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The generalized quantum mechanics of Einstein «deinterlaced» photon and Casimir force

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Abstract: Based on Majorana equations the e.-m. field as initially quantum object having isomorphic representation as quantum field of «deinterlaced» photon is considered. The calculation of Casimir force magnitude interpreted as consequence of an energy measurement of the generalized quantum field of a «deinterlaced» photon in the state corresponding to a «Feynman path» element is given. A metallic mirrors here plays role of classic apparatus measuring energy of this field.

Key-Words: generalized quantum mechanics, generalized path integral, Casimir force, Feynman paths, quantum field of a photon.

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1 Introduction

As known (see [10]), the Casimir forces are called the forces of attraction that arise between neutral metallic mirrors at a small distance among them. These forces have the quantum-electrodynamic origin; despite the smallness of them, their dependence on distance between mirrors (inverse proportionality of 4th degree) has been experimentally determined. Here the e.-m. field in the article is understood as a fundamentally quantum object that has no classic limit.

Note. *Majorana equations of e.-m. field (see [1]) not contain Planck constant h as well as the usual Maxwell equations. However the photon spin is not* 1*, but is h, which is why there should be a multiplier h in the l.h.s. and r.h.s. in the Majorana equations; these equations turn out to be equivalent to the previous ones only when $h \neq 0$ (for $h = 0$ the Maxwell equations just disappear).*

Therefore the Maxwell equations describes a quantum object not dependencies on quantity $h \neq 0$ and therefore that has no classic analogue, so that all laws of classic e.-d. is interpreted as means of such quantum field. This fundamental property the quantumness of e.-m. field existing in two isomorphic forms representing physically radically different quantum field (see [4]). One of them (the «deinterlaced» photon field) is responsible for the origin of Casimir force.

In the article, Casimir force is interpreted as result of a energy macroscopic measurement of quantum e.-m. field of the «deinterlaced» photon corresponding to small element of «Feynman path» of the photon, in field of which a mirrors are placed, that are such a classic apparatus.

Since the considered problem is obviously one-dimensionally the generalized Green function of arbitrary e.-m. field in its usual and isomorphic forms is constructed and studied.

In the article uses terminology and notation of monographs [6, 7, 8]; in calculations, we assume the light velocity $c = 1$.

2 Generalized Green function of one-dimensional quantum e.-m. field as functional on bump functions

Recall that Maxwell equations of e.-m. field in Majorana variables [1] for photon $\xi_t(x) = E_t(x) + iH_t(x)$ and antiphoton $\bar{\xi}_t(x) = E_t(x) - iH_t(x)$ are become equations

$$\begin{aligned} i\tfrac{\partial}{\partial t}\xi_t(x) &= (S,\hat{p})\xi_t(x), \\ -i\tfrac{\partial}{\partial t}\bar{\xi}_t(x) &= (S,\hat{p})\bar{\xi}_t(x), \end{aligned} \tag{1}$$

where $\hat{p} = \frac{1}{i}\nabla$, and S is photon spin operators (infinitesimal operators of rotation around coordinate axes in representation corresponding to

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weight 1, see [5]).

$$S_1 = \begin{pmatrix} 0, & 0, & 0 \\ 0, & 0, & -i \\ 0, & i, & 0 \end{pmatrix}, S_2 = \begin{pmatrix} 0, & 0, & i \\ 0, & 0, & 0 \\ -i, & 0, & 0 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} 0, & -i, & 0 \\ i, & 0, & 0 \\ 0, & 0, & 0 \end{pmatrix}.$$

$\xi_t(x)$, $\bar{\xi}_t(x)$ are supposed independent, see [1], that is why the Green function of these equations is represented by direct product of 3×3 matrices and their solution is column of 6 elements. Therefore, for the momentum representation of these equations are as follows:

$$i\frac{\partial}{\partial t}\tilde{\xi}_t(p) = (S,p)\tilde{\xi}_t(p),$$

$$-i\frac{\partial}{\partial t}\bar{\tilde{\xi}}_t(p) = (S,p)\bar{\tilde{\xi}}_t(p).$$

We will consider a quantum e.-m. field corresponding to states of a photon on the z axis, in which the coordinates x, y of photon is any (and hence the momentum $p_x = p_y = 0$ corresponding to them).

The problem on the quantum e.-m. field of an antiphoton is solved similarly. Almost to the end of the article will be investigated only a photon.

Such e.-m. fields in momentum representation, it is easy to see, satisfies the Shrödinger equations

$$i\frac{\partial}{\partial t}\begin{pmatrix} \tilde{\xi}_t^{(1)}(p_z) \\ \tilde{\xi}_t^{(2)}(p_z) \end{pmatrix} = S_z p_z \begin{pmatrix} \tilde{\xi}_t^{(1)}(p_z) \\ \tilde{\xi}_t^{(2)}(p_z) \end{pmatrix},$$

$$\frac{\partial}{\partial t}\tilde{\xi}_t^{(3)}(p_z) = 0,$$

the last component is «frozen» and does not participate in evolution.

It is clear that a generalized Green function of this equation is

$$\tilde{\Xi}_t(p_z) = \exp\left(t\begin{pmatrix} 0, & -p_z \\ p_z, & 0 \end{pmatrix}\right).$$

Bearing in mind the constructing of the coordinate representation the generalized Green function of this field as functional on columns of bump functions $\varphi(z) \in K$ (as Fourier preimage of solution in the momentum representation as functional $\int \bar{\tilde{\Xi}}_t \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} dp_z$ on columns of analytic functions $\psi(p) \in Z$ (см. [6]). Here

$$\int \bar{\Xi}_t \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} dz = \frac{1}{2\pi}\int \bar{\tilde{\Xi}}_t \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} dp_z$$

Let us consider the momentum representation of generalized Green function of considered e.-m. field of a photon in details.

Remark that the momentum representation of the solution contains in power coefficient a Hermit matrix, therefore which can be reduced to diagonal view by unitary transform $\tilde{Q}(p_z)$. Therefore we have

$$\tilde{\Xi}_t(p_z) = \exp\left(-i\begin{pmatrix} 0, & -itp_z \\ itp_z, & 0 \end{pmatrix}\right) =$$
$$= \tilde{Q}^{+}(p_z)\begin{pmatrix} \exp(-it|p_z|), & 0 \\ 0, & \exp(it|p_z|) \end{pmatrix}\tilde{Q}(p_z), \quad (2)$$

where

$$\tilde{Q}(p_z) = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\,\mathrm{sgn}\,p_z, & 1 \\ i\,\mathrm{sgn}\,p_z, & 1 \end{pmatrix}.$$

But the Fourier preimages of numeric functionals $\exp(it|p_z|)$ and $\mathrm{sgn}\,p_z$ on Z are the quantum Cauchy functional (see [3, 2])

$$C_{it}(z) = \tfrac{1}{2}\left(\delta(t-z) + \delta(t+z)\right) + + \tfrac{i}{2\pi}\left(\tfrac{1}{t-z} + \tfrac{1}{t+z}\right) \quad (3)$$

and, accordingly, the functional $-\frac{i}{\pi z}$ (see [6], p. 360 formula 19) on bump functions $\varphi(z) \in K$.

Note. *Previously it was shown that quantum Cauchy process (see [2]) is the correct analytic continuation in time of the Cauchy process transition probability* $\frac{1}{\pi}\cdot\frac{t}{t^2+z^2}$ *(see [9]) from the real semiaxis to the imaginary axis. Remark that formula (3) can also be obtained by a simple calculation using improper integrals.*

Hence, it easy to see, that

$$\tilde{\Xi}_t(p_z) =$$
$$\frac{1}{2}\begin{pmatrix} e^{-it|p_z|} + e^{it|p_z|}, & \mathrm{sgn}(p_z)\cdot 2i\sin(t|p_z|) \\ -\mathrm{sgn}(p_z)\cdot 2i\sin(t|p_z|), & e^{it|p_z|} + e^{-it|p_z|} \end{pmatrix} =$$
$$= \begin{pmatrix} \cos(tp_z), & \sin(tp_z) \\ -\sin(tp_z), & \cos(tp_z) \end{pmatrix}.$$

Hence, it easy to see, that we have coordinate representation the generalized Green function $\int \bar{\Xi}_t \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} dz$ of Maxwell–Majorana equations, where

$$\Xi_t(z) =$$
$$= \frac{1}{2}\begin{pmatrix} \delta(t-z) + \delta(t+z), & i\left(\delta(t-z) - \delta(t+z)\right) \\ -i\left(\delta(t-z) - \delta(t+z)\right), & \delta(t-z) + \delta(t+z) \end{pmatrix}. \quad (4)$$

Thus the generalized Green function of the studied e.-m. field is concentrated in points $z = \pm t$, which are the wave front, describing the initial state evolution of the field concentrated in point $z = 0$.

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We shall return to relation (2) that has a physical interpretation.

Indeed, the matrix diagonalization underlying this equality is interpreted as cut-off the spin interaction between components of the considered generalized e.-m. field and the rise in result a new field with the momentum representation of the generalized Green function

$$\tilde{\breve{\Xi}}_t(p_z) = \begin{pmatrix} \exp(-it|p_z|), & 0 \\ 0, & \exp(it|p_z|) \end{pmatrix}$$

with «deinterlaced» components. Here the unitary operator $\tilde{Q}(p_z)$ realizes both «deinterlacing» and «interlacing» of the field components.

Moreover the coordinate representation the generalized Green function of this field with «deinterlaced» components is $\int \bar{\breve{\Xi}}_t \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} dz$ (see (4)), where

$$\breve{\Xi}_t(z) = \begin{pmatrix} C_{-it}(z), & 0 \\ 0, & C_{it}(z) \end{pmatrix}. \tag{5}$$

Let us compare those generalized quantum e.-m. fields with unitary equivalent Green functionals $\Xi_t(z)$ and $\breve{\Xi}_t(z)$. Since

$$\int \bar{\Xi}_t(z) \begin{pmatrix} \varphi_1(z) \\ \varphi_2(z) \end{pmatrix} dz = \\ = \frac{1}{2} \begin{pmatrix} \varphi_1(t) + \varphi_1(-t) + i\left(\varphi_2(t) - \varphi_2(-t)\right) \\ -i\left(\varphi_1(t) - \varphi_1(-t)\right) + \varphi_2(t) + \varphi_2(-t) \end{pmatrix},$$

then this functional is concentrated in points $z = \pm t$ of a quantum e.-m. field wave front. At the same time the functional

$$\int \bar{\breve{\Xi}}_t(z) \begin{pmatrix} \varphi_1(z) \\ \varphi_2(z) \end{pmatrix} dz = \\ = \frac{1}{2} \begin{pmatrix} \frac{\varphi_1(t)+\varphi_1(-t)}{2} + \frac{i}{2\pi} v.p. \int \left(\frac{\varphi_1(z)}{t-z} + \frac{\varphi_1(z)}{t+z} \right) dz \\ \frac{\varphi_2(t)+\varphi_2(-t)}{2} - \frac{i}{2\pi} v.p. \int \left(\frac{\varphi_2(z)}{t-z} + \frac{\varphi_2(z)}{t+z} \right) dz \end{pmatrix}$$

is concentrated on the exterior of wave front (outside) in $R^{(1)}$, see 4rd section of present work.

Remark. So far, a bump functions have played a role of coordinates, in which it turned out to be possible to work with Green functionals of generalized quantum e.-m. fields. It turns out that with help of complex bump functions $\psi(z) = \varphi(z) + i\phi(z)$ ($\varphi(z), \phi(z) \in K$) we can construct a Shrödinger equations corresponding to Green functionals of quantum e.-m. fields and therefore their physically interpreted solutions.

Indeed, the existence of integral

$$\int \bar{\Xi}_t(z - z_0) \begin{pmatrix} \psi_1(z_0) \\ \psi_2(z_0) \end{pmatrix} dz_0 = \begin{pmatrix} \psi_1(z) \\ \psi_2(z) \end{pmatrix}$$

interpreting as solution of a Cauchy problem for a Shrödinger type equation with Hamiltonian

$$\hat{\mathrm{H}} = \begin{pmatrix} 0, & i\frac{\partial}{\partial z} \\ -i\frac{\partial}{\partial z}, & 0 \end{pmatrix}$$

(Cf. (1)).

From this point on to simplify the recording it will be considered one of two independent components of the quantum e.-m. field with Shrödinger equation and its physically interpreted solution

About similar procedure for $\breve{\Xi}_t(z)$ and $C_{it}(z)$ see 4rd section of present work.

Note. *The passage from $\Xi_t(z)$ to $\breve{\Xi}_t(z)$ by means of unitary (and therefore isomorphic) transform is the fundamental rebuilding of the quantum e.-m. field, representing its a completely new face. If the solution representation in the wave form (4) was known, then the constructed isomorphic solution (5) has a diffuse character that allowed to discover and construct the generalized quantum measure in the space of a photon «Feynman paths» with Hilbert instantaneous velocities (see [2]).*

3 Generalized functional integral corresponding to generalized Green function of constructed quantum e.-m. field of «deinterlaced» photon

Recall that the generalized quantum Cauchy process $C_{it}(z)$ — see (4), (and therefore the $\breve{\Xi}_t(z)$) is continuable to a generalized countably additive complex measure in the space dual the space of Hilbert instantaneous velocities of a photon, that is turned out to be a compact part of the continuous function space, see [2, 8].

The countably additivity of the quantum generalized Cauchy measure yields to possibility of quantum-theoretic expansion the state term of a photon to states on trajectories («Feynman paths»), continuous trajectories with Hilbert derivative.

We will assume that $\Delta t_1, \ldots, \Delta t_n$ ($t_0 = 0$, $t_n = t$) is certain partitioning of a time interval $[0, t]$, and Δz_j ($j = 1, \ldots, n$) is shifts of the photon at the appropriate time intervals.

Using the kernel theorem (see [8]) we have

$$\int \left(\prod_{j=1}^{n} \bar{C}_{i\Delta t_j}(\Delta z_j) \right) \times \\ \times \varphi(\Delta z_1, \ldots, \Delta z_n)\, dz_j \ldots dz_n,$$

where $\varphi(z_1, \ldots, z_n)$ is bump functions of n variables.

This allows to interpret that functional as a generalized state of the photon located sequentially on n

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segments Δz_j at the appropriate time intervals Δt_j. This implies that the convolution of these states

$$\int \left(\prod_{j=1}^{n} \bar{C}_{i\Delta t_j}(\Delta z_j) \right) \times \\ \times \varphi(\Delta z_1 + \ldots + \Delta z_n)\, dz_j \ldots dz_n = \\ = \int \bar{C}_{it}(z)\varphi(z)\, dz$$

is the state of the photon at the last moment of time.

The existence of the generalized Cauchy measure gives possibility of passage to the limit in the writing of the quantum Cauchy process and passage to the generalized functional functional integral on «Feynman paths»

$$\lim_{\max \Delta t_j \to 0} \int \left(\prod_{j=1}^{n} \bar{C}_{i\Delta t_j}(\Delta z_j) \right) \times \\ \times \varphi(\Delta z_1 + \ldots + \Delta z_n)\, dz_j \ldots dz_n = \\ = \int_{\{z_\tau\}} \left(\prod_{\tau=0}^{t} \bar{C}_{id\tau}(dz(\tau)) \right) \times \\ \times \varphi[z(\tau)] \prod_{\tau=0}^{t} dz_\tau = \\ = \int \bar{C}_{it}(z)\varphi(z)\, dz,$$

where $\{z_\tau\}$ is the support of generalized quantum Cauchy measure — «Feynman paths» — set of continuous trajectories on $[0, t]$, which are compact in topology of uniform convergence, dz_τ is differential at constant time, $dz(\tau) = \dot{z}(\tau)d\tau$ ($\dot{z}(\tau) \in L_2(0, t)$), and $\varphi[z(\tau)]$ are bump functionals on $\{z_\tau\}$.

Remarkably, those solution forms of the quantum fields, belonging to isomorphic Hilbert spaces with common scalar product, that allowed to discover the existence of such measure with similar properties in usual form of the quantum e.-m. field, on average (in eikonal approximation) giving the equations of classic electrodynamics.

4 Physic attribution of quantum e.-m. field of «deinterlaced» photon and Casimir forces

We consider the structure of the quantum field of «deinterlaced» photon with the generalized Green function $C_{it}(z)$ (see (3)) with the Hamilton functional $\int \hat{\hat{\mathrm{H}}}(z)\psi_0(z)\, dz = \frac{1}{\pi} \int z^{-2}\psi_0(z)\, dz$ and integral Shrödinger equation

$$i\frac{\partial}{\partial t}\psi_t(z) = \frac{1}{\pi} \int \alpha^{-2}\psi_0(z-\alpha)\, d\alpha.$$

with Cauchy problem solution on each small time interval Δt

$$\psi_{\Delta t}(z) = \psi_0(z) + i\Delta t \int \hat{\hat{\mathrm{H}}}(z-\alpha)\psi_0(\alpha)\, d\alpha - \\ -i\Delta t \int \hat{\hat{\mathrm{H}}}(\alpha)\psi_0(z-\alpha)\, d\alpha$$

Taking into account the definition of functional α^{-2}

$$v.p. \int \alpha^{-2}\psi_0(z-\alpha)\, d\alpha = \\ = \lim_{\varepsilon\to 0} \left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty} \right) \alpha^{-2}\psi_0(z-\alpha)\, d\alpha$$

where ε is any (see [6], p. 52 formula 7), in our problem, where $\varepsilon > 0$, we have

$$v.p. \int \alpha^{-2}\psi_0(z-\alpha)\, d\alpha = \\ = \left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty} \right) \alpha^{-2}\psi_0(z-\alpha)\, d\alpha \Bigg|_{\varepsilon\to 0},$$

where different from zero the result is obtained only on the even by α the bump functions $\psi_0(z-\alpha)$.

Therefore, we have the solution of Cauchy problem for Shrödinger equation on small time interval Δt

$$\psi_{\Delta t}(z) = \psi_0(z) - \\ - \frac{i\Delta t}{\pi} \left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty} \right) \alpha^{-2}\psi_0(z-\alpha)\, d\alpha \Bigg|_{\varepsilon\to 0}$$

Note, that «deinterlaced» photon, according to this formula, being in any localized state $\psi_0(z)$ at $t = 0$, at the very first moment it goes beyond the limits of the classic light cone $z = t$, $t < \Delta t$, on which it was located at $t = 0$, and filling at once whole coordinate space outside the small ε-vicinity of the origin of coordinates.

Therefore the Hamiltonian of such functional is generalized function

$$\hat{\mathrm{H}}_\varepsilon(z) = \frac{1}{\pi} \begin{cases} z^{-2}, & -\infty < z < -\varepsilon \\ 0, & -\varepsilon \le z \le \varepsilon \\ z^{-2}, & \varepsilon < z < \infty \end{cases}$$

Remark, that the constructed solution $\psi_{\Delta t}(z)$ conveniently to take as the «deinterlaced» photon state, in which mirrors are introduced, that are mentioned in «Introduction», at points $\pm\varepsilon$, since such mirrors does not deform such quantum field.

Thus the wave functional, in which mirrors are introduced,

$$\Delta_t\psi_0(z) = \\ = \frac{-i\Delta t}{\pi} \left(\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{\infty} \right) \alpha^{-2}\psi_0(z-\alpha)\, d\alpha \Bigg|_{\varepsilon\neq 0} = \\ = \psi_\varepsilon(z)\big|_{\varepsilon\neq 0} \quad (6)$$

and 2ε is distance between mirrors.

Moreover, since the operator $\frac{\partial}{\partial\varepsilon}$ (meaning the simultaneous moving asunder the mirrors), acting on

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wave functional $\psi_\varepsilon(z)$

$$\frac{\partial}{\partial\varepsilon}\frac{-i\Delta t}{\pi}\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha^{-2}\psi_0(z-\alpha)\,d\alpha = \\ =\frac{i\Delta t}{\pi}(\overleftarrow{\varepsilon}^{\,-2}+\overrightarrow{\varepsilon}^{\,-2})\alpha^{-2}\psi_0(z-\alpha) \tag{7}$$

(arrows mean direction of forces acting on left and right mirror) has the meaning of the force operator acting on mirrors from the side of the quantum field (Casimir forces) and moving asunder the mirrors. Here the a negative energy of the quantum field of «deinterlaced» photon corresponds to Casimir forces, contrary to positive energy of the quantum field of usual photon.

Remark, that the operator $\frac{\partial}{\partial\varepsilon}$ acts on an element of the Hilbert space $\psi_\varepsilon(z)$ (see (6)), is interpreted as a projection operator of this state to one of summand (mutually non-orthogonal vectors of Hilbert space) of integral sum.

Therefore the average of Casimir operator $\frac{\partial}{\partial\varepsilon}$ on quantum field in the state $\psi_\varepsilon(z)$ is

$$\overline{\frac{\partial}{\partial\varepsilon}}=\frac{\int\bar{\psi_\varepsilon}(z)\frac{\partial}{\partial\varepsilon}\psi_\varepsilon(z)\,dz}{\int\bar{\psi_\varepsilon}(z)\psi_\varepsilon(z)\,dz}= \\ =\frac{\int\bar{\psi_\varepsilon}(z)(\overleftarrow{\varepsilon}^{\,-2}+\overrightarrow{\varepsilon}^{\,-2})\psi_0(z-\varepsilon)\,dz}{\int\bar{\psi_\varepsilon}(z)\psi_\varepsilon(z)\,dz}.$$

For calculation of this fraction we will write the integrand vectors in the trigonometric (orthonormal) basis. Since

$$\int\bar{\psi_0}(z-\alpha)\psi_0(z-\alpha')\,dz= \\ =\int\exp\Big(-ip(\alpha-\alpha')\Big)\left|\tilde{\psi_0}(p)\right|^2dp,$$

we have (see (7))

$$\overline{\frac{\partial}{\partial\varepsilon}}=(\overleftarrow{\varepsilon}^{\,-2}+\overrightarrow{\varepsilon}^{\,-2})\times \\ \times\left(\int\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha^{-2}e^{-ip(\alpha-\varepsilon)}\left|\tilde{\psi_0}(p)\right|^2d\alpha\,dp\right)\times \\ \times\left(\int\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha^{-2}\exp(-ip\alpha)\left|\tilde{\psi_0}(p)\right|^2\times \\ \times\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha'^{-2}\exp(-ip\alpha')\,d\alpha\,d\alpha'\,dp\right)^{-1}= \\ =(\overleftarrow{\varepsilon}^{\,-2}+\overrightarrow{\varepsilon}^{\,-2})\times$$

$$\times\frac{\int\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha^{-2}\exp(-ip(\alpha-\varepsilon))\left|\tilde{\psi_0}(p)\right|^2d\alpha\,dp}{\int\left|\left(\int\limits_{-\infty}^{-\varepsilon}+\int\limits_{\varepsilon}^{\infty}\right)\alpha^{-2}\exp(-ip\alpha)\,d\alpha\right|^2\left|\tilde{\psi_0}(p)\right|^2dp}=$$

$$=(\overleftarrow{\varepsilon}^{\,-2}+\overrightarrow{\varepsilon}^{\,-2})\times$$

$$\times\frac{\int\limits_0^{\infty}\int\limits_{\varepsilon}^{\infty}\alpha^{-2}\cos(p\alpha)\cos(p\varepsilon)\left|\tilde{\psi_0}(p)\right|^2d\alpha\,dp}{2\int\limits_0^{\infty}\left(\int\limits_{\varepsilon}^{\infty}\alpha^{-2}\cos(p\alpha)\,d\alpha\right)^2\left|\tilde{\psi_0}(p)\right|^2dp}$$

Since this fraction is interpreted in the geometry of a Hilbert space as the square cosine of the angle between the vectors of this space $\frac{\partial}{\partial\varepsilon}\psi_\varepsilon(z)$ and $\psi_\varepsilon(z)$, the value of this fraction does not depend on the choice of bump function $\psi_0(z)$, that is why any bump function can be used to numerically find the value of this fraction, for example, $\psi_0(z)=\exp(-z^2)$.

Recall that the real quantum e.-m. field in the state of «deinterlaced» photons (taking into account the states of photon-antiphoton) contains two independent components.

Recall also that the influence of the field corresponding to element dt of a «Feynman path» $z(t)$ is taken into account, while the whole plane ($z=0$) is exist, which is why we have that «Casimir forces»corresponding each area unit of the plane $z=0$ is

$$\left(2\overline{\frac{\partial}{\partial\varepsilon}}\right)^{-2}.$$

At the same time, there is a continuous «tissue» of the quantum field $\psi_0(z)$ of «deinterlaced» photon, where the Casimir forces do not appears absolutely. That is interpreted as the presence of negative pressure forces into this environment stretching that «tissue». Therefore the Casimir forces measured in known experiment are exactly equal and opposite to the considered ones in present work.

5 Conclusion

The work shows that the real existence of Casimir forces indicates the reality of the existence of two unitary equivalent, but fundamentally different physically forms of the quantum e.-m. field, one of which (the known) is described by the Maxwell–Majorana equations with the solutions inside the light cone, contrary to other form — the quantum field, the field of «deinterlaced» photon, existing outside the light cone and responsible for Casimir forces.

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