full-cycle or half-cycle discrete Fourier transform (DFT) of the current and voltage signals and require the sampling of the signals in a time interval exceeding one complete cycle period of the fundamental component. Excellent comparison of performance of the Fourier and Walsh filters has also been done in [3].

In this paper two novel filtering algorithms for filtering the DC offset and harmonics of the fundamental frequency from the total current and voltage signals are proposed.

One of the proposed algorithms utilizes the samples of the signal over a time interval slightly greater than one complete cycle of the fundamental component but is computationally less demanding than the existing Fourier filters and hence, can be more useful in digital distance relaying applications.

The second algorithm is based on sampling the signal at arbitrary locations and hence the samples can be acquired during a time interval much less than the full cycle period of the fundamental component. The calculation of the fundamental component in the second method can therefore be done quickly even before the completion of one full cycle period of the signal, and hence is more suitable in practical applications keeping in mind the fact that the time elapsed between the occurrence of a fault and the operation of the digital distance relays should not exceed the time corresponding to one and a half cycle of the power line fundamental frequency [1]-[2].

It is also shown that the Fourier filters existing in the literature [1]-[5] for the removal of DC offset in current and voltage signals can also be interpreted as the weighted moving-average filters. These Fourier filters, although having similarity with the discrete Fourier transform (DFT) computation of a signal, do not precisely involve the computation of the DFT coefficients of the sliding windowed discrete-time sampled signal. Lastly, we present the simulation results to validate the proposed algorithms both under noisy and noiseless conditions.

The rest of the paper is organized as follows. In section II, we present an algorithm which requires sampling of the current and voltage signals in a time interval slightly exceeding one complete cycle period of the fundamental component T_0. In section III, we describe the second algorithm based on sampling the signal at arbitrary locations in a time interval much less than the full-cycle period of the fundamental component. Discussion related to the Fourier filters is presented in section IV. Simulation results are presented in section V. The paper is concluded in section VI.

2 A novel algorithm for DC offset removal under fault conditions

The current/voltage signals under fault conditions contain DC offset, subsynchronous resonance term and many harmonic and transient terms. The fault current /voltage signals have been analyzed by considering the presence of different types of transient signals in them [1]-[2].

To be consistent with the notations existing in the literature, the continuous-time current/voltage signal under fault conditions, denoted as $z_a(t)$, is assumed to contain $N' = (N_0 - 2)$ harmonics of the fundamental frequency $\Omega_0 = 2\pi f_0 = 2\pi / T_0$ (f_0 is generally 50 or 60 Hz in most of the countries of the world) along with an exponentially decaying factor. Hence, it can be expressed as [1, Eqn. 10]

$$z_a(t) = A_0 + \sum_{n=1}^{N_0-2} A_n \cos(n\Omega_0 t + \theta_n) + Ae^{-t/\tau}, \qquad (1)$$

where A_0 denotes the DC offset term and A_n and $\theta_n,\ n = 1, 2, ..., N_0 - 2$ denote the amplitude and phase of the $n\,th$ harmonic frequency component respectively, τ represents the time-constant of the circuit. This signal $z_a(t)$ is uniformly sampled with time interval $\Delta T = T_0 / N$. Therefore the total number of samples in one cycle period T_0 will be N. It may be noted here that in general, $N \neq N_0$ although, in a special case N can be taken equal to N_0.

The sampled signal $z(k)$ can, therefore, be expressed as

$$\begin{aligned} z(k) &= z_a(k\Delta T) \\ &= A_0 + \sum_{n=1}^{N_0-2} A_n \cos(nk\pi / M + \theta_n) + Ae^{-k\Delta T/\tau}, \forall k \in \mathbb{N} \end{aligned}$$

, (2)

where $M = N/2$. Using the fact that $\cos(n\pi / M) = \cos(n\pi(N+1)/M)$, it can be easily shown that

$z(N+1)-z(1)=A\left(e^{-N\Delta T/\tau}-1\right)e^{-\Delta T/\tau}$, and

$$z(N+2)-z(2)=A\left(e^{-N\Delta T/\tau}-1\right)e^{-2\Delta T/\tau}. \quad (3)$$

From (3), we obtain the relation

$$e^{-\Delta T/\tau}=\frac{z(N+2)-z(2)}{z(N+1)-z(1)}=U. \quad (4)$$

Substituting the value of $e^{-\Delta T/\tau}$ from (4) in (3), one can obtain the value of A as given by

$$A=\frac{z(N+1)-z(1)}{U(U^N-1)}=\frac{z(N+2)-z(2)}{U^2(U^N-1)}. \quad (5)$$

We also observe that

$$\sum_{k=0}^{N-1}\cos(nk\pi/M+\theta_n)=0 \text{ for } n=1,2,...,N_0-2. \quad (6)$$

Therefore the value of the A_0 can be obtained by finding the sum of the signal samples $z(k)$ in (2) over N samples as given by

$$\sum_{k=0}^{N-1}z(k)=NA_0+A(1-e^{-N\Delta T/\tau})/(1-e^{-\Delta T/\tau}). \quad (7)$$

Hence for $k>N+2$, we can obtain a signal $x(k)$ expressed as $x(k)=z(k)-A_0-Ae^{-k\Delta T/\tau}$, which is free from the DC offset and exponential terms.

It can be noted that (4), (5) and (7) do not involve the multiplication of signal samples with the cosine or sine terms and hence, are computationally less demanding as compared to the corresponding expressions of the Fourier filters [1]-[2] existing in the literature. However, this algorithm also utilizes the samples of the signal over a time interval that is slightly greater than one complete cycle of the fundamental component similar to the algorithms presented in [1] and [2].

It may also be mentioned here that there is no direct relation between the total number of harmonics present in the signal and the total number of samples taken in one complete cycle time T_0 of the signal as is generally assumed in the literature [1]-[5]. In other words, $N\neq N_0$ in general, although, in a special case N can be taken equal to N_0.

The amplitude of the fundamental component of the signal A_1 in (1) can then be obtained (by recognizing that other harmonic term will not contribute to summation over a period of N) as follows:

$A_1\cos\theta_1=\frac{1}{M}\sum_{r=1}^{N}x(r)\cos(r\pi/M)$, and

$$A_1\sin\theta_1=\frac{-1}{M}\sum_{r=1}^{N}x(r)\sin(r\pi/M). \quad (8)$$

The above algorithm can be easily extended for the cases where fault current or voltage signals contain multiple transient terms or subsynchronous resonance terms as discussed here. For such cases the fault current and voltage signals can be modeled using the expression [1]

$$z_a(t)=A_0+\sum_{n=1}^{N_0-2}A_n\cos(n\Omega_0 t+\theta_n)+Ae^{-t/\tau}\cos(\tilde{\Omega}t+\phi), \quad (9)$$

which implies that the sampled signal can be expressed as

$$z(k)=z_a(k\Delta T)=A_0+\sum_{n=1}^{N_0-2}A_n\cos(nk\pi/M+\theta_n)+Ae^{-k\Delta T/\tau}\cos(\tilde{\Omega}k\Delta T+\phi),\forall k\in\mathbb{N}. \quad (10)$$

Using (10) and the fact that $\cos(n\pi/M)=\cos(n\pi(N+1)/M)$, it can be easily shown that

$$z(N+1)-z(1)=A\left(e^{-N\Delta T/\tau}\cos\left[\tilde{\omega}(N+1)+\phi\right]-\cos\left(\tilde{\omega}+\phi\right)\right)e^{-\Delta T/\tau}=r_1, \quad (11)$$

$$z(N+2)-z(2)=A\left(e^{-N\Delta T/\tau}\cos\left[\tilde{\omega}(N+2)+\phi\right]-\cos\left(2\tilde{\omega}+\phi\right)\right)e^{-2\Delta T/\tau}=r_2, \quad (12)$$

$$
\begin{aligned}
&z(N+3)-z(3) \\
&= A\left(e^{-N\Delta T/\tau}\cos\left[\tilde{\omega}(N+3)+\phi\right]-\cos\left(3\tilde{\omega}+\phi\right)\right)e^{-3\Delta T/\tau} \\
&= r_3
\end{aligned}
$$

, (13)

$$
\begin{aligned}
&z(N+4)-z(4) \\
&= A\left(e^{-N\Delta T/\tau}\cos\left[\tilde{\omega}(N+4)+\phi\right]-\cos\left(4\tilde{\omega}+\phi\right)\right)e^{-4\Delta T/\tau} \\
&= r_4
\end{aligned}
$$

, (14)

where $\tilde{\omega}=\tilde{\Omega}\Delta T$.

It can be observed that (11) to (14) are similar to (20) to (23) of [1] but these do not involve the multiplication of signal samples with the cosine or sine terms and hence are computationally less demanding as compared to the Fourier filters [1] existing in the literature. The procedure for finding the values of $e^{-\Delta T/\tau}$, $\tilde{\omega}$ from (11) to (14) is similar to that followed in [1]. Specifically,

$$e^{-\Delta T/\tau}=\sqrt{\frac{(r_3)^2-r_2r_4}{(r_2)^2-r_1r_3}}=X\ , \tag{15}$$

$$\cos\tilde{\omega}=\left(\frac{r_1r_4-r_2r_3}{-2e^{-\Delta T/\tau}\left[(r_2)^2-r_1r_3\right]}\right), \tag{16}$$

From (11), we can also write,

$$\frac{z(2)-z(1)}{z(3)-z(1)}\triangleq\alpha=\frac{X\cos(2\tilde{\omega}+\phi)-\cos(\tilde{\omega}+\phi)}{X^2\cos(2\tilde{\omega}+\phi)-\cos(\tilde{\omega}+\phi)}$$

$$\tan(\tilde{\omega}+\phi)=\frac{X\cos\tilde{\omega}+\alpha-1-\alpha X^2\cos 2\tilde{\omega}}{X\sin\tilde{\omega}-\alpha X^2\sin 2\tilde{\omega}}$$

$$A=r_1/\left(Xe^{-N\Delta T/\tau}\cos\left[\tilde{\omega}(N+1)+\phi\right]-X\cos\left(\tilde{\omega}+\phi\right)\right)$$

Hence for $k>N+4$, we can obtain a signal $x(k)$ as given by

$x(k)=z(k)-A_0-Ae^{-k\Delta T/\tau}\cos(\tilde{\Omega}k\Delta T+\phi)$,

which is free from the DC offset and subsynchronous resonance terms. The value of the fundamental component of the signal in (9) can then be obtained using (8).

3 Algorithm based on sampling at arbitrary locations

The algorithm presented in this section is based on sampling the signal at arbitrary locations and hence, the signal samples can be acquired during a time interval much less than the full-cycle period of the fundamental component. The calculation of the fundamental component can, therefore, be done quickly even before the completion of one full-cycle period of the signal, and hence is more suitable in practical applications keeping in mind the stringent requirements related to the time elapsed between the occurrence of a fault and the operation of the digital distance relays as discussed in the introduction here.

Here, the signal $z_a(t)$ expressed in (1) is sampled at a set of known time instants $t_{0,}\ t_1,\ldots,\ t_{J-1}$ (not necessarily uniformly spaced) allowing us to form a set of simultaneous linear equations, which can be solved for all the unknown quantities including the fundamental component of the signal $z_a(t)$. The system of simultaneous linear equations obtained by sampling the signal $z_a(t)$ can be compactly written as a matrix product

$$\underline{Z}=\underline{R}\ \underline{P}\ , \tag{17}$$

where

$$\underline{Z}=\begin{pmatrix}z_a(t_0) & z_a(t_1) & \cdots\ z_a(t_{J-1}) & z_a(t_J)\end{pmatrix}^T$$

with superscript T standing for transposition of the matrix,

$$\underline{R}=\begin{pmatrix}
1\ \cos\Omega_0t_0 & \sin\Omega_0t_0 & \cos 2\Omega_0t_0 & \sin 2\Omega_0t_0 & \cdots & \cos N'\Omega_0t_0 & \sin N'\Omega_0t_0 & e^{-t_0/\tau} \\
1\ \cos\Omega_0t_1 & \sin\Omega_0t_1 & \cos 2\Omega_0t_1 & \sin 2\Omega_0t_1 & \cdots & \cos N'\Omega_0t_1 & \sin N'\Omega_0t_1 & e^{-t_1/\tau} \\
1\ \cos\Omega_0t_2 & \sin\Omega_0t_2 & \cos 2\Omega_0t_2 & \sin 2\Omega_0t_2 & \cdots & \cos N'\Omega_0t_2 & \sin N'\Omega_0t_2 & e^{-t_2/\tau} \\
\vdots & \vdots & & & \vdots & & & \\
1\ \cos\Omega_0t_{J-1} & \sin\Omega_0t_{J-1} & \cos 2\Omega_0t_{J-1} & \sin 2\Omega_0t_{J-1} & \cdots & \cos N'\Omega_0t_{J-1} & \sin N'\Omega_0t_{J-1} & e^{-t_{J-1}/\tau} \\
1\ \cos\Omega_0t_J & \sin\Omega_0t_J & \cos 2\Omega_0t_J & \sin 2\Omega_0t_J & \cdots & \cos N'\Omega_0t_J & \sin N'\Omega_0t_J & e^{-t_J/\tau}
\end{pmatrix}$$

, (18)

$$\underline{P}=\begin{pmatrix}A_0 & A_1\cos\theta_1 & \text{-}A_1\sin\theta_1 & A_2\cos\theta_2 & \text{-}A_2\sin\theta_2 & \cdots & A_{N'}\cos\theta_{N'} & \text{-}A_{N'}\sin\theta_{N'} & A\end{pmatrix}^T$$

, (19)

and $N'=N_0-2$ with $J=2N_0-3$. Therefore, the matrix $\underline{P}$ can be written as $\underline{P}=\underline{R}^{-1}\underline{Z}$.

It must be emphasized that the matrix $\underline{R}^{-1}$ can be computed in advance and the multiplication of its second and third rows with the observation vector $\underline{Z}$ will give us the unknown quantity of interest, i.e., $A_1 \cos\theta$ and $-A_1 \sin\theta_1$. Given the maximum frequency of samplers available today in MHz or even higher range, the time required for taking sufficient number of samples and the multiplication of rows with the observation vector described above will not exceed half-cycle time of 50 /60 Hz signal.

4 Discussion on the Fourier filters

In this section we point out that the Fourier filters existing in the literature for removal of DC offset in current and voltage signals can also be interpreted as the weighted moving-average (WMA) filters.

The Fourier filters, although having similarity with the discrete Fourier transform (DFT) computation of a signal, do not precisely involve the computation of the DFT coefficients of the sliding windowed sampled signal, as discussed below.

Moreover, there is no direct relation between the highest order of the harmonic term present in the signal given in (1), i.e., $(N_0 - 2)$ and the total point N in the full-cycle or half-cycle DFT computation, as is generally assumed in the literature [1]-[4]. Let us consider the real and imaginary parts of the DFT as defined in [1]-[4]:

$$Z_{real(k)} = \frac{1}{M} \sum_{r=k-N+1}^{k} z(r)\cos(r\pi / M), \qquad (20)$$

and

$$Z_{imag(k)} = \frac{-1}{M} \sum_{r=k-N+1}^{k} z(r)\sin(r\pi / M), \qquad (21)$$

where $z(0) = z(-1) = z(-2) = ... = z(-N+1) = 0$.

On the other hand, an M -point moving-average filter operating on a signal $z(k)$ produces the output $y(k)$ as defined by [7]

$$y(k) = \frac{1}{M}\sum_{r=0}^{M-1} z(k-r) = \frac{1}{M}\sum_{r=k-M+1}^{k} z(r), \qquad (22)$$

It is clear from (20) to (22) that the sequences $Z_{real(k)}$ and $Z_{imag(k)}$ can also be interpreted as twice the output of a N -point moving-average filter with input signals $z(k)\cos(k\pi / M)$ and $z(k)\sin(k\pi / M)$ respectively. Hence, the filtering operation as defined in (20) and (21) can be interpreted as the weighted moving-average filtering [7] of the signal $z(k)$.

Now we consider the standard definition of the real and imaginary part of the N -point DFT as given by [6]-[7]

$$Z_{real}(k) = \sum_{r=0}^{N-1} z(r)\cos(r\pi k / M), k = 0,1,2,..., N-1$$

, and

$$Z_{imag}(k) = -\sum_{r=0}^{N-1} z(r)\sin(r\pi k / M), k = 0,1,2,..., N-1$$

. (23)

It may be noted from (20) to (23) that $Z_{real(k)} \neq Z_{real}(k)$ and $Z_{imag(k)} \neq Z_{imag}(k)$ for $k = 0,1,2,..., N-2$. But for $k = N-1$, we can write the relations $Z_{real(k)} = Z_{real}(k)/M$ and $Z_{imag(k)} = Z_{imag}(k)/M$. Similarly, using (20) for $k = N$, we obtain

$$Z_{real(N)} = \frac{1}{M}\sum_{r=1}^{N} z(r)\cos(r\pi / M), \qquad (24)$$

which is equal to the real part of the N -point DFT, as defined in (23), of the signal $\{z(N), z(1), z(2), ..., z(N-1)\}$ (within a constant $1/M$) and not the N -point DFT of the signal $\{z(0), z(1), z(2), ..., z(N-1)\}$.

Similarly, $Z_{real(N+1)} = \frac{1}{M}\sum_{r=2}^{N+1} z(r)\cos(r\pi / M)$ is the real part of the N -point DFT of the signal $\{z(N), z(N+1), z(2), ..., z(N-1)\}$.

It is clear from this discussion that for $k \geq N$, the formulae $A_1 = \sqrt{Z_{real(k)}^2 + Z_{imag(k)}^2}$ and $\tan\theta_1 = Z_{imag(k)} / Z_{real(k)}$ as given in (5) and (6) of [1] and (8) in [2] are not precisely the amplitude and phase terms corresponding to the fundamental component of the signal given in (1).

In the light of above, it may be noted in passing that

(13), (14) and (15) of [1] need correction although (16) and (17) are correct. The correct versions of (13) to (15) of [1] are as given by

$$Z_{real(N)} = \frac{1}{M}\sum_{r=1}^{N} z(r)\cos(r\pi/M),$$

$$Z_{real(N+1)} = \frac{1}{M}\sum_{r=2}^{N+1} z(r)\cos(r\pi/M)$$

$$= Z_{real(N)} + \frac{1}{M}\left[z(N+1) - z(1)\right]\cos(\pi/M),$$

$$Z_{real(N+2)} = \frac{1}{M}\sum_{r=3}^{N+2} z(r)\cos(r\pi/M), \qquad (25)$$

where we have used the fact that $\cos(\pi/M) = \cos(\pi(N+1)/M)$. Substituting the value of $z(N+1)$ and $z(1)$ from (2) in (23) and simplifying we obtain (16) to (17) of [1]. Similar remark will hold for Type II and Type III signals discussed in [1]. It may lastly be noted from (1) that $z_a(0) \neq 0$ as required in (7) of [1].

In summary, we can treat the Fourier filters as WMA filters and we should take cautions while interpreting the output of the Fourier filters for $k \geq N$ as amplitude and phase of the fundamental component term of the fault current /voltage signals.

5 Simulation results

The simulations of the proposed algorithms are carried out in MATLAB using the analog signal as given by

$$z_a(t) = A_0 + \sum_{n=1}^{N_0-2} A_n \cos(n\Omega_0 t + \theta_n) + Ae^{-t/\tau}, \quad (26)$$

with signal parameters $A_0 = 100$, $N_0 = 12$, $A = 200$, $\Omega_0 = 100\pi$, $A_n = 300/n$ and $\theta_n = 5n\pi/180$. It is sampled at a rate of twelve samples per cycle time of 50 Hz signal. The sampled signal can be written as

$$z(k) = 100 + \sum_{n=1}^{N_0-2} \frac{300}{n}\cos(nk\pi/6 + 5n\pi/180) + 200e^{-k/80},$$

$$\forall k \in \mathbb{N} \qquad (27)$$

The signal in (27) is plotted in Fig. 1.

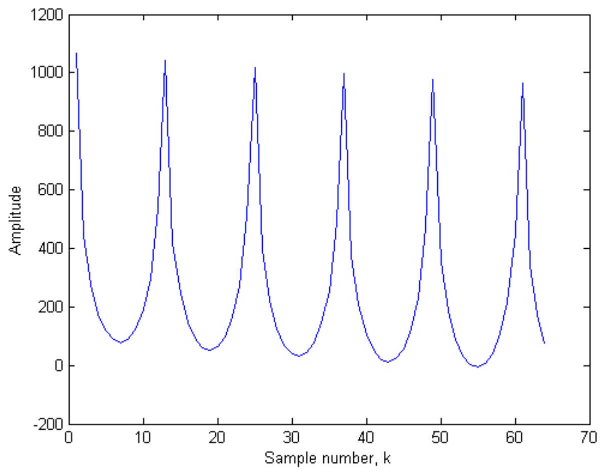

Fig. 1. Signal $z(k)$

A comment regarding the choice of sampling rate is in order here. The sampling frequency must be at least twice the highest frequency present in the input signal, if we want to reconstruct the signal back from the sampled signal. Here our goal is to estimate the amplitude and phase of the fundamental frequency component of the signal and eliminate the harmonics of it and hence the sampling frequency can be safely chosen to satisfy the condition

$$\Omega_s = \frac{2\pi}{\Delta T} \geq 2\Omega_0, \qquad (28)$$

where we have neglected the frequency spectrum of the term $Ae^{-k\Delta T/\tau}$ in (26) beyond $2\Omega_0$, which is reasonably a good assumption.

Using the samples of the signal in (27), the simulations are performed for the algorithm presented in section II and it is observed that the estimated values of the DC term A_0, amplitude of the exponentially decaying term A and the amplitude of the fundamental component of the signal A_1 using the proposed technique are exactly equal to their respective true values for $N_0 \leq 12$

under noise free case.

The simulation results for $N_0 > 12$ are also presented in Fig. 2. It can be noted from it that the estimated values of the signal parameters are very close to their true values even up to $N_0 = 100$ in (27).

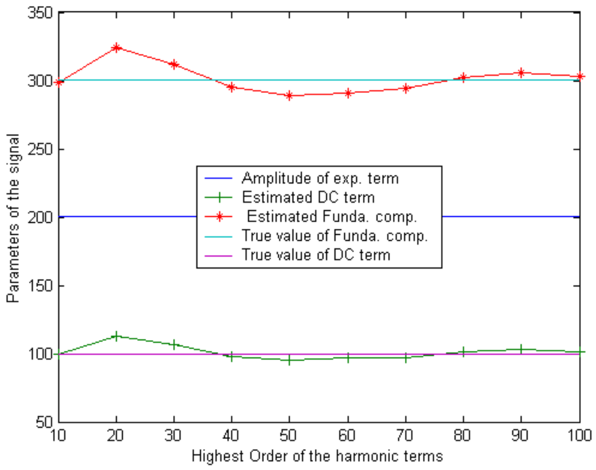

Fig. 2. Estimated parameters of the signal $z(k)$ using proposed algorithm as a function of the order of the harmonics N_0.

The simulation results for $N_0 = 4$ using the solution of simultaneous linear equation presented in (17) also give true estimate of the signal parameters that are exactly equal to their respective values under noise free case. To study the effect of robustness of the algorithms, a white Gaussian noise signal was added in (27) for different values of signal-to-noise ratio (SNR).

The results of simulations performed under additive-white Gaussian Noise (AWGN) condition for both the algorithms are shown in Fig. 3. It can be seen from it that the performance of both the algorithms is almost same under large SNR condition but for SNR below 20dB, the performance of the simultaneous linear equation method is inferior to that of the first method. However, it is common for the power transmission systems to have an SNR much greater than 20dB and hence both the approaches are useful.

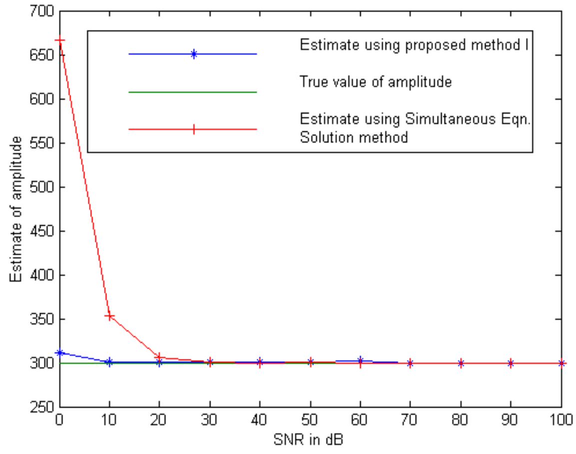

Fig. 3. Estimated amplitude of the fundamental component of the signal $z(k)$ using the proposed algorithm in section II and simultaneous equation solution method as a function of SNR in dB for $N_0 = 4$.

The simulations are also performed for the proposed algorithm presented in section III using the signal

$$z_{1a}(t) = A_0 + \sum_{n=1}^{N_0-2} A_n \cos(n\Omega_0 t + \theta_n) + Ae^{-t/\tau}\cos(\tilde{\Omega}t + \phi)' \quad (29)$$

with signal parameters $A_0 = 100$, $A = 200$, $\Omega_0 = 100\pi$, $A_n = 300/n$ and $\theta_n = 5n\pi/180$, $\tilde{\Omega} = 500\pi$ rad/sec, $\phi = 0$, $\tau = 33.33$ ms for various values of N_0 in (21) using a fixed matrix $\underline{R}$ (computed for $N' = 2$). The signal in (29) is sampled at a rate of twelve samples per cycle time of 50 Hz signal. The sampled signal thus obtained can be written as

$$z(k) = 100 + \sum_{n=1}^{N_0-2} \frac{300}{n}\cos(nk\pi/6 + 5n\pi/180) + 200e^{-k/300}\cos(5k\pi/6), \forall k \in \mathbb{N} \quad (30)$$

The results of the simulations are presented below in Table 1.

Table 1
Estimated amplitude of the DC and harmonic component of the signal $z(k)$ using the proposed algorithm II using matrix $\underline{R}$ having a fixed value of $N' = 2$.

N_0	3	4	5	6
Amplitude of the first Harmonic	300	300	393.4605	393.4605
Amplitude of the second Harmonic	4.4939e-014	150	147.9695	119.0349
DC term	100	100	-4.7103e04	-4.7103e04

It is clear from the results presented in Table 1 that the estimated values of the signal parameters are equal to their true values for the case upto $N_0 = 4$, but for larger values of N_0 the results are not close to their true values. This is because the entries of the matrix in (10) are dependent on the order of the harmonic and the estimated results are poor if the true value of the order of the harmonics present in the signal exceeds the order assumed in the entries of the matrix in (10).
A comment regarding the computational cost of each algorithm is in order here. The method presented in section 2 involving (5) and (7) requires of the order of N number of additions and multiplications for computing the value of $x(k)$ from $z(k)$. The method discussed in section 3 requires of the order of N^2 number of additions and multiplications for computing the value of $x(k)$ from $z(k)$.

6 Conclusions

Two novel filtering algorithms for the removal of DC offset, subsynchronous resonance terms in the current and voltage signals under fault condition are presented. Both the algorithms are computationally less demanding than the existing Fourier filters but the second algorithm based on sampling the signal at arbitrary locations is faster than the other algorithms and hence is more attractive. It is also shown that the Fourier filters existing in the literature for the removal of DC offset in current and voltage signals are essentially weighted moving-average filters. Moreover, the Fourier filters although looking similar to the discrete Fourier transform (DFT) computation of a signal, do not precisely involve the computation of the DFT coefficients of the sliding windowed sampled signal. Lastly, it is clear from the simulation results that the performance of the proposed algorithms is almost identical under noiseless as well as noisy conditions having SNR greater than 20dB.

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