Computation of Time Delay Margins for Stability of a Single-Area Load Frequency Control System with Communication Delays

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Abstract: - Time delays have become unavoidable in power systems since communication links are extensively used for sending and receiving control signals. This paper investigates the effect of time delays on the stability of a single-area load Frequency Control (LFC) system. A direct and exact method to compute delay margins is presented. The delay margin is the maximum amount of the time delay that the system can tolerate before it becomes unstable for a given operating point.The proposed method starts with the determination of all possible purely imaginary characteristic roots for any positive time delay. To achieve this, Rekasius substitution is first used to convert the transcendental characteristic equation of the LFC system into a polynomial. Then, Routh stability criterion is applied to determine the critical root, the corresponding oscillation frequency and the delay margin for stability. For a wide range of controller gains, delay margins of LFC system are determined to find out the qualitative effect of controller gains on the delay margin. Finally, theoretical delay margin results are verified by using the time-domain simulation capabilities of Matlab/Simulink.

Key-Words: -Load Frequency Control System, Time Delay, Stability, Delay Margin

1 Introduction

With the extensive use of open communication infrastructure and phasor measurement units (PMU) in the wide-area measurement/monitoring systems (WAMS), time delays have become inevitable in electric power systems, and raise concerns about the system dynamic response [1, 2]. The total time delay consisting of measurement and communication delays in power systems has a destabilizing impact, reduces the effectiveness of control system damping and leads to unacceptable performance such as loss of synchronism and instability [3-6]. In this paper, we focus on the effect of time delays on the stability performance of a single-area LFC system. The main goal of the LFC system is to maintain a reasonably uniform frequency in an interconnected power system consisting of several pools [7].

Traditionally, dedicated communication links were used to sending and receiving control signals. For this reason, in stability analysis it was reasonable to neglect time delays associated with the communication network. However, communications delays significantly increase when an open and distributed communication network is used to send control signals [3, 4, 6, 26]. It was reported that communication delays in LFC systems can be in the range of 5-15 sec [6]. The size of communication delays mainly depends on the physical media of communication (such as fiber-optic-cables, digital microwave links, power lines, telephone lines and satellite links [1]) as well as several other factors including the phasor package size, transmission protocol employed and communication network load (congested or idle). As a result, these delays may fluctuate randomly in a certain range. Therefore, it is essential to estimate the maximum amount of time delay known as the delay margin that the system could tolerate without becoming unstable. Such knowledge on the delay margin (upper bound in the time delay) will be helpful in the controller design for cases where uncertainty in the delay is unavoidable.

There are mainly two types of theoretical methods to compute delay margins of time-delayed dynamical systems. The first group of methods is basically frequency domain approaches that aim to determine the critical eigenvalues or roots of the system. The common starting point of these direct methods is the determination of all the imaginary roots of the characteristic equation. The existing frequency domain procedures can be classified into the following five distinguishable approaches:

i) Schur-Cohn (Hermite matrix formation) [8-10]
ii) Elimination of transcendental terms in the characteristic equation [11]
iii) Matrix pencil, Kronecker sum method [8-10, 12]
iv) Kronecker multiplication and elementary transformation [13]
v) Rekasius substitution [14-16].

These methods demand numerical procedures of different complexity and they may result in different precisions in computing imaginary roots. A detailed comparison of these methods, demonstrating their strengths and weakness can be found in [17]. Among these methods, the method reported in [9] is the only method that has been effectively used to estimate the delay margin for automatic generation control systems with commensurate time delays [6]. The exact method based on Rekasius substitution presented in [15] has been applied only into small-signal stability analysis of power system to compute delay margins [18]. Finally, the method presented in [11] has been successfully applied to stability analysis of time-delayed generator excitation control system [19-21].

The second group of methods includes indirect time-domain methods based onLyapunov stability theory and linear matrix inequalities (LMIs) techniques [22]–[24] and has been applied to calculate the delay margin of the wide-area damping controller [25] and LFC system [26]. Even though indirect methods can deal with both constant and time-varying delays, they give conservative delay margin results. On the other hand, the frequency domain methods could give accurate delay margin results [19, 20].

This paper utilizes a frequency-domain approach based on Rekasius substitutionreported in [14-17] to compute the delay margins of LFC system. The proposed method first uses Rekasius substitution [14] to convert the transcendental characteristic equation of the LFC system into an algebraic polynomial, which is then analyzed relatively easily for cases with purely imaginary roots. This procedure does not use any approximation or transformation to eliminate the transcendentality of the characteristic equation. Therefore, it is exact and the purely imaginary roots of the new algebraic polynomial coincide with the purely imaginary roots of the transcendental characteristic equation exactly. As a result, this method reduces the stability problem effectively to one free of delay, which in turns requires calculating only imaginary roots of a single-variable polynomial. For this reason, Routh stability criterion is then used to determine the critical root, the corresponding oscillation frequency and the delay margin for stability.

Time delay margins are computed for a wide range of proportional and integral (PI) controller gains. The accuracy of theoretical delay margins is validated by using Matlab/Simulink to demonstrate the effectiveness of the proposed method. Finally, delay margins results are compared with those obtained by an indirect method based on Lyapunov stability theory [26]. It is observed that the proposed method gives more accurate delay margin results for LFC system.

2 Time-Delayed LFC System

The block diagram of a single-area LFC system is shown in Fig. 1. The conventional LFC model is modified to take into account the communication time delay into the control loop [4], [7], [26]. All generators are assumed to be equipped with a non-reheat turbine. The PI controller, which is the load frequency controllers used currently in industry, is included in the model.

The state space equation model of LFC system could be represented as follows [4, 7]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + F\Delta P_d \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

The state variables, inputs, outputs and system matrices are defined as

$$x(t) = \begin{bmatrix} \Delta f & \Delta P_m & \Delta P_v & \int ACE \end{bmatrix}^T$$

$$y(t) = \begin{bmatrix} ACE & \int ACE \end{bmatrix}^T$$

$$A = \begin{bmatrix} -\dfrac{D}{M} & \dfrac{1}{M} & 0 & 0 \\ 0 & -\dfrac{1}{T_{ch}} & \dfrac{1}{T_{ch}} & 0 \\ -\dfrac{1}{RT_g} & 0 & -\dfrac{1}{T_g} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & \dfrac{1}{T_g} & 0 \end{bmatrix}^T$$

$$F = \begin{bmatrix} -\dfrac{1}{M} & 0 & 0 & 0 \end{bmatrix}^T,$$

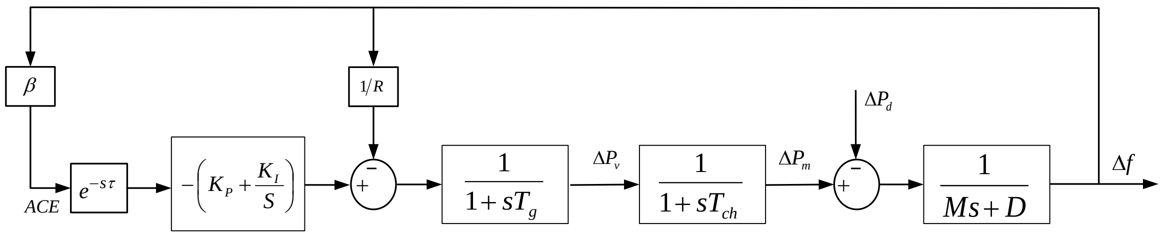

Fig. 1 Block diagram of a single-area LFC system with a communication delay

where, $\Delta f, \Delta P_m, \Delta P_v, \Delta P_d$ represent the deviation of frequency, the generator mechanical output, valve position, and load, respectively. ACE and $\int ACE$ denote the area control error and its integral. Finally, M, D, T_g, T_{ch} and R are the moment of inertia of the generator, generator damping coefficient, time constant of the governor, time constant of the turbine, and speed drop, respectively.

Since there is no net tie-line power exchange in the single-area LFC system, the area control error (ACE) is defined as

$$ACE = \beta \Delta f \tag{2}$$

where β represents frequency bias factor. In order to simplify the stability analysis, all the time delays including the communication delays for transmission of control signal between the control center and power plant, and the delay for transmission of ACE are lumped together and represented by an exponential block $e^{-s\tau}$ in Fig. 1 [4, 26]. In this case, the input of PI controller is the ACE signal and PI controller output is

$$\begin{aligned} u(t) &= -K_P ACE - K_I \int ACE \\ &= -Ky(t-\tau) = -KCx(t-\tau) \end{aligned} \tag{3}$$

where $K = \begin{bmatrix} K_P & K_I \end{bmatrix}$, K_P and K_I denote proportional and integral controller gains, respectively.

Substituting the input signal given in (3) into (1) results in the following closed-loop model of the LFC system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau) + F\Delta P_d \\ y(t) &= Cx(t) \end{aligned} \tag{4}$$

where

$$A_d = -BKC = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{K_P\beta}{T_g} & 0 & -\frac{K_I}{T_g} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

For stability analysis, it is necessary to obtain the characteristic equation of time-delayed LFC system. This could be easily achieved by using the following:

$$\begin{aligned} \Delta(s,\tau) &= \det\left[sI - A - A_d e^{s\tau} \right] \\ &= P(s) + Q(s)e^{-s\tau} = 0 \end{aligned} \tag{5}$$

where, τ is the total time delay, $P(s)$ and $Q(s)$ are polynomials in s with real coefficients given below:

$$\begin{aligned} P(s) &= p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s \\ Q(s) &= q_1 s + q_0 \end{aligned} \tag{6}$$

$$\begin{aligned} & p_4 = RT_g T_{ch} M, \\ & p_3 = MRT_{ch} + RDT_g T_{ch} + RT_g M, \\ & p_2 = MR + RDT_{ch} + RT_g D, \\ & p_1 = RD + 1,\ q_1 = \beta RK_P,\ q_0 = \beta RK_I \end{aligned} \tag{7}$$

The location of the roots of the characteristic equation of (5) must be determined to analyze the stability of LFC system with a time delay. In the following section, first, the stability problem is described and the stability analysis of the single-area LFC system using Rekasius substitution is presented.

3 Stability Analysis

3.1 Delay-Dependent Stability and Delay Margin

For stability analysis of the LFC system, it is essential to determine conditions on the delay such that the LFC system will be stable. The stability is characterized by roots of the characteristic equation defined by (5). Please note that there exists an exponential term $e^{-s\tau}$ in the characteristic equation, which makes the stability analysis complicated. The characteristic equation may have infinitely many roots because of the exponential term and thus, the computation of these roots becomes a difficult task. However, for stability analysis of the LFC system, there is no need to determine all roots. It is sufficient to evaluate the variation of roots of the system characteristic equation with respect to the time delay. As with the delay-free system (i.e., $\tau = 0$), the stability of the LFC system depends on the locations of the roots of system's characteristic equation defined by (5). It is obvious that these roots are a function of the time delay τ. As τ changes, location of some of the roots may change. For LFC system to be asymptotically stable, all the roots of the characteristic equation of (5) must lie in the left half of the complex plane. In other words

$$\Delta(s,\tau) \neq 0, \quad \forall s \in C^{+} \tag{8}$$

where C^{+} represents the right half plane of the complex plane.

Depending on system parameters, there are two different possible types of asymptotic stability situations due to the time delay τ [8]:

i) *Delay-independentstability*: The characteristic equation of (5) is said to be *delay-independent stable* if the stability condition of (8) holds for all positive and finite values of the delay, $\tau \in [0,\infty)$.

ii) *Delay-dependent stability*: The characteristic equation of (5) is said to be *delay-dependentstable* if the condition of (8) holds for some values of delays belonging in the delay interval, $\tau \in [0,\tau^{*})$ and is violated for other values of delay $\tau \geq \tau^{*}$.

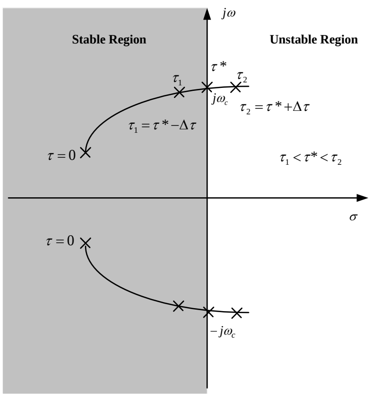

Fig. 2 Illustration of movement of characteristic equation roots with respect to the time delay

In the delay-dependent case, the roots of the characteristic equation move as the time delay τ increases starting from $\tau = 0$. Figure 2 illustrates the movement of the roots. Note that the delay–free system ($\tau = 0$) is assumed to be stable. This is a realistic assumption since for the practical values of system parameters the LFC system is stable when the total delay is neglected. Observe that as the time delay τ is increased, a pair of complex roots moves in the left half of the complex plane. For a finite value of $\tau > 0$, they cross the imaginary axis and pass to the right half plane. The time delay value τ^{*} at which the characteristic equation has purely imaginary roots is the upper bound on the delay size and is defined as the delay margin. The system will be stable for any given delay less or equal to this margin, $\tau \leq \tau^{*}$.

The following section presents the proposed method based on Rekasius substitutionthat allows us to evaluate the delay dependency of stability and enables us to compute delay margins for the delay-dependent case.

3.2 Computation of Delay Margin using Rekasius Substitution Method

A necessary and sufficient condition for the system to be asymptotically stable is that all the roots of the characteristic equation of the LFC system given in (5), lie in the left half of the complex plane. In the single delay case, the problem is to find values of τ^{*} for which the characteristic equation of (5) has roots (if any) on the imaginary axis of the *s-plane*. Clearly, $\Delta(s,\tau) = 0$ is an implicit function of *s* and τ which may, or may not, cross the imaginary axis.

Assume for simplicity that $\Delta(s,0) = 0$ has all its roots in the left half plane. That is, the delay-free system is stable. Observe that the characteristic equation of (5) has an exponential transcendentality feature because of the term $e^{-s\tau}$. This results in infinitely many finite roots, which makes the determination of the roots and delay margin a difficult task. However, this problem could be easily overcome by using an exact substitution for the transcendental term suggested by Rekasius [14]. This substitution is given as;

$$e^{-s\tau} = \frac{1-Ts}{1+Ts} \quad \tau \in \Re^{+}, \;\; T \in \Re \tag{9}$$

and is defined only for $s=j\omega_c$. It should be pointed out that the equation (9) is an exact substitution, not an approximation, when the characteristic equation of (5) has roots on the imaginary axis. Further, (9) gives the following mapping condition relating ω_c and T [14, 16]:

$$\tau^* = \frac{2}{\omega_c}\left[Tan^{-1}(\omega_c T) \pm \ell\pi\right] \quad \ell = 0,1,2,... \qquad (10)$$

This equation describes an asymmetric mapping in which one T is mapped into infinitely many τ^*'s for a given ω_c. Inversely, for the same ω_c, one particular τ^* corresponds to one T only. The substitution of (9) into (5) results in an augmented characteristic equation as;

$$\Delta(s,T) = a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 \quad (11)$$

where

$$\begin{aligned} &a_5 = Tp_4,\ a_4 = p_4 + Tp_3, \\ &a_3 = p_3 + Tp_2,\ a_2 = p_2 + T(p_1 - q_1), \quad (12) \\ &a_1 = p_1 + q_1 - q_0 T,\ \ a_0 = q_0 \end{aligned}$$

This method reduces the stability problem effectively to one free of delay, which in turns requires calculating only roots of a single-variable polynomial. It is obvious from (11) that after Rekasius substitution the system characteristic equation of (5) has become an ordinary polynomial whose coefficients are parameterized in T only. Note that $T \in \Re$, thus it can also be negative. It must be noted that the 4th order characteristic equation with delay given in (5) is now converted into a 5th order polynomial given in (11) without transcendentality. It is clear that these two equations $\Delta(s,\tau)$ and $\Delta(s,T)$ possess exactly the same imaginary roots and there is no correspondence between the remaining roots. Since these two equations have a perfect coincidence with respect to the imaginary roots, we prefer solving the simpler $\left\{(T_k,\omega_{ck}) \text{ for } \Delta(s,T)=0\right\}$ instead of solving $\left\{(\tau^*_{k\ell},\omega_{ck}) \text{ for } \Delta(s,\tau)=0\right\}$. The question is to determine all $T \in \Re$ values, which causes imaginary roots of $s = j\omega_c$ of the augmented characteristic equation $\Delta(s,T)=0$. For this purpose, Routh-Hurwitz criterion could be utilized. To determine the values of substitution parameter T, we need to form the Routh array based on (11) and set the only term $R_{11}(T)$ in the s^1 row to zero [15, 16, 27]. The Routh's array is obtained as;

$$\begin{array}{llll} s^5 & a_5 & a_3 & a_1 \\ s^4 & a_4 & a_2 & a_0 \\ s^3 & R_{31} & R_{32} & 0 \\ s^2 & R_{21} & R_{22} & 0 \\ s^1 & R_{11} & 0 & 0 \\ s^0 & R_{22} & 0 & 0 \end{array} \qquad (13)$$

where

$$\begin{aligned} &R_{31} = \frac{a_4 a_3 - a_5 a_2}{a_4},\ R_{32} = \frac{a_4 a_1 - a_5 a_0}{a_4},\ R_{22} = a_0 \\ &R_{21} = \frac{R_{31} a_2 - a_3 a_4}{R_{31}},\ R_{11} = \frac{R_{21} R_{32} - R_{31} R_{22}}{R_{21}} \end{aligned} \qquad (14)$$

By setting the term $R_{11}(T)$ in the s^1 row to zero we obtain the following 7th order polynomial of T as;

$$t_7 T^7 + t_6 T^6 + ... + t_1 T + t_0 = 0 \qquad (15)$$

The roots of this polynomial may easily be determined by standard methods. Depending on the roots of (15), the following situation may occur:

i) The polynomial of (15) does not have any real roots, which implies that the characteristic equation of (5) does not have any roots on the $j\omega$-axis. In that case, the system is stable for all $\tau \geq 0$, indicating that the system is *delay-independent stable*.

ii) The polynomial of (15) has at least one positive or negative real root, which implies that the characteristic equation of (5) has at least a pair of complex roots on the $j\omega$-axis. In that case, the system is *delay-dependent stable*.

The polynomial given by (15) may have at most seven real roots, $T_c = \{T_1, T_2, ..., T_7\}$. Once this set of real roots is determined, the corresponding crossing frequencies $s = \pm j\omega_c$ can be found using the auxiliary equation, which is formed by the s^2 row of the Routh's array. For a real $T_i \in T_c$ $i = 1,2,...,7$, the auxiliary equation is given as follows;

$$R_{21}(T_i)s^2 + R_{22}(T_i) = 0 \qquad (16)$$

It must be mentioned here that in order for (16) to yield imaginary roots $s = \pm j\omega_c$, the following additional sign agreement condition has to be satisfied also [15];

Table 1 Delay margin results obtained by the proposed method for various values K_P and K_I

$\tau^*(s)$	K_I						
K_P	0.05	0.1	0.15	**0.2**	0.4	**0.6**	1.0
0	30.9151	15.2014	9.9595	7.3354	3.3816	2.0421	0.9229
0.05	31.8750	15.6813	10.2794	7.5752	3.5014	2.1218	0.9704
0.1	32.7509	16.1192	10.5712	**7.7940**	3.6103	2.1938	1.0124
0.2	34.2258	16.8562	11.0621	8.1616	3.7922	2.3127	1.0785
0.4	35.8338	17.6579	11.5940	8.5578	3.9802	2.4255	1.1183
0.6	34.9216	17.1950	11.2776	8.3121	3.8260	**2.2811**	0.9474
1.0	0.5954	0.5857	0.5753	0.5643	0.5158	0.4634	0.3610

Table 2 Delay margin results obtained by the method [26] for various values K_P and K_I

$\tau^*(s)$	K_I						
K_P	0.05	0.1	0.15	**0.2**	0.4	**0.6**	1.0
0	27.927	13.778	9.056	6.692	3.124	1.910	0.886
0.05	27.874	14.061	9.284	6.866	3.215	1.974	0.927
0.1	27.038	13.682	9.220	**6.941**	3.290	2.029	0.963
0.2	25.114	12.760	8.617	6.535	3.320	2.108	1.016
0.4	20.364	10.426	7.065	5.384	2.832	1.912	1.017
0.6	14.618	7.477	5.1567	3.958	2.130	**1.475**	0.827
1.0	0.546	0.538	0.530	0.522	0.482	0.438	0.348

$$R_{21}R_{22} > 0 \tag{17}$$

Observe that the coefficient R_{21} is a function of $T_i \in T_c$ and R_{22} is a positive constant coefficient since $R_{22} = a_0 = q_0$. For this reason, the auxiliary equation will yield imaginary roots, for positive R_{21} only. For those $T_i \in T_c$ values, the crossing frequencies are obtained from (16) as;

$$\omega_c = \sqrt{\frac{R_{22}}{R_{21}(T_i)}} \tag{18}$$

Observe that we can determine at most seven different crossing frequencies $\{\omega_{c1}, \omega_{c2}, ..., \omega_{c7}\}$ corresponding to $T_c = \{T_1, T_2, ..., T_7\}$. Substituting ω_{ci} and T_i for $i = 1, 2, ..., 7$ into (10), we can further get the corresponding time delays $\{\tau_1^*, \tau_2^*, ..., \tau_7^*\}$. According to the definition of delay margin, the minimum of those time delays will be the system delay margin.

4 Results

In this section, for various values P and PI controller gains, delay margins are determined. The accuracy of theoretical delay margin results is confirmed by using Matlab/Simulink [28]. The LFC system parameters used in this paper is as follows [26]:

$$T_{ch} = 0.3\,s, T_g = 0.1\,s, R = 0.05,$$
$$D = 1.0, \beta = 21, M = 10\,s$$

4.1 Theoretical Results

First, we choose a typical PI controller gains $K_P = 0.6; K_I = 0.6\ s^{-1}$ to demonstrate the delay margin computation. The process of the delay margin computation consists of the following five steps:

Step 1: Determine the characteristic equation of time delayed LFC system using (5), (6) and (7). This equation is found to be:

$$\Delta(s,\tau) = (0.015s^4 + 0.2015s^3 + 0.52s^2 + 1.05s) + (0.63s + 0.63)e^{-s\tau} = 0$$

Step 2: Apply Rekasius substitution given by (9) into (5) in order to obtain the augmented characteristic equation given by (11) and (12). The coefficients of this equation are found to be

$a_5 = 0.015T;\ a_4 = 0.2015T + 0.015;$
$a_3 = 0.52T + 0.2015;\ a_2 = 0.42T + 0.52;$
$a_1 = -0.63T + 1.68;\ a_0 = 0.63$

Step 3: Compute elements of Routh table given by (14) and determine the values of T_c using (15). Only three roots among the seven roots of (15) are found to be real. All the roots are given below.

$T_1 = 1.61878;\ T_2 = 0.13167 + j0.48238;$
$T_3 = 0.13167 - j0.48238;\ T_4 = -0.09500;$
$T_5 = -0.07444;\ T_6 = -0.07444 + j0.000001;$
$T_7 = -0.07444 - j0.000001$

Step 4: Compute R_{21} for all real T values and check their sign. Note that for only $T_1 = 1.61878$, R_{21} is positive and its value is $R_{21} = 0.98072$. For this reason, the remaining real roots of T are not taken into account since they will not result in imaginary roots for the characteristic equation of the LFC system defined by (5) or equivalently the auxiliary equation given in (16).

Step 5: Compute the crossing frequency ω_c using (18) and the corresponding delay margin using (10). They are found to be $\omega_c = 0.80149$ rad/s and. $\tau^* = 2.2811$ s. This result indicates that the delay margin is about $\tau^* = 2.28112$ s for $K_P = 0.6$, $K_I = 0.6\ s^{-1}$. When time delay exceeds this value, the LFC system will become unstable.

Delay margin results for various values of PI controller gains (K_P, K_I) are summarized in Table 1 and shown in Fig. 3. Results indicate that the delay margin τ^* decreases when K_I is increased for fixed K_P values. As a result, it could be stated that the increase of K_I results in a less stable LFC system. The effect of K_P on the delay margin has two tendencies when K_I is fixed. The delay margin increases with the increase of K_P when K_P belongs to the interval of $K_P = 0 - 0.4$. On the other hand, the delay margin decreases with the increase in K_P for $K_P \geq 0.6$. This kind of effect of K_P on the delay margin has also been observed in the time-delayed excitation control system [20].

When delay margin (τ^*) results are compared with those of [26] presented in Table 2, it is observed that τ^* values are larger than those obtained by the method reported in [26]. The comparison clearly indicates that indirect time-domain methods based onLyapunov stability theory gives more conservative delay margin results. The conservative delay margin estimation of [26] could be clearly seen in Fig. 4 and 5 in which that delay margin results are compared for fixed values of $K_P = 0.2$ and $K_P = 0.6$ as K_I is changed from $K_I = 0.05$ to $K_I = 1$. Please note that delay margin results of [26] shown by the dashed line is always lower than those of the proposed method. Time-domain simulations given in the following section will validate the accuracy of delay margin results.

The effect of the Integral (I) controller only is also investigated when $K_P = 0$. Figure 6and Table 3 show delay margin results for $K_I = 0.05 - 1$ It is clear that the delay margin decreases with the increase of K_I. Moreover, delay margins are larger than those obtained by the method reported in [26].

Table 3 Delay margin results for various values K_I

K_I	$\tau^*(s)$	
	Proposed Method	Method of [26]
0.05	30.9151	27.9268
0.1	15.2014	13.7775
0.15	9.9595	9.0560
0.2	7.3354	6.6915
0.4	3.3816	3.1241
0.6	**2.0421**	**1.9104**
1.0	0.9229	0.8858

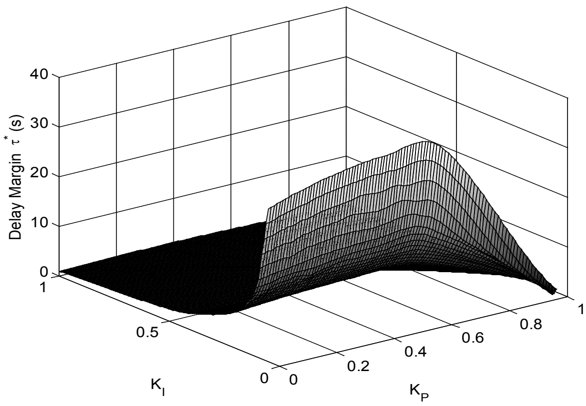

Fig. 3 Variation of the delay margin with respect to K_P and K_I

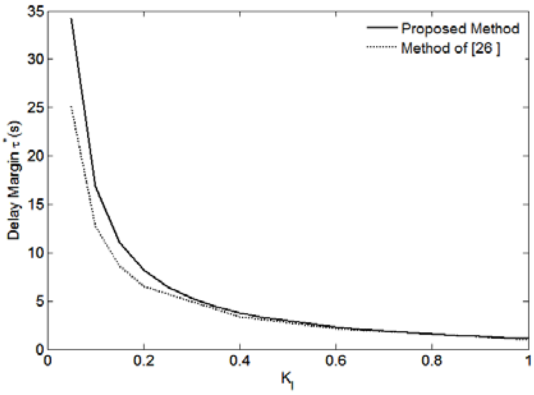

Fig. 4 Variation of the delay margin with respect to K_I ($K_P = 0.2$)

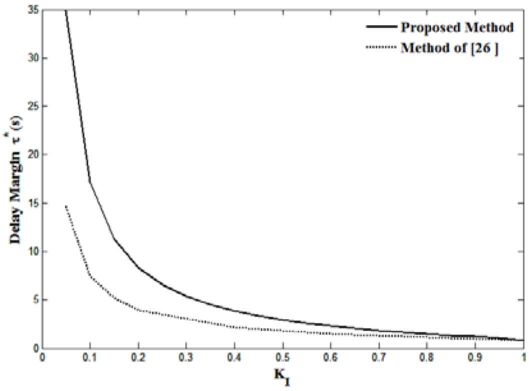

Fig. 5 Variation of the delay margin with respect to K_I ($K_P = 0.6$)

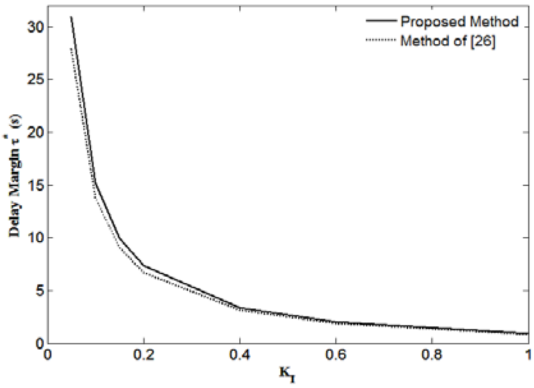

Fig. 6 Variation of the delay margin with respect to K_I ($K_P = 0$)

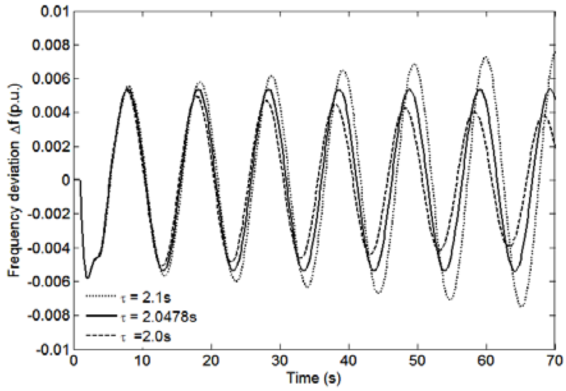

Fig. 7 Frequency deviation for different time delays ($K_P = 0, K_I = 0.6$)

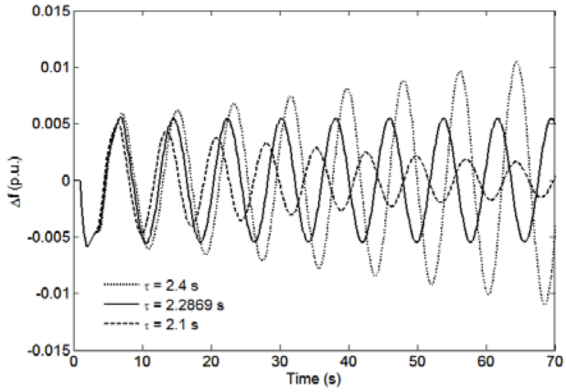

Fig. 8 Frequency deviation for different time delays ($K_P = 0.6, K_I = 0.6$)

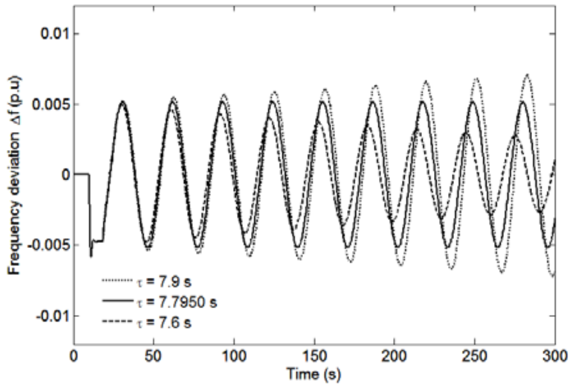

Fig. 9 Frequency deviation for different time delays ($K_P = 0.1, K_I = 0.2$)

4.1 Verification of Theoretical Results

The simulations are carried out using Matlab/Simulink to verify theoretical delay margin results. For illustrative purposes, three different sets of controller gains are chosen as: $(K_P = 0, K_I = 0.6)$ $(K_P = 0.6, K_I = 0.6)$ and $(K_P = 0.1, K_I = 0.2)$.

For $(K_P = 0, K_I = 0.6)$ controller gains, it is clear from Table 3 that the delay margin is computed as $\tau^* = 2.0421\ s$ by the proposed method and $\tau^* = 1.9104\ s$ by the method [26]. However, the delay margin obtained by using simulation is found to be $\tau^* = 2.0478\ s$. Figure 7 shows simulation results for $\tau^* = 2.0478\ s$ as well as for two other delay values $(\tau = 2.0\ s,\ \tau = 2.1\ s)$. The LFC system is marginally stable $\tau^* = 2.0478\ s$ since sustained oscillations are observed in the frequency deviation. When the time delay is less than $\tau^* = 2.0478\ s$, the LFC system is expected to be stable. Figure 7 also presents such a stable simulation result for $\tau = 2.0\ s$. On the other hand, when the time delay is larger than $\tau^* = 2.0478\ s$, the system becomes unstable since it has growing oscillations, as shown in Fig. 7 for $\tau = 2.1\ s$.

When the theoretical delay margins of the proposed methods ($\tau^* = 2.0421\ s$) and that of [26] ($\tau^* = 1.9104\ s$) are compared with the one determined by using simulation ($\tau^* = 2.0478\ s$), it could be concluded that the theoretical delay margin obtained by the proposed method is in close agreement with the simulation result. The relative percentage error observed in delay margin results are 0.278 % for the proposed method and 6.710 % for the method of [26], clearly indicating the accuracy the proposed method.

For ($K_P = 0.6, K_I = 0.6$) controller gain, Table 1 and 2 show that delay margins of the proposed method and method of [26] are $\tau^* = 2.2811\ s$ and $\tau^* = 1.475\ s$. On the other hand, the delay margin is found to be $\tau^* = 2.2869\ s$ by using the simulation approach. The relative percentage error between the theoretical delay margin result and simulation result is 0.254 % while it is 35.502 % for the method [26]. The low percentage error observed in the proposed method validates again its accuracy similar to the previous case. Figure 8 presents the frequency deviation for three different time delays, $\tau = 2.1\ s$, $\tau^* = 2.2869\ s$ and $\tau = 2.4\ s$, illustrating, stable, marginally stable and unstable cases, respectively.

Finally, simulation results are depicted in Fig. 9 for PI controller gains of $K_P = 0.1, K_I = 0.2$. In this case, from Table 1 and 2, the delay margins are $\tau^* = 7.7940\ s$ and $\tau^* = 6.9410\ s$ for the proposed method and method of [26], respectively while it is determined as $\tau^* = 7.7950\ s$ by the simulation. The frequency deviations are presented in Fig. 9 for $\tau = 7.6\ s$ (stable), $\tau^* = 7.7950\ s$ (marginally stable) and $\tau = 7.9\ s$ (unstable), which validates the accuracy of delay margins computed by the proposed method.

5 Conclusions

This paper has analyzed the stability of the single area LFC system that contains communication delays. A theoretical method based on Rekasius substitution has been proposed to compute the delay margins. The accuracy of delay margin results is proved using time domain simulation capabilities of Matlab/Simulink. It has been observed that the relative percentage error between the theoretical delay margin results and ones determined by simulation are negligible. Therefore, the proposed method is an effective method to estimate delay margins of LFC systems. Moreover, delay margin computation and simulation studies clearly indicate that the proposed method provides more accurate delay margins as compared to the methods based on Lyapunov stability and linear matrix inequality techniques. The stability analysis of multi-area LFC systems with delays is put in perspective as future work.

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