Optimal Design of a Two-Stage Helical Gearbox via NSGA-II and MAIRCA: Balancing Transmission Efficiency and Structural Compactness

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Abstract: - A hybrid optimization and decision-making framework for the best design of a two-stage helical gearbox is presented in this research. The primary objective is to simultaneously maximize mechanical efficiency under different transmission ratio conditions and minimize the entire gearbox volume, indicating structural compactness. The bi-objective optimization problem is solved using the Non-dominated Sorting Genetic Algorithm II (NSGA-II), which produces a variety of Pareto-optimal solutions. The Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) technique is incorporated as a multi-criteria decision-making tool to determine the most balanced design from the Pareto front. To ensure that the solutions are practically feasible, a comprehensive computer-based model is created to assess gear geometry, load distribution, and power losses. The findings show that the suggested NSGA-II-MAIRCA method successfully manages the volume and efficiency trade-off, producing high-performance and small gearbox layouts. For gearbox designers looking for effective solutions under performance and space constraints, this work provides useful data.

Key-Words: - NSGA-II, MAIRCA, Pareto front, two-stage helical gearbox, multi-objective optimization, gearbox volume, and gearbox efficiency.

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1 Introduction

Since they can effectively transfer torque and power while being small and smooth in operation, helical gearboxes are essential parts of many industrial applications. Gearboxes are crucial intermediary mechanisms that adjust rotational speed and torque between motors and mechanical loads in any application, from wind turbines and manufacturing machines to automobile drivetrains and aircraft propulsion systems. When a high gear ratio is needed without affecting transmission stability or adding unnecessary noise, two-stage helical gearboxes in particular are widely utilized. A more flexible trade-off between size, efficiency, durability, and manufacturability is made possible by their configuration, which consists of two sequential meshing stages. However, the growing need for mechanical systems that are both space-efficient and energy-efficient has resulted in strict guidelines for gearbox design, requiring simultaneous optimization of structural compactness and transmission efficiency.

Gear system design is fundamentally a multi-objective problem that requires striking a balance between competing goals, such as optimizing transmission efficiency and power density while minimizing weight or cross-sectional area. Conventional gear design approaches frequently depend on empirical formulas or trial-and-error procedures, which might not be enough for managing intricate trade-offs between several competing goals. Several optimization strategies have been put out in recent decades to get around these restrictions.

The study in [1] carried out one of the first thorough studies on gear system optimization, comparing several evolutionary methods for the best gear system design. Their findings highlighted the advantages of multi-objective genetic algorithms in generating well-distributed Pareto-optimal solutions

and investigating intricate design domains. Similar to this, the work in [2] introduced a multi-objective formulation of gear design problems by implementing an optimal engineering design model for spur gear sets.

The creation of the NSGA-II algorithm in [3], which upgraded previous genetic algorithms by adding elitism and crowding distance mechanisms for improved convergence and solution diversity, was a significant breakthrough in the field. Since then, NSGA-II has emerged as the de facto standard for multi-objective optimization (MOO), particularly in applications related to mechanical design. A fundamental approach to using genetic algorithms (GAs) to multi-objective engineering optimization problems is possibly found in the work of the authors of [4]. The authors note that many engineering tasks in the actual world have essentially competing goals, where improving one criterion may cause another to become less satisfactory. The study promotes the usage of solution sets, also referred to as Pareto-optimal sets, where each solution achieves a balanced trade-off without being dominated by others, in order to address this. It provides a thorough overview of how specific fitness assignment schemes and diversity preservation strategies can be used to adapt evolutionary algorithms for multi-objective settings.

With an emphasis on gear ratios and shaft configurations, the study in [5] presented a GA-based technique for creating gear trains. In the meantime, the authors of [6] optimized a two-stage helical gearbox while considering durability, volume, and efficiency using the Taguchi method and grey relational analysis. Their work underscored the benefit of hybridizing statistical and decision-making tools for gearbox design.

Epicyclical gear trains have also been optimized using genetic approaches. The researchers in [7] introduced a genetic multi-objective model to address power transmission and material consumption trade-offs. More specifically, the work in [8] conducted a comprehensive study on the optimization of a two-stage spur gearbox using NSGA-II and several decision-making methods, achieving significant improvement in gear efficiency and cost-effectiveness.

In the realm of helical gearboxes, the authors of [9] formulated a multi-objective model to enhance efficiency and reduce the bottom cross-sectional area, addressing both performance and structural compactness. They demonstrated that evolutionary algorithms could simultaneously minimize housing size while maintaining optimal power flow. The study in [10] presents a comprehensive approach to the optimal design of two-stage helical gear trains through the application of multi-objective optimization techniques. Recognizing the limitations of manual gear design—particularly its iterative, time-consuming nature due to the need to simultaneously address kinematic and strength requirements-the authors explored three different optimization methodologies: the MATLAB Optimization Toolbox, a standard genetic algorithm (GA), and the Non-dominated Sorting Genetic Algorithm II (NSGA-II). While the first two methods approached the problem in a sequential manner—first minimizing gear volume and then evaluating load-carrying capacity-NSGA-II treated the problem as a true multi-objective optimization task. Design variables included face width, module, and number of teeth, with constraints applied to bending strength, surface fatigue resistance, and interference avoidance.

The study in [11] addresses the multi-objective optimization of a two-stage helical gearbox featuring double gears in the first stage-a configuration that introduces additional design complexity. The authors applied the MARCOS (Measurement Alternatives and Ranking according to Compromise Solution) method to simultaneously minimize the bottom area of the gearbox and maximize its transmission efficiency. Three key design parameters were considered: the first-stage gear ratio and the face width coefficients (Xba) for both the first and second stages. To determine the relative importance of these criteria, the study employed the MEREC method to compute objective weights prior to applying MARCOS for final decision-making. This combination of weighting and decision-making methods represents a structured and effective approach to solving multi-objective design problems. Another contribution in [12] focused on planetary gear trains and used genetic algorithms to optimize gear geometry and mechanical performance. The applicability of multi-objective frameworks across different gear types reinforces the versatility of genetic-based strategies.

The authors of [13] also demonstrated the use of the MARCOS method for general two-stage helical gearbox optimization, while their later work [14] incorporated multi-criteria decision-making (MCDM) techniques into the optimization process. These studies highlighted the importance of integrating decision-making steps into the Pareto analysis phase to assist designers in selecting the most suitable compromise solution.

The work of [15] presents a targeted multi-objective optimization approach aimed at enhancing the reliability of the planet carrier in wind turbine gearboxes—an essential component subjected to complex loading conditions. The study focused on minimizing stress concentration, structural

deformation, and component mass through structural dimension optimization. A parametric finite element model was developed and simulated using ANSYS Workbench to capture the effects of key design variables on the mechanical behavior of the planet carrier. To improve computational efficiency, a polynomial response surface was constructed to approximate the finite element analysis results, and a screening algorithm was employed to guide the optimization process. The proposed method successfully identified global optimal solutions based on the minimum principle applied to the response surface. Simulation results validated the effectiveness of the approach, showing significant improvements in both the structural stability and weight reduction of the planet carrier. The framework in [16] tackled the issue of vibration and noise in electric bus gearboxes using a comprehensive optimization framework, demonstrating the practical implications of multi-objective analysis in real-world applications.

The study in [17] addresses the complex challenge of optimizing multi-speed gearbox designs involving mixed discrete and continuous variables, along with multiple conflicting objectives. Traditionally, such problems are decomposed into simpler subproblems due to the limitations of classical optimization methods, which often rely on explicit mathematical formulations to derive solutions. In contrast, this study demonstrates the effectiveness of using the NSGA-II algorithm to directly solve the original, full-scale multi-objective problem without simplification. The algorithm successfully handles a combination of real-valued and discrete decision variables, accommodates multiple constraints, and generates a diverse set of nondominated solutions in a single simulation run. For instance, the authors of [18] applied the TOPSIS decision-making method in conjunction with NSGA-II to evaluate gearbox designs with double gear-sets in the second stage. This integration allowed for better selection of compromise solutions based on closeness coefficients.

In high-precision applications, optimization also extends to bearings. The authors of [19] optimized high-speed angular contact ball bearings for aircraft gearboxes using NSGA-III, ensuring minimal temperature rise and maximum fatigue life. Similarly, advanced machine learning techniques have been introduced for fault diagnosis and performance optimization. The study in [20] employed a multi-task deep learning framework to detect faults in gearboxes using one-dimensional convolutional neural networks, thus adding a predictive maintenance dimension to design optimization.

Gearbox design is becoming more and more influenced by environmental factors. The authors of [21] used a multi-objective model to optimize gear shifting algorithms in order to balance fuel consumption, emissions, and performance. A robust multi-objective optimization framework for gear microgeometry design is provided in the work of [22], highlighting the essential function that microgeometry plays in improving the tooth flank contact pattern and enhancing gear performance. The primary objectives were to minimize the peak-to-peak transmission error (PPTE) to reduce noise, vibration, and harshness (NVH), while simultaneously lowering contact and root stresses to improve gear durability and prevent tooth failure. The authors combined Design of Experiments (DOE) using the LDP software with meta-modeling techniques in MATLAB, employing squared exponential Gaussian regression to approximate objective functions. A key strength of this work lies in its robustness analysis, where torque variations were considered through probability distributions and worst-case scenarios to ensure the reliability of the design under uncertain conditions. The final multi-objective optimization yielded a well-defined Pareto front, from which the Rank 1 design achieved significant performance gains compared to a traditional manually selected baseline.

The combination of NSGA-II with decision-making tools has gained popularity in recent studies. The study in [23] proposes a comprehensive multi-objective optimization framework for spur gear design, addressing the inherent complexity arising from the presence of multiple conflicting objectives and mixed decision variables. Utilizing the NSGA-II algorithm, the model incorporates gear module, number of teeth, and transmission ratio as design variables, while aiming to optimize center distance, bearing capacity coefficient, and meshing efficiency. To identify the most suitable compromise solutions from the generated Pareto front, the study integrates three decision-making techniques: Shannon Entropy, LINMAP, and TOPSIS. Among these, the LINMAP-based selection consistently yielded solutions with lower deviation indices, suggesting superior balance across objectives. The work in [24] tackled the stochastic optimization of synchronizer mechanisms in gearboxes, accounting for variability in driver behavior and manufacturing tolerances. Most recently, the authors of [25] optimized an aeroengine accessory gearbox transmission using a heuristic approach, reaffirming the effectiveness of hybrid MOO strategies in aerospace applications.

Despite these advancements, a limited number of studies focus explicitly on balancing transmission efficiency and structural compactness using NSGA-II and advanced MCDM methods such as MAIRCA. Moreover, the application of such hybrid strategies to

two-stage helical gearboxes—a configuration that is both common and mechanically efficient—remains underexplored. The current study addresses this research gap by proposing a novel framework for optimizing a two-stage helical gearbox using NSGA-II for Pareto front generation and MAIRCA (Measurement of Alternatives and Ranking according to Compromise Solution) for final decision-making. The study aims to simultaneously minimize the cross-sectional area (as a proxy for structural compactness) and maximize transmission efficiency, which are often conflicting objectives in gearbox design.

2 Optimization Problem

2.1 Gearbox Volume Calculation

When designing and optimizing a two-stage helical gearbox, housing volume prediction is essential, especially when material efficiency and compactness are important factors. The gearbox housing's total volume V can be calculated using (Figure 1):

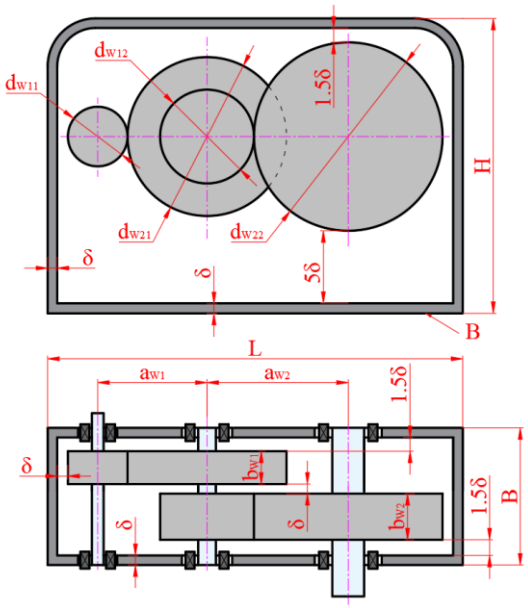


Fig. 1: Schema for finding gearbox volume
Source: created by the authors

$V_{gb} = L \cdot B \cdot H(1)$

Where L, B, and H denote the total length, breadth, and height of the gearbox case, respectively. The dimensions are dependent upon the geometry of the gear arrangement and are determined using the following formulas:

$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta(2)$
$B = b_{w1} + b_{w2} + 6 \cdot \delta(3)$
$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot \delta(4)$

In the above equations:

1. $b_{w1}$and b_{w2} denote the gear face widths of the first and the second stage.
2. $d_{w1i}$and d_{w2i} (i=1÷2) are the pitch diameter of the pinion, and the pitch diameter of the gear in stage i, respectively.
3. δ represents the additional clearance space (also known as casing allowance), which compensates for manufacturing tolerances, assembly space, and thermal expansion. According to the recommendation from [26], δ typically ranges between 7 mm and 10 mm to ensure adequate spacing.

The above parameters can be calculated using the following expressions:

$b_{wi} = X_{bai} \cdot a_{wi}(5)$
$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1)(6)$
$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1)(7)$

In which, X_{bai} and a_{wi} (i=1÷2) are the face width coefficient and the center distance of stage i; a_{wi} can be found by [26]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})}(8)$$

Where T_{1i} (i=1÷2) denotes the pinion torque of stage i, which can be calculated by:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})}(9)$$

2.2 Gearbox Efficiency Determination

The efficiency of a two-stage helical gearbox is an important performance indicator, representing the effectiveness of input power transmission through the system with low losses. The calculation is carried out using the subsequent expression:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}}(10)$$

In which, η_{gb} is the total gearbox efficiency (in %), P_{in} denotes the overall input power delivered to the gearbox, and P_l is the total power loss suffered inside the transmission system. According to [27], the total power loss P_l comprises four main components:

$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0}(11)$

Each component corresponds to a distinct source of mechanical loss in the gearbox, as detailed below.

a) Power Loss Due to Gear Meshing (P_{lg})

Gear meshing losses account for the inefficiency in power transmission between gear pairs. This loss is computed by summing the individual losses from each gear stage:

$P_{lg} = \sum_{i=1}^{2} P_{lgi}(12)$

For each gear stage iii, the power loss is defined as:

$P_{lgi} = P_{gi} \cdot (1 - \eta_{gi})(13)$

Here, P_{gi} is the transmitted power at stage i, and η_{gi} is the corresponding gear efficiency, which can be evaluated using the following expression derived

from [27]:

$$\eta_{gi} = 1 - \left(\frac{1+1/u_i}{\beta_{ai}+\beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot \left(\beta_{ai}^2 + \beta_{ri}^2\right) (14)$$

The coefficients β_{ai} and β_{ri} represent geometric terms related to the active flanks of the gear teeth and are given by:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} (15)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} (16)$$

The sliding friction coefficient f is a function of the relative sliding velocity v between meshing surfaces and is determined by empirical formulas [6]:

- When $v \leq 0.424$ (m/s):

$$f = -0.0877 \cdot v + 0.0525 (17)$$

- When $v > 0.424$ (m/s):

$$f = 0.0028 \cdot v + 0.0104 (18)$$

These expressions ensure that both low- and high-speed contact conditions are accurately captured in the frictional loss modeling.

b) Power Loss Due to Bearings (P_{lb})

The power dissipated in rolling-element bearings—mainly due to friction between the balls and the raceways—is estimated as:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i (19)$$

where f_b=0.0011 is the friction coefficient specific to angular-contact radial ball bearings [28], F_i is the applied radial load on bearing i, and v_i is the relative rotational speed at that location.

c) Power Loss Due to Seals (P_s)

Friction at the shaft seals also contributes to energy loss. Assuming two shaft seals in the system, the total seal-related loss is:

$$P_s = \sum_{i=1}^{2} P_{si} (20)$$

Each individual loss term P_{si} is calculated based on oil temperature, viscosity, and shaft geometry using the empirical relation:

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} (21)$$

Here, t_{oil} denotes oil temperature (°C), VG_{40} is the kinematic viscosity at 40°C, d_s is the shaft diameter at the seal location, and n is the shaft speed in revolutions per minute.

d) Power Loss Due to Oil Churning (P_{Z0})

When gear pairs rotate inside a lubricating medium (e.g., an oil bath), they incur additional torque resistance due to oil churning effects. The associated power loss is given by:

$$P_{Z0} = \sum_{i=1}^{k} T_{Hi} \cdot \frac{\pi \cdot n_i}{30} (22)$$

where k=2 is the number of gear stages, n_i is the rotational speed of gear stage i, and T_{Hi} is the hydrodynamic torque due to churning:

$$T_{Hi} = C_{Spi} \cdot C_{1i} \cdot e^{\frac{C_{2i} \cdot v}{v_{t0}}} (23)$$

The stage-specific correction factor C_{Spi} accounts for oil immersion conditions. For stage 1, C_{Spi}=1 (complete immersion). For stage 2, C_{Spi} is computed as:

$$C_{Spi} = \left(\frac{4 \cdot e_{max}}{3 \cdot h_C}\right)^{1.5} \cdot \frac{2 \cdot h_{Ci}}{l_h} (24)$$

with the mesh length l_{hi} estimated using:

$$l_{hi} = (1.2 \div 2.0) \cdot d_{a2i} (25)$$

The empirical constants C_{1i} and C_{2i} reflect gear geometry and are given by:

$$C_{1i} = 0.063 \cdot \left(\frac{e_{1i}+e_{2i}}{e_0}\right) + 0.0128 \cdot \left(\frac{b_i}{b_0}\right) (26)$$

$$C_{2i} = \frac{e_{1i}+e_{2i}}{80 \cdot e_0} + 0.2 (27)$$

With e_0=b_0=10 mm as the reference values for tooth thickness and face width.

2.3 Objective functions and constraints

This study formulates a bi-objective optimization problem that aims to simultaneously minimize the gearbox volume and maximize the mechanical efficiency. To accommodate both objectives in a minimization framework suitable for evolutionary algorithms, the second objective is expressed as minimizing the negative of efficiency. The problem is stated as:

$$\min f_1(X) = V_{gb} (28)$$

$$\min f_2(X) = 1 - \eta_{gb} (29)$$

Design Variables

The three decision variables considered are: u_1, X_{ba1}, and X_{ba2}. These variables directly affect the center distances, gear diameters, casing dimensions, and therefore the objectives.

Constraints

The optimization is subject to the following constraints:

$$1 \leq u_i \leq 9 (30)$$

$$0.25 \leq Xba_i \leq 0.4 (31)$$

This formulation enables a comprehensive search for optimal gearbox designs using NSGA-II to generate the Pareto front, followed by MAIRCA to support final solution selection.

3 Methodology

3.1 NSGA-II method

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed in this study to solve the bi-objective optimization problem of minimizing gearbox volume while maximizing mechanical efficiency. NSGA-II is a well-established evolutionary algorithm known for its ability to maintain solution diversity and converge efficiently

toward the Pareto front.

The key steps of NSGA-II include:

1. Initial Population Generation: A population of design solutions is randomly generated within the bounds of the decision variables u_1, X_{ba1}, and X_{ba2}.
2. Non-dominated Sorting: Individuals are classified into different Pareto fronts based on dominance relationships. The first front contains the non-dominated solutions.
3. Crowding Distance Assignment: A crowding distance metric is used to maintain diversity among solutions on the same front.
4. Selection, Crossover, and Mutation: Tournament selection is applied based on Pareto rank and crowding distance. Simulated binary crossover and polynomial mutation are used to generate offspring.
5. Population Update with Elitism: The current population and offspring are combined, and the best solutions based on non-domination and crowding distance are retained.

This process continues over multiple generations until convergence criteria are met. The final result is a Pareto front illustrating the trade-offs between volume and efficiency.

Implementation characteristics of NSGA-II: The optimization technique was carried out in MATLAB with a specially created NSGA-II algorithm. The population size was set to 100, while the maximum number of generations was limited to 200. Simulated Binary Crossover (SBX) and polynomial mutation have been used with a crossover probability of 0.9 and a mutation probability of 0.1. The distribution indices for crossover and mutation were set to 20. The lower and upper bounds of the design variables were defined as $u_1 \in [1,10]$, $X_{ba1} \in [0.25,0.40]$, and $X_{ba2} \in [0.25,0.40]$. In addition, the second-stage transmission ratio was calculated as $u_2 = u_h/u_1$, and infeasible solutions with $u_2 < 1$or $u_2 > 9$were penalized by assigning very large objective values.

3.2 MAIRCA Method

The Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) technique is adopted in this study to support multi-criteria decision-making (MCDM) and identify the most balanced design from the Pareto-optimal set. MAIRCA allows for structured evaluation based on both decision-maker preferences and the actual performance of alternatives. The technique is implemented according to the principles described in [29], as follows:

Step 1: Constructing the Decision Matrix

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} (32)$$

In which, x_{mn} is the result of criterion n in variant m.

Step 2: Calculating initial preferences P_{A_j} by:

$$P_{A_j} = \frac{1}{m}, \text{j} = 1, 2, \ldots, \text{n} \quad (33)$$

Step 3: Computing the ideal theoretical performance matrix by:

$$t_{p_{ij}} = P_{A_j} \cdot w_j, \text{i} = 1, 2, \ldots, \text{m; j} = 1, 2, \ldots \quad (34)$$

In which, w_j denotes the weight of the j^{th} criterion.

Step 4: Normalizing the real performance value

+) For benefit-type criteria (e.g., gearbox efficiency):

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left(\frac{x_{ij} - x_i^-}{x_i^+ - x_i^-}\right) (35)$$

+) For cost-type criteria (e.g., gearbox volume):

$$t_{r_{ij}} = t_{p_{ij}} \cdot \left(\frac{x_{ij} - x_i^+}{x_i^- - x_i^+}\right) (36)$$

In (36), x_j^+ and x_j^- represent the best and worst values for criterion j, respectively.

Step 5: Calculating the deviation matrix

$$g_{ij} = t_{p_{ij}} - t_{r_{ij}} (37)$$

Step 6: Finding the final score

$$Q_i = \sum_{i=1}^{m} g_{ij} (38)$$

- Ranking the options' order by minimizing Qi.

This hybrid framework, combining NSGA-II for Pareto optimization and MAIRCA for final decision-making, enables effective navigation of the trade-off between gearbox structural compactness and mechanical performance.

3.3 Entropy Method for Criteria Weight Determination

The Entropy method was employed in this study to objectively determine the weights of the criteria prior to applying the MAIRCA method. This approach evaluates the relative importance of each criterion based on the dispersion of data in the Pareto-optimal solution set [30]. The procedure is described as follows.

Step 1: Computing normalized values

The normalized value of each indicator is calculated as:

$$p_{ij} = \frac{x_{ij}}{m + \sum_{i=1}^{m} x_{ij}^2} (39)$$

Step 2: Calculating the entropy of each criterion

The entropy value for each criterion is determined by:

$$me_j = -\sum_{i=1}^{m} [p_{ij} \times \ln(p_{ij})] -$$

$$\left(1-\sum_{i=1}^{m} p_{ij}\right) \times \ln\left(1-\sum_{i=1}^{m} p_{ij}\right) (40)$$

Step 3: Determining criteria weights

The weight of each criterion is obtained as:

$$w_j = \frac{1-me_j}{\sum_{j=1}^{m}(1-me_j)} (41)$$

In this study, the Entropy method was applied to the Pareto-optimal solutions obtained for each transmission ratio u_h. The resulting weights were then used as input parameters in the MAIRCA method to ensure an objective and data-driven decision-making process.

4 Results and Discussions

4.1 Relationship Between Transmission Ratio and Optimal First-Stage Gear Ratio

Figure 2 shows the linear relationship between the overall transmission ratio u_h and the mean optimal gear ratio u_1 in the first stage, extracted from the NSGA-II Pareto sets. The following regression equation was found (with R^2=0.9479):

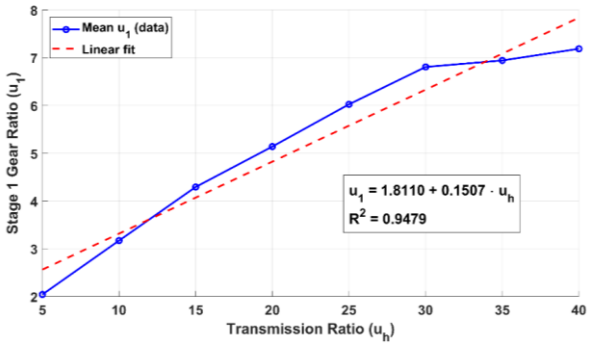


Fig. 2: Linear relationship between u_h and optimal u_1 from Pareto optimal solutions

Source: created by the authors

$$u_1 = 1.8110 + 0.1507 \cdot u_h (42)$$

The strong coefficient of determination indicates a clear trend: as u_h increases, the optimizer tends to allocate more transmission ratio to the first stage. This is likely due to efficiency benefits and mechanical stability gained when higher torque is absorbed earlier in the gear train.

To validate these findings, Figure 3 shows the linear regression using values selected by the MAIRCA method. The MAIRCA-based trend is (with R^2=0.9780):

$$u_1 = 2.0286 + 0.1887 \cdot u_h (43)$$

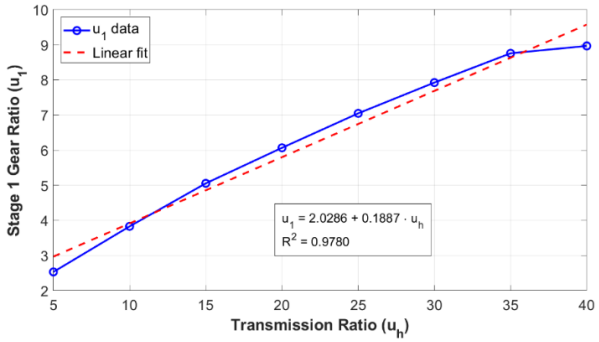


Fig. 3: Linear relationship between u_h and optimal u_1 from the MAIRCA method

Source: created by the authors

This trend confirms that MAIRCA prioritizes slightly more aggressive first-stage ratios, possibly due to its systematic weighting of efficiency over volume. The higher R^2 value suggests that MAIRCA-based selections are more stable and consistent across different u_h.

4.2 Efficiency-to-Volume Ratio Analysis

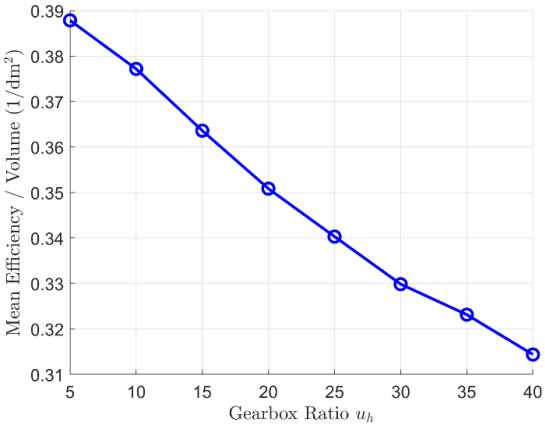


Fig. 4: Variation of a verage efficiency-to-volume ratio with respect to overall transmission ratio uh

Source: created by the authors

To further investigate the space–performance trade-off, Figure 4 illustrates the average ratio between efficiency and volume (1/dm²) across different values of u_h. The results show a clear decreasing trend:

1. At u_h=5, the mean efficiency/volume is highest (~0.39 1/dm²),
2. At u_h=40, the ratio drops to ~0.31 1/dm².

This suggests that achieving high performance at larger reductions requires disproportionately more structural volume. Therefore, lower u_h values offer better energy-to-size efficiency, which is desirable in compact systems such as electric vehicles and aerospace gearboxes.

4.3 Pareto Fronts of Volume vs. Efficiency

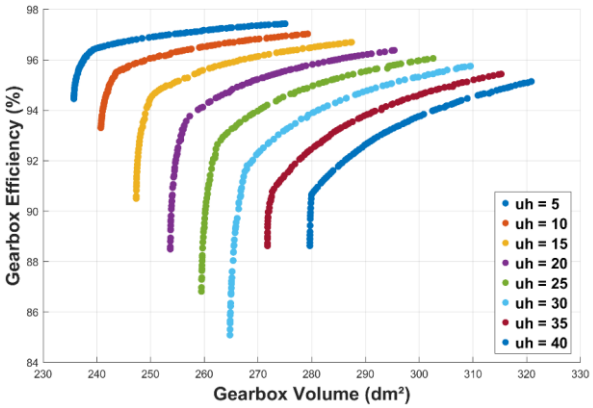


Fig. 5: Pareto fronts of gearbox efficiency versus volume for different u_h

Source: created by the authors

Figure 5 displays the Pareto fronts for all values of uh, plotting gearbox volume against efficiency. Each colored curve indicates a specific uh value and depicts the trade-off between two conflicting aims

For lower u_h (e.g., 5–15), the Pareto fronts shift upward and leftward, showing that it is possible to achieve both high efficiency and low volume.

1. As u_h increases (e.g., 30–40), the fronts bend downward and rightward, indicating increasing design difficulty in maintaining both objectives simultaneously.

This reinforces the role of NSGA-II in generating a diverse set of alternatives and underscores the value of integrating MCDM techniques like MAIRCA for solution selection.

4.4 Analysis of Trends in Volume and Efficiency Relative to Transmission Ratio

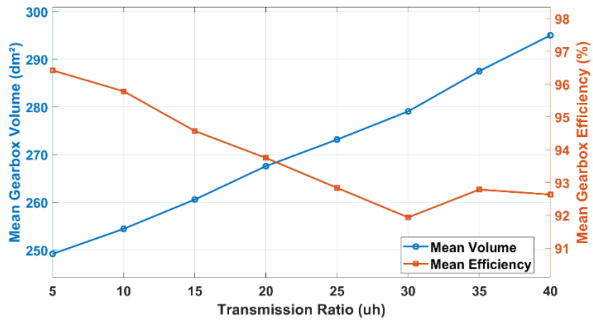


Fig. 6: Variation of mean gearbox volume and efficiency with respect to u_h

Source: created by the authors

To further investigate the overall behavior of gearbox characteristics across varying transmission ratios u_h, Figure 6 illustrates the average gearbox volume and average efficiency as functions of u_h, using dual y-axes for clarity.

1. The blue curve (left axis) represents the mean gearbox volume (dm³),
2. The orange curve (right axis) shows the corresponding mean efficiency (%).

The results reveal two clear and opposite trends:

1. As u_h increases from 5 to 40, the average volume increases steadily from approximately 250 dm³ to nearly 295 dm³. This aligns with the geometrical expansion of gear stages necessary to meet increased torque and reduce requirements.
2. The average efficiency indicates a declining trend, decreasing from approximately 97.5% to roughly 92.8%. This reduction is defined by enhanced power losses resulting from lengthened shafts, enlarged gears, and increased rotational mass in high-ratio setups.

The rate of get-involved efficiency reduces beyond u_h=30, indicating that additional increases in volume yield a reduction in transmission ratio while suffering substantial structural costs. This asymmetry in the trade-off reinforces the need for a balanced design framework like NSGA-II + MAIRCA, particularly for applications where both size and performance are tightly constrained.

Comparison of efficiency with literature and industrial data:

The efficiency values obtained in this study (ranging from 90.7% to 95.9%) were compared with reported values in both academic and industrial sources to assess their physical plausibility.

According to [31], the overall efficiency of optimized two-stage gear reducers can reach values between 97.2% and 98.1% under idealized design and operating conditions. In contrast, practical industrial gearboxes typically exhibit lower efficiencies due to additional losses such as manufacturing tolerances, lubrication conditions, and assembly factors. As reported in [32], the overall efficiency of real cylindrical gear reducers generally falls within the range of 95–96%.

The efficiency range obtained in the present study is therefore consistent with these references. In particular:

1. The upper bound (~95.9%) aligns closely with typical industrial gearbox performance [32],
2. The slightly lower values (down to ~90.7%) can be attributed to higher transmission ratios and increased cumulative

losses in multi-stage configurations,

3. The difference between the present results and the idealized values reported in [31] reflects the inclusion of more comprehensive loss components in the proposed model.

These comparisons confirm that the developed model provides realistic and physically meaningful efficiency predictions, while capturing the expected trade-off between transmission efficiency and structural compactness.

4.5 MAIRCA-Based Optimal Design Selection

Table 1. summarizes the best gearbox configurations selected using the MAIRCA method for each value of uh. The corresponding gear ratios, face width coefficients, volumes, and efficiencies are presented

Table 1. Optimal helical gearbox configurations selected by the MAIRCA method

u_h	u_1	X_{ba1}	X_{ba2}	V dm³)	η (%)
5	2.53	0.29	0.400	237.17	95.87
10	3.83	0.25	0.400	243.58	95.51
15	5.06	0.26	0.400	249.95	94.47
20	6.07	0.25	0.400	256.63	93.57
25	7.05	0.25	0.400	262.46	92.65
30	7.92	0.25	0.400	267.95	91.81
35	8.76	0.25	0.400	272.95	90.88
40	8.97	0.25	0.400	280.09	90.70

Source: created by the authors

The results illustrate a consistent preference for higher u_1 values and wider second-stage gears (X_{ba2}=0.40) across all configurations. As u_h increases, the efficiency gradually decreases while volume increases. The decrease in performance has been reduced, indicating the successful application of MAIRCA in balancing structural and functional objectives.

The results verify that the hybrid NSGA-II–MAIRCA framework enables practical and reasonable design options for two-stage helical gearboxes within multi-criteria restrictions.

Sensitivity analysis of MAIRCA weights

To evaluate the robustness of the decision-making process, a sensitivity analysis was performed by perturbing the entropy-based weights within ±10% while maintaining the normalization condition.

The results reveal that, for most transmission ratios, the selected solution remains unchanged over a wide range of weight variations. In particular, complete stability was observed for $u_h = 20$and $u_h = 25$, while only a single transition between two solutions occurred for other cases.

This trend illustrates that the MAIRCA ranking is robust, with numerous nearby Pareto solutions potentially revealing closely aligned Qi_values, so establishing a zone of nearly optimal trade-off options instead of a singular definitive optimum.

5 Conclusion

This research introduced a comprehensive framework that integrates the NSGA-II with the MAIRCA method to enhance the design of a two-stage helical gearbox. The double goals were to reduce gearbox volume, indicating structural compactness, and to enhance mechanical efficiency, which denotes energy performance.

A comprehensive mathematical model was developed to compute the gearbox volume based on gear geometry, face width, and casing dimensions, while accounting for mechanical power losses arising from gear meshing, bearings, seals, and oil churning. The NSGA-II algorithm was employed to generate a diverse Pareto front for each transmission ratio u_h [5], [40], capturing the inherent trade-off between volume and efficiency.

Results revealed that as the transmission ratio increases:

1. The volume of the gearbox rises nonlinearly, indicating the necessity for larger structural casings to meet heightened reduction requirements.
2. Efficiency steadily reduces, illustrating the mechanical and geometric constraints linked to higher gear ratios.
3. The efficiency-to-volume ratio decreases steadily with u_h, indicating that compact and energy-efficient designs are more feasible at lower reductions.

To facilitate rational selection among Pareto-optimal solutions, the MAIRCA method was applied, demonstrating its capability to consistently select well-balanced designs that reflect designer preferences. Linear regression analyses confirmed the stability and predictability of design trends across the full range of u_h values.

In conclusion, the proposed NSGA-II–MAIRCA approach provides an effective and systematic methodology for multi-objective gearbox design, offering both global search capability and decision-level refinement. This framework is highly adaptable to other mechanical systems with conflicting objectives.

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Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors confirm that all listed contributors have made significant, direct, and intellectual contributions to the conception, design, and execution of this research. Vu Duc Binh formulated the mathematical model for gearbox volume and conducted the computer-based evaluation. Hoang Xuan Tu and Nguyen Manh Cuong designed the optimization framework and carried out the NSGA-II algorithm simulations in MATLAB. Dinh Van Thanh developed the comprehensive power loss model encompassing gear meshing, bearings, seals, and churning losses. Nguyen Khac Minh supervised the project, implemented the MAIRCA-Entropy decision-making methodology, performed the sensitivity analysis, and prepared the final manuscript. All authors have reviewed, edited, and approved the final version of the manuscript and agreed to its submission. No ghostwriting or unlisted contributors were involved in the creation of this scientific article.

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Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this scientific article.

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