

Influx of random variables in the Unit Commitment problem

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Abstract: - With rapid strides in the realm of power restructuring, the Unit commitment problem (UCP) has transformed itself into a dysfunctional amalgam of several variables relating to power system operation; commercial ones of the system operators and stochastic parameters corresponding to non-deterministic behavior of power equipment and accessories. The older set of UCP formulations has undergone sea changes in scope and enormity. Present ones have expanded to include indeterminate real life situations that require several probability measures. Corresponding solution techniques that address these amplified functions also need to be fitter, robust and versatile to generate acceptable and realistic optimal solutions. While there are classes of solutions galore, the state-space method is amenable to the inclusion of stochastic variables for generating effective UCP solutions. This paper attempts to graft random processes into the state space analysis of a generating block using a suitable selective state merger method employing appropriate transition rates. This helps to arrive at a reliability index that can vet a good UCP solution.

Key Words- Unit Commitment, Reliability constraints, State space, State transition, State merger, Generator scheduling, Markov chains, Ergodic theorem

1. Introduction

A typical generation model consists of several generating units of varying hierarchy and ratings. The units are interconnected to assign a specified power rating to a block. Several such blocks cohere to form a local grid, often classified according to the geography of the region. The (UCP) is a major factor in the daily operation and planning of a power system. The objective of Unit commitment problem UCP is to determine the optimal set of generating units to be in service during each interval of the scheduling period (a day or a week ahead), to meet system demand and reserve requirements at minimal

production cost, subject to satisfying a large set of operating constraints [1]. The solution of the UC problem is a complex optimization problem involving both discrete and continuous variables. Generation levels for each feasible solution can be obtained by the economic dispatch procedure. While exhaustive enumeration of all feasible combinations of generating units will yield an ideal solution, it is often impractical in real time computing.

Research efforts have concentrated on efficient, suboptimal UC algorithms that can be applied realistically to power systems. Over the years, several classes of solutions have been applied to the solution of the UCP. This has ranged from the crude priority list method on the one hand to svelte soft

computing techniques covering Genetic Algorithms (GA), Artificial Neural Networks (ANN), Fuzzy logic (FL), Particle Swarm Optimization (PSO) and a whole battery of hybrid variants [1],[2],[3],[4]. However, the major constraint that comes across is the recurrence of random variables on account of the non-deterministic behavior of generating models. This manifests itself in the form of failure rates, repair rates, consequent probability distribution functions, cumulative distribution functions among others [4]. Since the associated probabilistic models do not lend themselves well to inclusive computation, it has been the norm to use simplistic probabilistic models, which is detrimental to accuracy of a good UCP solution [5].

Current work focuses on stochastic decomposition, multiband robustness [4], approximate dynamic programming for state-spaces [6], clustering techniques [8]. It is in this context that the state-space approach is adapted to the application of relevant probabilistic factors due to the very nature of probability of transition between states.

2. Problem Statement

The overall objective function of the UCP of N generating units for a scheduling time horizon T is given by [1],[9].

$$\text{Min } \sum_{i=1}^N \sum_{t=1}^T [U_{it} F_{it}(P_{it}) + V_{it} S_{it}] \quad (1)$$

U_{it} = Units status variable

P_{it} = Output power of unit i at time t.

$F_{it}(P_{it})$ = Production cost of a committed unit i (considered in quadratic form)

V_{it} = Start-up/shut-down variable

S_{it} = Start-up/shutdown variable (function of the down time of unit i)

PD_t = System peak demand at hour t (MW)

subject to the primary constraints

(a) Load demand constraints:

$$\sum_i^N U_{it} P_{it} = PD_t \quad (2)$$

(b) Spinning reserve:

Spinning reserve, R_t is the total amount of generation capacity specified by the system operator, when load demand rises excessively, from all synchronized (spinning)

units to meet any abnormal operating conditions.

$$\sum_i^N U_{it} P_{max,i} \gg (PD_t + R_t) \quad (3)$$

(c) Generation limits

$$U_{it} P_{min,i} \ll P_{it} \ll U_{it} P_{max,i} \quad (4)$$

where $P_{min,i}$ and $P_{max,i}$ are the extreme generation limits of unit i, respectively. There are secondary constraints relating to minimum up/down time, Unit initial status, Crew constraints, Unit availability, Unit de-rating among others. Unit Commitment (UC) strategies involve

- Number and magnitude of the de-rated states
- Transition between these de-rated states

It is the aim of this paper to segregate those states which fall within a desired band of power ratings and merge them on a selective basis.

3. The State Space Alternative

In the state-space method of reliability evaluation, a system is described by its states and by the possible transitions between them [7],[9]. A system state represents a particular condition where every component (among many) is in some given operating state of its own; working, de-rated, failed, in maintenance, or in some other condition of relevance. If the state of any of the components changes, (on account of performance, system or environment) the system enters another state. All of the possible states of a system make up the state space. The state space diagram illustrates the state space and the transition between states. A typical state space diagram depicting two independent components A and B is shown in Fig.1.

The state-space approach is characterized by the easy applicability of a Markov model for a repairable system (wherein the transitions can close in on themselves) subject to the condition that the probabilities of transition from any state to any other should not depend on the system states that were occupied earlier in the process [10],[11].

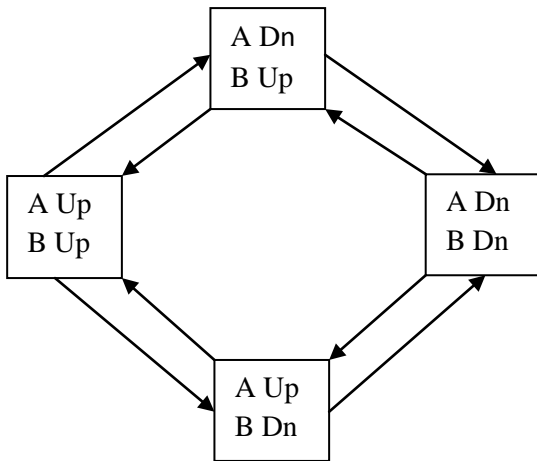


Fig.1 State space diagram - Two independent components

This is reflected by the intensity of transition, which is defined given by

$$q_{ij}(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} P[X(t + \Delta t) = j | X(t) = i] \quad (5)$$

Where $X(t)$ is the random variable representing the system state t ; similarly for $X(t+\Delta t)$. In cases where the transition intensities are time-independent, the preferred term is λ_{ij} , the expected no. of transitions from state i to state j per unit time spent in i .

$$P[X(t + \Delta t) = j | X(t) = i] \approx \lambda_{ij} \Delta t \text{ for small } \Delta t \quad (6)$$

If the events that cause change of system states (on account of failures, repairs, etc.) have exponential distributions, then the transition rate is also constant. Such processes which are based on constant transition rates are homogenous Markov process. To solve the state probabilities, the matrix differential equation

$\frac{dp(t)}{dt} = p(t)A$ must be solved, where $p(t)$ is a row vector consisting of the elements

$$\frac{dp_1(t)}{dt}, \frac{dp_2(t)}{dt}, \dots$$

$p(t)$ is a row vector consisting of the elements

$$p_1(t), p_2(t), \dots$$

A is the transition intensity matrix with the elements

$$a_{ij} = \lambda_{ij} \text{ for } i \neq j \text{ and } a_{ii} = -\sum_{j \neq i} \lambda_{ij} \quad (7)$$

The elements in each row of matrix A always add up to zero. If only the long-term

(steady state) probabilities are of interest, they can be obtained by the simpler task of solving the set of linear equations

$$pA = 0 \quad (8)$$

In such a case, an additional equation

$$\sum_i p_i = 1 \quad (9)$$

is required, asserting the fact that the probabilities of all possible states add up to zero.

4.State Frequency and Duration

The frequency of encountering a state i , f_i is defined as the expected number of arrivals into, or departures from i per unit time, computed over a long period. In order to relate the frequency, probability, and mean duration of a given system state, the history of a given system state will be regarded as consisting of two alternating periods, the stays in i , T_i and the stays outside i , \hat{T}_i [10]. Thus the system is represented by a two-state process whose state-space diagram is represented by Fig.2.

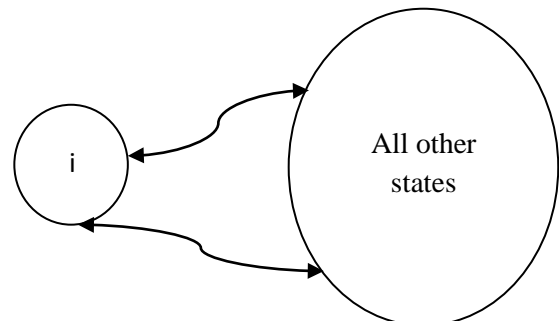


Fig.2 Two-state process

The mean cycle time, T_{ci}

$$= T_i + \hat{T}_i \quad (10)$$

In the long run, the state frequency, f_i equals the reciprocal of the mean cycle time.

$$f_i = \frac{1}{T_{ci}}$$

Since

$$p_i = \frac{T_i}{T_{ci}}, \text{ it follows that } f_i = \frac{p_i}{T_i}, \quad (11)$$

Next, a link between the frequencies f_i mean durations T_i , and the system transition rate will be attempted.

To begin with, the concept of the frequency of transfer from state i to state j is introduced.

- f_{ij} = expected no. of direct transfers from i to j per unit time = frequency of encountering state i
- T_i = duration of stay in i
- λ_{ij} = transition from state i to j
- p_i = prob. of stay in i

$$f_{ij} = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} P[(X(t + \Delta t) = j) \cap (X(t) = i)] \quad (12)$$

$$= \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} P[X(t + \Delta t) = j | X(t) = i] P[X(t) = i] \quad (13)$$

$$f_{ij} = \lambda_{ij} p_i \quad (14)$$

The transition rate is a conditional frequency, the condition being that the system resides in i . It follows that

$$f_i = \sum_{j \neq i} f_{ij} \quad (15)$$

Using (14),

$$f_i = p_i \sum_{j \neq i} \lambda_{ji} \quad (16)$$

Using (11),

$$T_i = \frac{1}{\sum_{i \neq j} \lambda_{ij}} \quad (17)$$

5.State Merger

In several applications, the solution of the state-space model for long term state probabilities can be simplified if certain sets of states are combined to form single states. However, in such cases, information on the transitions within the combined states will disappear in the solution. Hence these combinations are justified only if this risk is justified [7], [11]. By virtue of merger of states, a new process is generated with several new states (combined states) and new transitions (between the combined states) being created. In most cases, the new processes are not Markovian, since the durations of stays in the combined stays are not exponentially distributed.

To ensure that the new processes continue to be Markovian, with constant transition rates, lumpability (or mergeability) conditions have to be satisfied. A group of states can be merged, if the transition rate to

any other state or group of lumped states is the same from each state in the group. Consider Fig.3 where a number of states j are combined into a single state J .

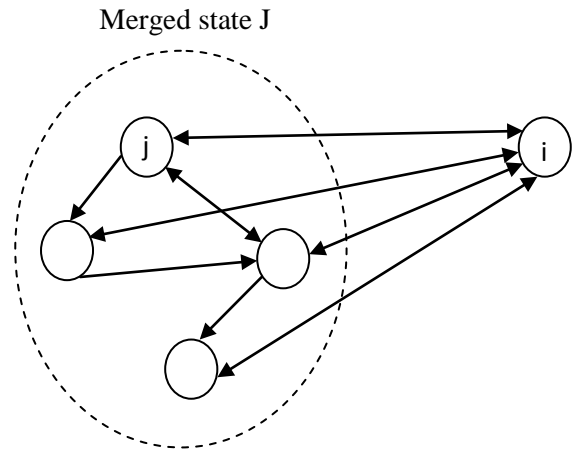


Fig.3 Combining states j into J

The probability of J , p_j is obtained by

$$p_j = \sum_{j \in J} p_j \quad (18)$$

The probabilities can be added up because the events of being in any of the states j are mutually exclusive. The frequency of J , f_j is the total of the frequencies of leaving a state j for a state I outside J , and therefore

$$f_j = \sum_{i \notin J} \sum_{j \in J} f_{ij} \quad (19)$$

Using (14),

$$f_j = \sum_{i \notin J} \sum_{j \in J} p_j \lambda_{ji} \quad (20)$$

$$f_j = \sum_{j \in J} (p_j \sum_{i \notin J} \lambda_{ji}) \quad (21)$$

For a direct solution of the state space obtained after combining the states j , the transition rates λ_{ji} and λ_{ij} are required. These rates are determined on the basis that the frequency of transfers from I to J must be the same as that from I to all the states j before their combination; similarly for J to i . Again resorting to (14),

$$p_i \lambda_{iJ} = p_i \sum_{j \in J} \lambda_{ij} \quad (22)$$

$$p_j \lambda_{ji} = \sum_{j \in J} p_j \lambda_{ji} \quad (23)$$

Thus

$$\lambda_{ij} = \sum_{j \in J} \lambda_{ij} \quad (24)$$

$$\lambda_{ji} = \frac{\sum_{j \in J} p_j \lambda_{ji}}{\sum_{j \in J} p_j} \quad (25)$$

For mergeability to be satisfied, λ_{ji} are the same for all j . In such a case, Eqn. 27 reduces to

$$\lambda_{ji} = \lambda_{ji} \text{ for all } j \in J \quad (26)$$

This can be generalized for the derivation of the transition rates between two combined states I and J. Such a case is represented in Fig. 4.

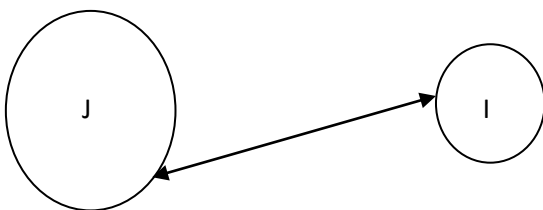


Fig.4 State merger

$$\lambda_{IJ} = \frac{\sum_{i \in I} p_i \sum_{j \in J} \lambda_{ij}}{\sum_{i \in I} p_i} \quad (27)$$

$$\lambda_{JI} = \frac{\sum_{j \in J} p_j \sum_{i \in J} \lambda_{ji}}{\sum_{j \in J} p_j} \quad (28)$$

If the conditions of mergeability are satisfied,

$$\lambda_{IJ} = \sum_{j \in J} \lambda_{ij} \quad i \in I$$

and $\lambda_{JI} = \sum_{j \in J} \lambda_{ji} \quad j \in J$ (29)

However, Eqn. 29 will not be applied, since the state probabilities are different, the formation of each state depending on the failure indices of a combination of generating units.

6. Case Study

A generating block with its base details are specified in Table-1. The data is representative of an Independent Power Producer employing hydel power. A constant-failure rate (CFR) model has been chosen [10]. A well-known characteristic of the CFR model, is that the time to failure of a component is independent on how long it has been operating. This property is consistent

with the completely random and independent nature of the failure process.

Table-1 Generating Block Data

1	2	3	4	5	6	7	8
Unit No.	Rating (MW)	Reliability	Failures/Year	λ Failure rate (Per year)	Repair period (days)	Days for Hot-start	μ Repair rate (Per Yr.)
1	1	0.98	1.0	0.003	5	1	0.167
2	2	0.97	0.5	0.001	6	1	0.143
3	5	0.95	0.6	0.002	9	2	0.091
4	10	0.93	1.1	0.003	14	3	0.059
5	10	0.91	0.9	0.002	15	3	0.056

It should be noted that Col.5 is derived from Col.4; Col.8 from Col.5 & 6. The no. of states S is given by the combinatorial figure [9].

$$S = C_0^5 + C_1^5 + C_2^5 + C_3^5 + C_4^5 + C_5^5 = 1 + 5 + 10 + 10 + 5 + 1 = 32$$

The state transition diagram is enumerated in Fig. 5. It displays six tiers, each corresponding to a set of states.

- Tier-1 : 1 state
- Tier-2 : 5 states
- Tier-3 : 10 states
- Tier-4 : 10 states
- Tier-5 : 5 states
- Tier-6 : 1 state

The unit codes have 5-bit string lengths, indicating the concatenation of the operation status of each of the 5 units. The tiers and their corresponding states were so chosen on a Boolean basis of the associated unit codes, such that transition occurs between only those states of adjacent tiers. Computation was performed on the unit codes, by assigning a 5-bit Gray code to each of the maximum possible 32 codes. However, when several units are involved, manual enumeration of tier ordering is difficult. This is obviated by the use of a MATLAB R.8.1 code developed by the authors. An increase in the number of units will not affect the proposed solution. In such a case, the planner will have a wider variety of selective merger possibilities.

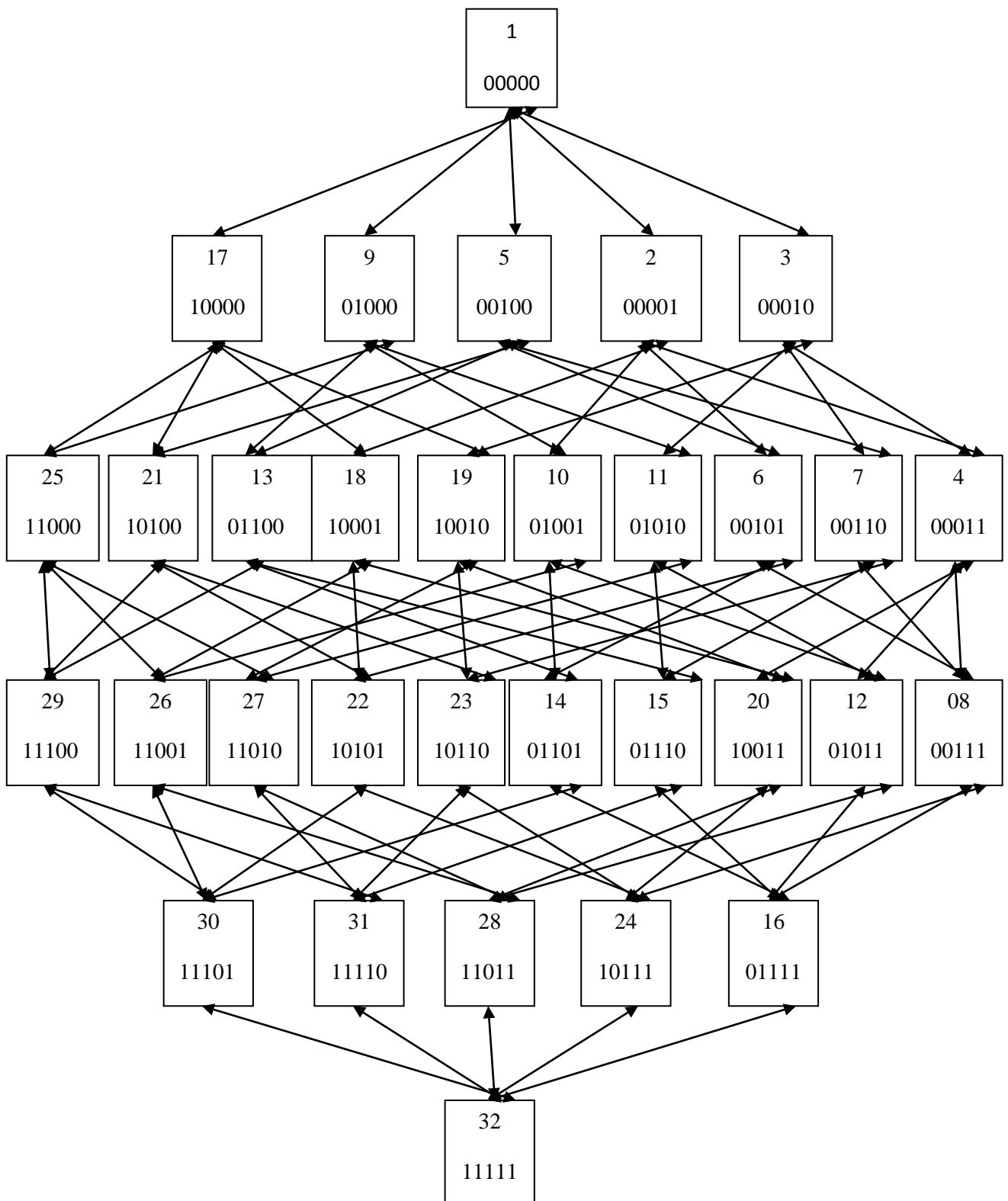


Fig.5 State transition - Tier wise

These are enumerated in Table-2, where the derated MW values have been sorted in ascending order.

Table - 2 State Details

S.No	State No.	No. of Units Up	Units Up	Derated MW	Code
1	1	0	-	0	00000
2	17	1	1	1	10000
3	9	1	2	2	01000
4	25	2	1,2	3	11000
5	5	1	3	5	00100
6	21	2	1,3	6	10100
7	13	2	2,3	7	01100
8	29	3	1,2,3	8	11100
9	2	1	5	10	00001
10	3	1	4		00010
11	18	2	1,5	11	10001
12	19	2	1,3		10010
13	10	2	2,5	12	01001
14	11	2	2,4		01010
15	26	3	1,2,5	13	11001
16	27	3	1,2,4		11010
17	6	2	3,5	15	00101
18	7	2	3,4		00110
19	22	3	1,2,5	16	10101
20	23	3	1,3,4		10110
21	14	3	2,3,5	17	01101
22	15	3	2,3,4		01110
23	30	4	1,2,3,5	18	11101
24	31	4	1,2,3,4		11110
25	4	2	4,5	20	00011
26	20	3	1,4,5	21	10011
27	12	3	2,4,5	22	01011
28	28	4	1,2,4,5	23	11011
29	8	3	3,4,5	25	00111
30	24	4	1,3,4,5	26	10111
31	16	4	2,3,4,5	27	01111
32	32	5	1,2,3,4,5	28	11111

For instance, S.No.30 reflects State - 24 when Units 1, 3, 4, 5 are active with a combined generation of 26 MW, the corresponding unit operation code being 10111. Each of these 32 states has an associated state probability, as listed in Table - 3. The probabilities have been determined using the data in Table - 1.

Table-3 State Probabilities

State	State prob.
1	0.000000189
2	0.000001911
3	0.000002511
4	0.000025389
5	0.000003591
6	0.000036309
7	0.000047709
8	0.000482391
9	0.000006111
10	0.000061789
11	0.000081189
12	0.000820911
13	0.000116109
14	0.001173991
15	0.001542591
16	0.015597309
17	0.000009261
18	0.000093639
19	0.000123039
20	0.001244061
21	0.000175959
22	0.001779141
23	0.002337741
24	0.023637159
25	0.000299439
26	0.003027661
27	0.003978261
28	0.040224639
29	0.005689341
30	0.057525559
31	0.075586959
32	0.764268141

Table - 4 segregates the entries of Table - 2, sorting being performed on the basis of the number of units being in the Up condition [9]. As an added measure, the cumulative probabilities for each block are added.

Table-4 State Details-Ordered

State	No. of Units Up	Derated MW	State prob.	Cum. Prob.
1	0	0	0.00000	0.00000
17	1	1	0.00001	0.00002
9		2	0.00001	
5		5	0.00000	
2		10	0.00000	
3		10	0.00000	
25	2	3	0.00030	0.00002
21		6	0.00018	
13		7	0.00012	
18		11	0.00009	
19		11	0.00012	
10		12	0.00006	
11		12	0.00008	
6		15	0.00004	
7		15	0.00005	
4		20	0.00003	
29	3	8	0.00569	0.02208
26		13	0.00303	
27		13	0.00398	
22		16	0.00178	
23		16	0.00234	
14		17	0.00117	
15		17	0.00154	
20		21	0.00124	
12	22	0.00082		
8	25	0.00048		
30	4	18	0.05753	0.21257
31		18	0.07559	
28		23	0.04022	
24		26	0.02364	
16	5	27	0.01560	0.76427
32		28	0.76427	
			1.00000	0.99896

There are 6 tiers corresponding to 1, 5, 10, 10, 5, 1 states with 0, 1, 2, 3, 4, 5 individual units respectively in an Up condition. This transition is depicted in Fig. 6. The specific transition rates have been derived using Eqns. 24, 25, 26, 27 and 28.

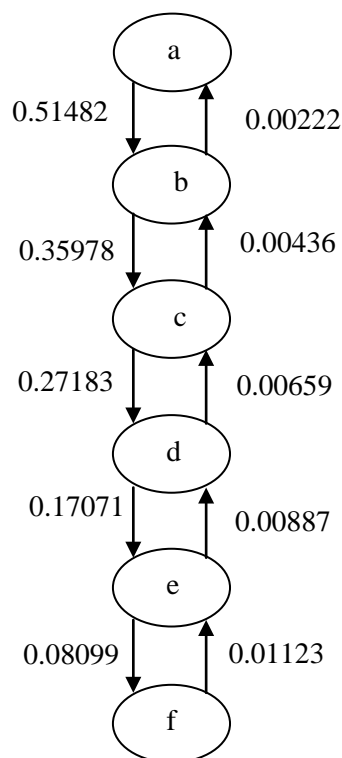


Fig.6 State merger with transition rates (per year basis)

The merged states a, b, c, d, e, f are detailed below.

- State a \equiv Tier-1 \equiv Merger of states with 0 units Up
~ State No.s 1
- State b \equiv Tier-2 \equiv Merger of states with 1 units Up
~ State No.s 7,9,5,2,3
- State c \equiv Tier-3 \equiv Merger of states with 2 units Up
~ State No.s 25, 21, 13,18,19,10,11,6, 7,4
- State d \equiv Tier-4 \equiv Merger of states with 3 units Up
~ State No.s 29,26,27, 22,23,14,15,20,12 ,8
- State e \equiv Tier-5 \equiv Merger of states with 4 units Up
~ State No.s 30,31,28, 24,16
- State f \equiv Tier-5 \equiv Merger of states with 5 units Up
~ State No. 32

A statistical summary of the above merger is listed in Table-5.

Table-5 Statistical Details of Merger

Merger	State	Units Up	Derated MW	Mean MW	σ
a	1	0	0	0.000	0.000
b	17,9,5,2,3	1	1,2,5,10	4.500	3.500
c	25,21,13,18,19,10,11,6,7,4	2	3,6,7,11,12,15,20	10.570	5.368
d	29,26,27,22,23,14,15,20,12,8	3	8,13,16,17,21,22,25	17.429	5.368
e	30,31,28,24,16	4	18,23,26,27	23.500	3.500
f	32	5	28	28.000	0.000

It can be seen that the mean derated MW ranges from 4.5 to 28 MW. The first entry does not qualify since it corresponds to a zero generation case. A striking observation is that the standard deviation of each merged state has mirror symmetry about the central derated case. The numerical values correspond to 3.500 and 5.368 MW. While this may be a coincidence for the given generating block data, it provides a path for investigation into more complicated state mergers.

7. SELECTIVE MERGER

Initially, we had reduced the state transition diagram in Fig.5 to the one in Fig.6 with a 6-tier merger. In the event that a generalized merger is required, a more stringent procedure is called for. Applying the concepts of state merger gleaned from Section 5 and the related set of Eqns.(18) through (29), we are now in a position to segregate the set of states to any desired pattern.

As an illustration, a specific condition is set; one that will decide the nature of the merger. Consider a condition where a certain power generation band of 3 MW between 17 and 20 MW is arbitrarily selected. The transition diagram in Fig.5 has been demarcated into three newer tiers and is put up as Fig.7. Likewise, Table-2 has been redrawn as Table-6 with this intent.

Table-6 Selective Merger

S.No	State No.	No. of Units Up	Units Up	Derated MW	Code
1	1	0	-	0	00000
2	17	1	1	1	10000
3	9	1	2	2	01000
4	25	2	1,2	3	11000
5	5	1	3	5	00100
6	21	2	1,3	6	10100
7	13	2	2,3	7	01100
8	29	3	1,2,3	8	11100
9	2	1	5	10	00001
10	3	1	4		00010
11	18	2	1,5	11	10001
12	19	2	1,3		10010
13	10	2	2,5	12	01001
14	11	2	2,4		01010
15	26	3	1,2,5	13	11001
16	27	3	1,2,4		11010
17	6	2	3,5	15	00101
18	7	2	3,4		00110
19	22	3	1,2,5	16	10101
20	23	3	1,3,4		10110
21	14	3	2,3,5	17	01101
22	15	3	2,3,4		01110
23	30	4	1,2,3,5	18	11101
24	31	4	1,2,3,4		11110
25	4	2	4,5	20	00011
26	20	3	1,4,5	21	10011
27	12	3	2,4,5	22	01011
28	28	4	1,2,4,5	23	11011
29	8	3	3,4,5	25	00111
30	24	4	1,3,4,5	26	10111
31	16	4	2,3,4,5	27	01111
32	32	5	1,2,3,4,5	28	11111

The shaded part of Table-6 indicates that, the selected band consists of five states, comprising of states from the erstwhile Tiers-2,3 and 4. Thus, a selective merger of states is applied, as illustrated in Fig.7.

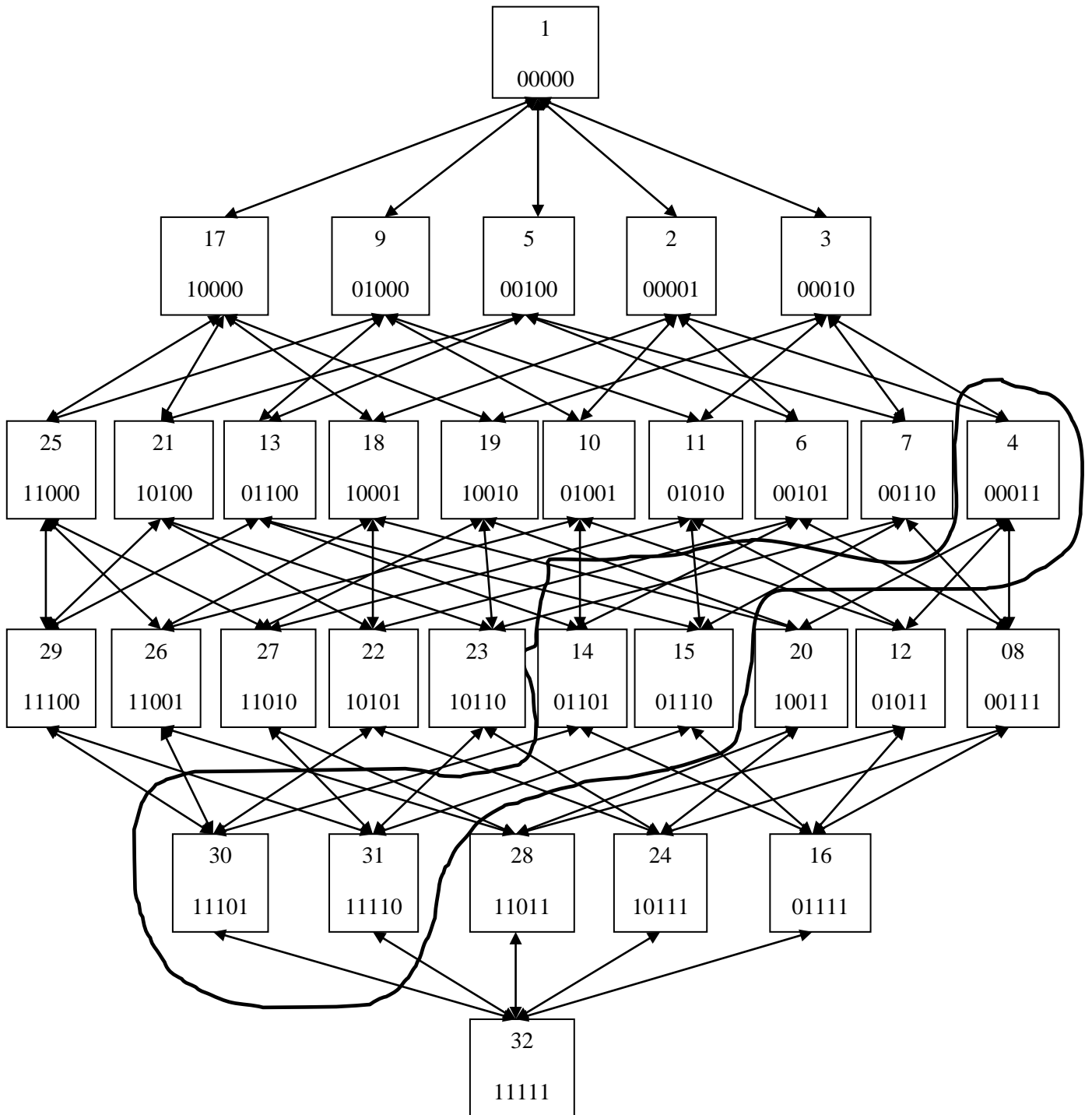


Fig.7 Selective merger

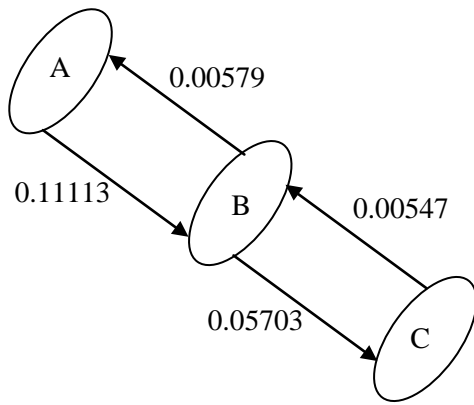


Fig.8 Selective merger with transition rates (per year basis)

The merged states A, B and C are detailed below.

- State A ≡ Tier-1 Merger of states with derated output 0-16 MW
State No.s 1,17,9,5,2,3,25,21,13,18,19,10,11,6,7,29,26,27,22,23
- State B ≡ Tier-2 Merger of states with required power band 17-20 MW
State No.s 4,14,15,30,31
- State C ≡ Tier-3 Merger of states with derated output 17-28 MW
State No.s 20,12,8,28,24,16,32

In this case, state B is the preferred state; it containing the required band. Transitions from state B to state C are preferred, the latter containing states having superior output ratings. However, transitions to state A moves the system to pessimistic power ratings. This is borne out by visual inspection of Table-6. The essential criteria in this case would be the net transition rate of State B and to a lesser extent, the expected duration of stay in the same state, T_B , which could lead to a frequency index too. The index T_B will serve as an effective reliability index, which can be used to rank UCP solutions.

$$T_B = \frac{1}{\sum(\text{Departure transition rates})} \quad (30)$$

$$T_B = \frac{1}{\lambda_{BA} + \lambda_{BC}} = \frac{1}{(0.00579 + 0.05703)} = 15.918 \text{ Years}$$

Where λ_{BA} = transition rate from state B to A,
 λ_{BC} = transition rate from state B to C.

8. CONCLUSIONS

The paper proposes a new method of selective state merger to assist a power grid generating pool in obtaining efficient unit commitment solutions. The frequency and duration approach has been used to lay the basis for state operations. When individual state transitions are considered, the probability of transition from any one state to another is characterized by the appropriate failure rate or repair rate. Basic probability theory has been adapted to lay the groundwork for state mergers. Selective mergers are considered by the system operator for a particular exigency. Since these mergers can be made to occur in a wide variety of ways, the specific exigency can force the system operator to create specific and selective mergers.

A case study for a five-unit system with simplified failure and repair data has been used to simulate the individual and cumulative probability figures. Subsequently selective state mergers have been carried out for a six-tier case (with an essential criteria corresponding to the number of operative generating units) and also for a three-tier case (with an essential criteria corresponding to a specific power band). In both these cases (as well as any other possible state merger), a definite action is required, leading to the probability of state modification being unity, the Markov chain to be decided appropriately. Since the tabling of the system UCP schedule is not the express purpose of this paper, a set of transition rates have been determined for both cases. These will assist the system operator in refining an optimal UCP schedule. The residence time in a particular state, T_B , is only indicative, though a seemingly preposterous one. Towards this end, it is expected that the use of a suitable ergodic theorem can complement the integration of the final transition rate into the selection of the final UCP schedule.

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