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Probability Models of Natural Catastrophe Losses

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Abstract: - In this article we describe parametric curve-fitting methods for modeling historical natural catastrophe losses. We summarize relevant theoretical results above Excess over Threshold Method (EOT) and provide its application to the data about total catastrophe losses in the world in period 1970-2014, published in No2/2015 Swiss Re study Sigma.

Key-Words: - Excess over Threshold Method, Generalized Pareto Distribution, losses, natural catastrophe

1 Introduction

Catastrophic events affect various regions of the world with increasing frequency and intensity (Fig. 1). Many regions are threatened by catastrophic risks large range, where extensive disruptions are frequently, sometimes more than once a year. Large catastrophic events can be caused by natural phenomena or are caused by man. It should be noted that many of the events and natural character are to a large extent influenced by human activity. This mainly concerns climate change, but also, for example, the influence of the mining industry. Serious events in recent years are often the result of terrorist acts.

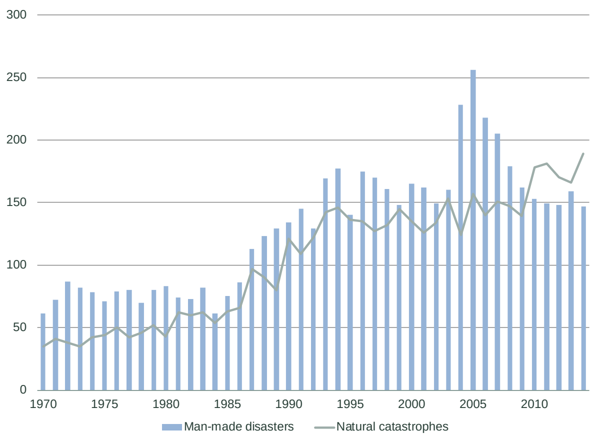

Fig.1 Number of catastrophic events 1970-2014

According to the latest *sigma* study, global insured losses from natural catastrophes and man-made disasters were USD 35 billion in 2014, down from USD 44 billion in 2013 and well below the USD 64 billion-average of the previous 10 years. There were 189 natural catastrophe events in 2014, the highest ever on sigma records, causing global economic losses of USD 110 billion. (Swiss Re sigma No 2/2015, p. 4)

Catastrophe modelling is a risk management tool that uses computer technology to help insurers, reinsurers and risk managers better assess the potential losses caused by natural and man-made catastrophes. The models use historical disaster information to simulate the characteristics of potential catastrophes and to determine the potential losses cost.

We are interested in probability modelling catastrophe losses, specifically the tails of loss severity distributions. Thus is of particular relevance in reinsurance if we are required to choose or price a high-excess layer. In this situation it is essential to find a good statistical model for the largest observed historical losses.

2 Problem Formulation

Suppose losses are the independent, identically distributed (*iid*) random variables $X_1, X_2, \ldots,$ with common distribution function

$$F_X(x) = P(X \le x), \text{ where } x > 0 \qquad (1)$$

The generalized Pareto Distribution (GPD) is the limit distribution of scaled excess of high thresholds.

2.1 GPD Theorem

Suppose $X_1, X_2, \ldots$ are *iid* with distribution *F*. Than

$$G(x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \qquad (2)$$

where

$$1+\xi\left(\frac{x-\mu}{\sigma}\right) > 0 \qquad (3)$$

is the limit distribution of the maxima $M_n = X_{1,n} = max(X_1, \ldots, X_n)$. Then for a large enough threshold *u*, the conditional distribution function of $Y = (X - u / X > u)$ is approximately

$$H(x) = 1 - \left(1+\frac{\xi x}{\tilde{\sigma}}\right)^{-1/\xi} \qquad (4)$$

defined on $\{x : x > 0 \text{ and } (1+\xi x/\tilde{\sigma}) > 0\}$, where $\tilde{\sigma} = \sigma + \xi(u-\mu)$.

The family of distributions defined by equation (4) is called the generalized Pareto family (GPD). For a fixed high threshold *u*, the two parameters are the shape parameter ξ and the scale parameter $\tilde{\sigma}$. For simpler notation, we may just use σ for the scale parameter if there is no confusion.

2.2 Excess over Threshold Method

The modelling using the excess over threshold method follows the assumptions and conclusions in GPD Theorem. Suppose $x_1, x_2, \ldots, x_n$ are raw observations independently from a common distribution *F*(*x*). Given a high threshold *u*, assume $x_{(1)}, x_{(2)}, \ldots, x_{(k)}$ are an observation that exceeds *u*. Here we define the ascendances as $x_i = x_{(i)} - u$ for $i = 1, 2, \ldots, k$.

By GPD Theorem x_i may be regarded as realization of independently random variable which follows a generalized Pareto family with unknown parameters ξ and σ. In case $\xi \neq 0$, the likelihood function can be obtained directly from (4):

$$L((\xi,\sigma/\mathbf{x})) = \prod_{i=1}^{k}\left[\frac{1}{\sigma}\left(1+\frac{\xi x_i}{\sigma}\right)^{-1/\xi-1}\right] \qquad (5)$$

3 Problem Solution

The analysis focus on 264 amounts of damage (in USD millions) of total natural catastrophes in time period from January 2010 to December 2014, published in Swiss Re sigma 2011-2015.

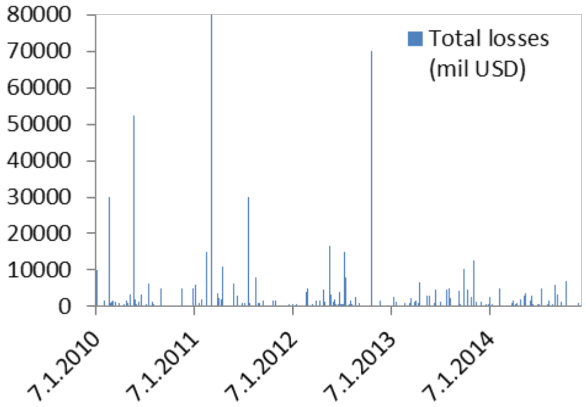

Fig.2 Chronologically arranged the total losses of natural catastrophes 2010-2014

The time series plot (Fig.2) allows us to identify the most extreme losses and their approximate times of occurrences.

We have fitted a generalized Pareto distribution using the maximum likelihood method for parameters estimation to the data above threshold of 3000 (Fig.3), above threshold of 5000 (Fig.5) and above threshold 8000 (Fig. 7).

These plots are useful for examining the distribution based on sample data. We have overlaid a theoretical distribution function on the same plot with empirical distribution of the sample to compare them.

The black stair lines on Fig.3, Fig.5 and Fig.7 show the empirical distribution functions of empirical sample data and the blue curves present the theoretical distribution function of the estimated generalized Pareto distributions for different thresholds. The red lines are the lower and upper bounds of the 95% confidence interval estimates of the distribution function. It can be seen that the estimated parametric distribution function falls inside the bands.

In Fig.3, Fig.5 and Fig. 8 we see the good fit of all three generalized Pareto distributions of total losses on natural catastrophes.

The QQ-plots (Fig.4, Fig.6, and Fig.8) against the generalized Pareto distributions is another way to examine visually the hypothesis that the losses which exceed a very high threshold come from estimated distributions.

Table 1 presents the parameters of the fitted generalized Pareto distributions on the data above three different thresholds. By *p*-values in this table we can state the best fit in the case of threshold $u = 5\,000$.

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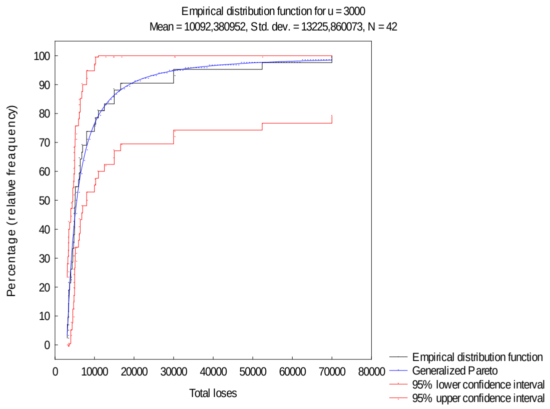

Fig.3 GPD fitted to 42 exceedances of the threshold 3000

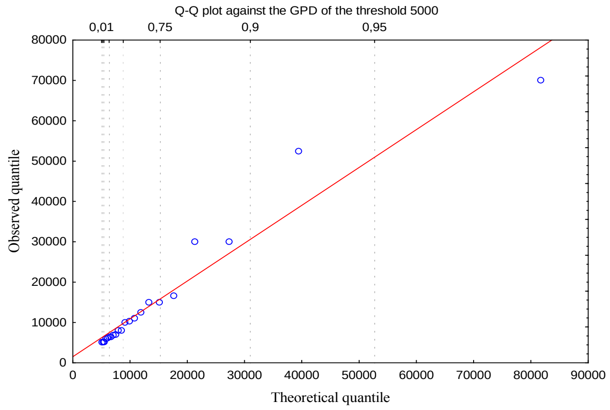

Fig.6 QQ-plot against the GPD fitted to 22 exceedances of the threshold 5000

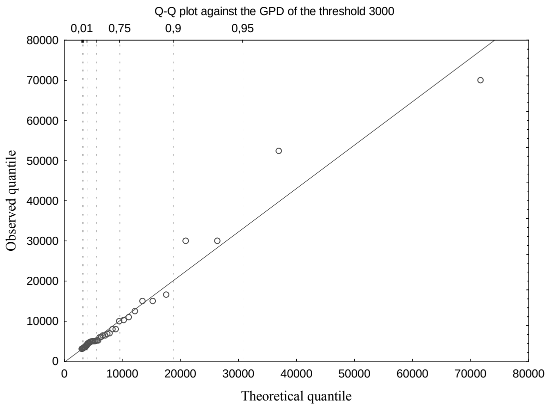

Fig.4 QQ-plot against the GPD fitted to 42 exceedances of the threshold 3000

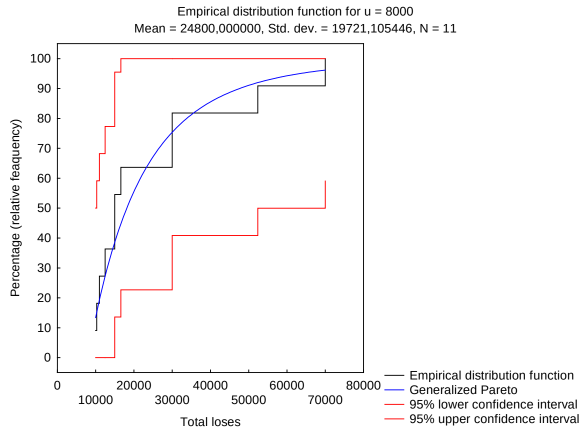

Fig.7 GPD fitted to 11 exceedances of the threshold 8000

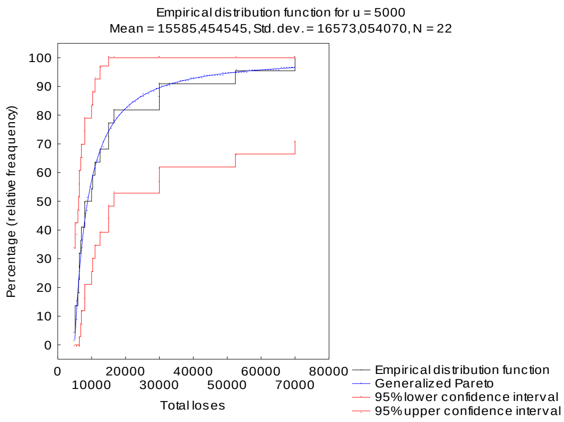

Fig.5 GPD fitted to 22 exceedances of the threshold 5000

Fig.8 QQ-plot against the GPD fitted to 11 exceedances of the threshold 8000

Table 1 Comparisons of estimated GPD for different thresholds

	$u = 3\ 000$	$u = 5\ 000$	$u = 8\ 000$
parametr ξ	2842.322	4195.862	13706.81
parametr σ	-0.67771	-0.75417	-0.19003
p-value	0.850026	0.959389	0.760575

4 Conclusion

We have shown that fitting the generalized Pareto distribution to natural catastrophic losses which exceed high thresholds is a useful method for estimating the tails of loss severity distributions. In our experience with several insurance datasets we have found consistently that the generalized Pareto distribution is a good approximation in the tail.

This is not altogether surprising. As we have explained, the method has solid foundations in the mathematical theory of the behaviour of extremes; it is not simply a question of ad hoc curve fitting.

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