# Bi-Objective Optimization of Two-Stage Helical Gearboxes with Dual-Gear First Stage: A Comparative NSGA-II– MCDM Framework

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*Abstract:* - The bi-objective optimization of two-stage helical gearboxes involves inherent trade-offs between structural compactness and transmission efficiency, particularly when a dual-gear first-stage configuration is adopted. In such cases, selecting a representative optimal design from a Pareto-optimal solution set remains a critical challenge. This paper proposes a comparative NSGA-II–MCDM framework for the bi-objective optimization of two-stage helical gearboxes with a dual-gear first stage, aiming to minimize the bottom area of the gearbox while maximizing its efficiency. First, the NSGA-II algorithm is employed to generate a diverse set of Pareto-optimal solutions across different transmission ratios. Subsequently, three multi-criteria decision-making (MCDM) methods – TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution), MARCOS (The Measurement Alternatives and Ranking according to Compromise Solution), and MABAC (The Multi-Attributive Border Approximation Area Comparison) - are independently applied to rank the Pareto-optimal solutions and select representative optimal designs. A regression-based analysis is then conducted to characterize the relationship between the overall transmission ratio and the first-stage transmission ratio associated with each MCDM method, providing insight into the structural tendencies induced by different decision-making strategies. Furthermore, a Spearman rank correlation analysis is performed to evaluate the consistency of ranking results among the considered MCDM methods. The results indicate strong and stable agreement between TOPSIS and MARCOS across the investigated transmission-ratio domain, whereas the rankings produced by MABAC exhibit lower correlation with the other methods, reflecting a distinct decision-making behavior rather than numerical inconsistency. Overall, the proposed framework not only enables effective identification of optimal gearbox designs but also offers a systematic approach for comparing decision-making strategies and assessing ranking consistency. The findings provide practical guidance for robust decision-making in multi-objective gearbox design and highlight the importance of method selection in MCDM-based optimization.

*Key-Words:* - Two-stage helical gearbox, Dual-gear first stage, Bi-objective optimization, NSGA-II, MABAC, Gearbox bottom area, Efficiency, Multi-criteria decision-making.

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## 1 Introduction

Gear transmission systems are indispensable components in mechanical engineering, widely used in automotive, aerospace, and industrial applications. The design of efficient and compact gearboxes remains a critical engineering challenge due to the inherent trade-offs between geometric compactness and power transmission performance. Traditional optimization methods for gearbox design have primarily focused on single-objective formulations such as minimizing weight or maximizing efficiency. However, such approaches fail to adequately capture the multi-dimensional nature of real-world gearbox

design problems, which often require balancing conflicting objectives.

The emergence of multi-objective evolutionary algorithms (MOEAs) has significantly advanced gearbox design optimization. Foundational works [1], [2] established the theoretical basis for MOEAs, particularly the NSGA-II algorithm, which is widely recognized for its elitist sorting and ability to generate diverse Pareto-optimal solutions efficiently. Early applications in gear design demonstrated the potential of genetic algorithms and swarm intelligence techniques for optimizing spur and helical gear systems, [3], [4]. Later studies incorporated tribological constraints and system-level performance factors, further enhancing design fidelity, [5].

Recent years have seen a proliferation of studies applying MOEAs and hybrid decision-making methods to gear transmission systems. For instance, the study in [6] employed Taguchi–Grey analysis for a two-stage helical gearbox, while the work [7] extended this approach to bevel-helical gearboxes. Multi-criteria decision-making (MCDM) frameworks such as TOPSIS, MARCOS, and EDAS have also been integrated with MOEAs to systematically rank Pareto-optimal solutions, thereby supporting practical engineering decision-making, [8]. These studies confirm that evolutionary optimization, when combined with decision-support techniques, provides robust and interpretable outcomes.

In terms of gearbox structures, most prior studies concentrated on spur or simple helical gear trains [9], [10], with limited attention to dual-gear first-stage configurations. Recent studies [11], [12], [13] recently investigated two-stage helical gearboxes, targeting objectives such as minimizing length or maximizing efficiency. Study [14] compared different MOEAs for gear system design, showing that no single algorithm universally outperforms others, hence motivating hybrid frameworks. More advanced studies, such as [15] and [16], incorporated noise, vibration, and dynamic constraints into optimization formulations, further highlighting the complexity of real-world gearbox design.

Despite these advances, several research gaps can still be identified. First, although the multi-objective optimization of two-stage helical gearboxes has been extensively investigated, studies focusing specifically on dual-gear first-stage configurations remain limited. Such configurations offer enhanced flexibility in transmission ratio distribution and improved potential for compact gearbox layouts, yet their optimization has not been sufficiently explored. Second, while various MCDM methods have been integrated with multi-objective evolutionary algorithms (MOEAs) to select compromise solutions from Pareto-optimal sets, most existing studies rely on a single decision-making technique, which may introduce method-dependent bias into the final design selection. A systematic comparison of different MCDM methods within the same optimization framework is therefore still lacking for gearbox design problems.

To address these gaps, this study proposes a comparative NSGA-II - MCDM framework for the bi-objective optimization of two-stage helical gearboxes with a dual-gear first stage. The considered objectives are: (i) minimizing the gearbox bottom area to achieve structural compactness, and (ii) maximizing gearbox efficiency to enhance transmission performance. The NSGA-II algorithm is first employed to generate a diverse set of Pareto-optimal solutions across different transmission ratios. Subsequently, three widely used MCDM methods - TOPSIS, MARCOS, and MABAC-are independently applied to rank the Pareto-optimal solutions and identify representative optimal designs.

By combining Pareto-based optimization with a comparative evaluation of multiple MCDM techniques, this study not only avoids reliance on a single decision-making method but also enables a deeper investigation into decision consistency and ranking behavior among different MCDM approaches. The proposed framework provides practical guidance for gearbox designers by clarifying how different decision-making strategies influence the selection of optimal solutions, thereby contributing to more transparent and reliable multi-objective gearbox design.

# 2 Optimization Problem

## 2.1 Determination of Gearbox Bottom Area

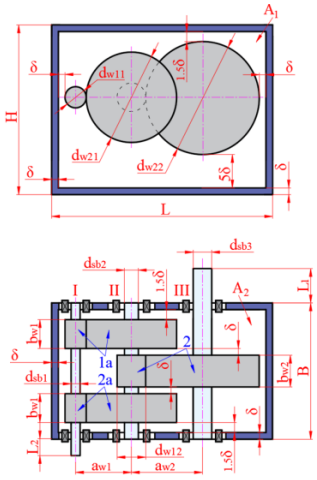



Fig. 1: For the determination of the gearbox bottom area

*Source: created by the authors*

For a two-stage helical gearbox with two gears in the first stage, the bottom area is calculated by (Figure 1):

$$A_b = (L \cdot B) \quad (1)$$

where L and B are determined by:

$$L = d_{w11} + \frac{d_{w21}}{2} + \frac{d_{w12}}{2} + d_{w22} + 4 \cdot \delta \quad (2)$$

$$B = b_{w1} + 2 \cdot b_{w2} + 7 \cdot \delta \quad (3)$$

In which, δ=7÷10 (mm) [17]; $b_{wi}$, $d_{w1i}$, $d_{w2i}$ are the gear width, the pitch diameter of the pinion, and the gear of i[th] stage (i=1÷2), respectively. These components can be found by:

$$b_{wi} = \mathrm{X}_{bai} \cdot a_{wi} \quad (4)$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \quad (5)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \quad (6)$$

where $a_{wi}$ (i=1÷2) is the center distance of stage i which is computed by [18]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot \frac{k_{H\beta}}{[AS_i]^2 \cdot u_i \cdot X_{bai}}} \quad (7)$$

In which, $X_{bai}$ is the wheel face width coefficient of stage i; $T_{1i}$ (i=1÷2) are the pinion torque of stage i which can be calculated by:

$$T_{11} = \frac{T_{out}}{\left(2 \cdot u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right)} \quad (8)$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (9)$$

## 2.2 Determination of Gearbox Efficiency

The total efficiency of the gearbox $\eta_{gb}$ is determined as:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (10)$$

In which, $P_l$ is the total power loss in the system and $P_{in}$ is the input power. The total loss $P_l$ is calculated by [19]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (11)$$

where $P_{lg}$, $P_{lb}$, $P_{ls}$, and $P_{Z0}$ are gear meshing loss, bearing friction loss, seal resistance loss, and loss due to idle motion, respectively. The detailed determination of these loss elements can be found as noted in [20].

## 2.3 Objectives Functions

The design of a two-stage helical gearbox with a dual-gear first stage is formulated as a bi-objective optimization problem, aiming to balance two conflicting design goals: achieving compactness while ensuring high efficiency. Accordingly, the following objective functions are defined:

- Minimization of gearbox bottom area:

$$\min f_1(X) = A_b \quad (12)$$

- Maximization of gearbox efficiency:

$$\max f_2(X) = \eta_{\mathrm{gb}} \quad (13)$$

## 2.4 Constrains

In this study, two primary constraints are imposed to ensure the feasibility and reliability of the gearbox design. These constraints are derived from the standard design requirements of gear transmissions.

The first constraint concerns the permissible range of gear ratios:

$$1 \le u_{\mathrm{i}} \le 9 \quad (14)$$

The second constraint relates to the wheel face width coefficient, expressed as:

$$0.25 \le Xba_{\mathrm{i}} \le 0.4 \quad (15)$$

# 3 Methodology

To solve the bi-objective optimization problem of the two-stage helical gearbox with a dual-gear first stage, a hybrid framework combining the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and MABAC method is employed. NSGA-II is first applied to generate a set of Pareto-optimal solutions. Subsequently, the MABAC method is used to evaluate and rank these solutions, thereby identifying the best compromise design in accordance with decision-making preferences.

## 3.1 NSGA-II method

The NSGA-II algorithm is one of the most widely applied multi-objective evolutionary algorithms due to its efficiency in handling conflicting objectives and generating diverse Pareto-optimal fronts. NSGA-II incorporates three main mechanisms:

1. Fast non-dominated sorting – The population is divided into different fronts based on Pareto dominance, ensuring elitism while maintaining computational efficiency.

2. Crowding distance assignment – A density estimation mechanism preserves diversity among solutions along the Pareto front.

3. Selection, crossover, and mutation – Genetic operators are employed to explore the solution space, while elitist selection guarantees that the best solutions are preserved across generations.

By combining these mechanisms, NSGA-II can effectively approximate the Pareto front of the gearbox optimization problem, providing a wide

range of feasible trade-off solutions between minimizing the bottom area and maximizing efficiency.

## 3.2 MABAC Method

The MABAC technique, originally introduced in 2015, ranks alternatives by measuring their distances relative to a "border approximation area." The implementation steps followed in this study are synthesized as follows, [21]:

Step 1: Create the initial decision-making matrix:

$$X = \begin{bmatrix} r_{11} & \cdots & r_{1j} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_1 & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix}_{m\times n} \quad (16)$$

where m is the number of alternatives and n is the number of criteria.

Step 2: Normalize the decision matrix:

$$r_{ij}^{*} = \frac{r_{ij} - r_i^{-}}{r_i^{+} - r_i^{-}} \quad (17)$$

$$r_{ij}^{*} = \frac{r_{ij} - r_i^{+}}{r_i^{-} - r_i^{+}} \quad (18)$$

where i=1, 2, ..., m and j=1, 2, ..., n. Equation (17) is applied to benefit criteria, while, and (18) is applied to cost criteria. Here, $r_i^{+} = max(r_1, r_2, \dots, r_m)$ and $r_i^{-} = min(r_1, r_2, \dots, r_m)$.

Step 3: Calculate the weighted matrix elements by:

$$v_{ij} = w_j + w_j \times r_{ij}^{*} \quad (19)$$

Step 4: Compute the border approximation area matrix:

$$g_j = \left(\prod_{i=1}^{m} v_{ij}\right)^{1/m} \quad (20)$$

In which j=1, 2, ..., n.

Step 5: Calculate the distance between the alternatives and the border approximation area by:

$$q_{ij} = v_{ij} - g_i \quad (21)$$

where i=1,2, ..., m and j=1,2, ..., n.

Step 6: Aggregate the distances for each alternative:

$S_i = \sum_{j=1}^{n} q_{ij}$ i = 1, 2, …, m (22)

Step 7: Rank the options by maximizing S.

## 3.3 TOPSIS Method

The TOPSIS is a widely used MCDM method that ranks alternatives based on their relative closeness to an ideal solution. In this study, TOPSIS is employed to rank the Pareto-optimal gearbox designs generated by the NSGA-II algorithm. The fundamental principle of TOPSIS is that the selected alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution.

To apply the TOPSIS method effectively, the following computational steps are carried out in a systematic order, [22].

First, the decision matrix is constructed as:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (23)$$

where $x_{mn}$represents the value of the $n$-th criterion corresponding to the $m$-th alternative.

Next, the decision matrix is normalized to eliminate the influence of different measurement units. The normalized value $k_{ij}$is calculated as

$$k_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad (24)$$

Subsequently, the weighted normalized decision matrix is obtained by incorporating the relative importance of each criterion:

$$l_{ij} = w_j \times k_{ij} \quad (25)$$

where $w_j$denotes the weight of the $j$-th criterion.

The positive ideal solution $A^{+}$and the negative ideal solution $A^{-}$are then determined as:

$$A^{+} = \{l_1^{+}, l_2^{+}, \dots, l_j^{+}, \dots, l_n^{+}\} \quad (26)$$

$$A^{-} = \{l_1^{-}, l_2^{-}, \dots, l_j^{-}, \dots, l_n^{-}\} \quad (27)$$

where $l_j^{+}$and $l_j^{-}$denote the best and worst values of the $j$-th criterion ($j = 1,2, \dots, n$), respectively.

The separation distances of each alternative from the positive ideal solution and the negative ideal solution are computed as:

$$D_i^{+} = \sqrt{\sum_{j=1}^{n} \left(l_{ij} - l_j^{+}\right)^2} \quad (28)$$

$$D_i^{-} = \sqrt{\sum_{j=1}^{n} \left(l_{ij} - l_j^{-}\right)^2} \quad (29)$$

where $i = 1,2, \dots, m$.

Finally, the relative closeness coefficient $R_i$of each alternative is calculated as

$$R_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (30)$$

with $0 \le R_i \le 1$. A higher value of $R_i$indicates that the corresponding alternative is closer to the ideal solution and farther from the anti-ideal solution. Therefore, the ranking of alternatives is determined by maximizing the closeness coefficient $R_i$.

## 3.4 MARCOS Method

The Measurement Alternatives and Ranking according to Compromise Solution (MARCOS) method is an advanced multi-criteria decision-making (MCDM) technique that evaluates alternatives based on their relative utility with respect to both the ideal and anti-ideal solutions. In this study, the MARCOS method is applied to rank the Pareto-optimal gearbox designs obtained from the NSGA-II algorithm. The fundamental concept of MARCOS lies in determining the degree of utility of each alternative by simultaneously considering its closeness to the ideal solution and its distance from the anti-ideal solution.

To implement the MARCOS technique, the following computational steps are strictly followed, [23].

Step 1: Construction of the initial decision-making matrix.
The initial decision matrix is defined as:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \quad (31)$$

where $m$and $n$denote the number of alternatives and criteria, respectively.

Step 2: Extension of the decision matrix. The initial matrix is expanded by incorporating the anti-ideal solution (AAI) and the ideal solution (AI), resulting in the extended matrix

$$X = \begin{matrix} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{matrix} \begin{bmatrix} x_{aa1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{ai1} & \cdots & x_{ain} \end{bmatrix} \quad (32)$$

In Eq. (32), the definitions of the anti-ideal and ideal solutions depend on the nature of each criterion. For the gearbox efficiency objective, $AAI = \min(x_{ij})$and $AI = \max(x_{ij})$. Conversely, for the gearbox length objective, $AAI = \max(x_{ij})$and $AI = \min(x_{ij})$, where $i = 1,2,\dots,m$and $j = 1,2,\dots,n$.

Step 3: Normalization of the extended matrix. The extended decision matrix is normalized to obtain the normalized matrix $U = [u_{ij}]_{m\times n}$ according to the following expressions:

$$u_{ij} = \frac{x_{AI}}{} x_{ij} \quad (33)$$

$$u_{ij} = x_{ij} / x_{AI} \quad (34)$$

Equation (33) is applied to the gearbox bottom area criterion (cost-type objective), whereas Eq. (34) is used for the gearbox efficiency criterion (benefit-type objective).

Step 4: Determination of the weighted normalized matrix.
The weighted normalized matrix $C = [c_{ij}]_{m\times n}$is computed as:

$$c_{ij} = u_{ij} \cdot w_j \quad (35)$$

where $w_j$ represents the weight coefficient associated with the $j$-th criterion.

Step 5: Calculation of the utility degrees.
The utility degrees of the alternatives relative to the anti-ideal and ideal solutions are calculated as

$$K_i^- = \frac{S_i}{S_{AAI}} \quad (36)$$

$$K_i^+ = S_i / S_{AI} \quad (37)$$

where $S_i$is defined as:

$$S_i = \sum_{i=1}^{m} c_{ij} \quad (38)$$

Step 6: Computation of the overall utility function. The overall utility function of each alternative is expressed as:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1-f(K_i^+)}{f(K_i^+)} + \frac{1-f(K_i^-)}{f(K_i^-)}} \quad (39)$$

In Eq. (39), the utility functions associated with the anti-ideal and ideal solutions are given by

$$f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^i} \quad (40)$$

$$f(K_i^+) = K_i^- / (K_i^+ + K_i^i) \quad (41)$$

Step 7: Ranking of alternatives.

Finally, the alternatives are ranked by maximizing the overall utility function $f(K_i)$, with higher values indicating more preferable gearbox designs.

# 4 Results and Discussions

## 4.1 Optimal Solution Analysis based on Multiple MCDM Methods

This subsection presents a comparative analysis of the optimal solutions selected from the Pareto-optimal set using three MCDM methods, namely MABAC, TOPSIS, and MARCOS. The objective is not only to identify the optimal design alternatives corresponding to each method but also to investigate the structural characteristics and decision tendencies induced by different MCDM strategies within the NSGA-II–based optimization framework.

### 4.1.1 Optimal Solutions Selected by MABAC, TOPSIS, and MARCOS

Based on the Pareto-optimal solutions generated by the NSGA-II algorithm, three representative optimal gearbox designs were selected using the MABAC, TOPSIS, and MARCOS methods, respectively. The corresponding values of the first-stage transmission ratio, the overall transmission ratio, and the two objective functions-bottom area and gearbox efficiency-are summarized in Table 1.

As can be observed from Table 1, the optimal solutions obtained by the three MCDM methods do not coincide, although they are all located on the same Pareto front. The TOPSIS- and MARCOS-based solutions exhibit relatively similar objective values, particularly in terms of efficiency, whereas the MABAC-based solution shows a different balance between compactness and efficiency. This initial observation suggests that different MCDM methods may emphasize distinct trade-off preferences when selecting an optimal design from the same Pareto-optimal set.

### 4.1.2 Regression-Based Characterization of Optimal Transmission Ratios

To further characterize the relationship between the overall transmission ratio and the first-stage transmission ratio selected by each MCDM method, a linear regression analysis was conducted. For each method, the optimal first-stage ratio $u_1$was regressed against the corresponding overall ratio $u_h$using an ordinary least squares approach, resulting in regression models of the form:

$$u_1 = \mathrm{a} + \mathrm{b} \cdot \mathrm{u}_h \qquad (42)$$

The predicted-versus-actual relationships for the three MCDM methods are illustrated in Figure 2(a)–(c), while the estimated regression parameters and goodness-of-fit indicators are summarized in Table 2. The results indicate that all three regression models exhibit a strong linear relationship between $u_h$and $u_1$, with coefficients of determination exceeding 0.97. This confirms that, regardless of the decision-making strategy, the allocation of the first-stage transmission ratio follows a consistent structural trend with respect to the overall ratio.

Table 1. Optimal gearbox solutions by different MCDM methods from the Pareto-optimal set

| Method | $u_h$ | $u_1$ | $X_{ba1}$ | $X_{ba2}$ | Ab ($mm^2$) | ηgb (%) |
|---|---|---|---|---|---|---|
| MABAC | 5 | 1.88 | 0.25 | 0.26 | 66907.67 | 96.09 |
| | 10 | 2.85 | 0.25 | 0.29 | 65238.41 | 95.51 |
| | 15 | 3.34 | 0.25 | 0.26 | 65164.71 | 94.85 |
| | 20 | 4.50 | 0.25 | 0.29 | 66291.23 | 94.41 |
| | 25 | 5.26 | 0.25 | 0.28 | 67090.80 | 93.85 |
| | 30 | 5.95 | 0.25 | 0.28 | 67938.97 | 93.31 |
| | 35 | 6.36 | 0.25 | 0.32 | 68970.50 | 92.89 |
| | 40 | 7.10 | 0.25 | 0.30 | 69634.91 | 92.29 |
| TOPSIS | 5 | 1.66 | 0.25 | 0.25 | 66709.05 | 96.06 |
| | 10 | 2.54 | 0.25 | 0.25 | 64807.79 | 95.46 |
| | 15 | 3.37 | 0.25 | 0.26 | 65160.61 | 94.84 |
| | 20 | 4.08 | 0.25 | 0.27 | 65974.92 | 94.24 |
| | 25 | 4.43 | 0.25 | 0.25 | 66677.86 | 93.49 |
| | 30 | 5.01 | 0.25 | 0.25 | 67528.18 | 92.93 |
| | 35 | 5.43 | 0.25 | 0.25 | 68385.06 | 92.39 |
| | 40 | 5.92 | 0.25 | 0.25 | 69189.11 | 91.85 |
| MARCOS | 5 | 1.68 | 0.25 | 0.25 | 66708.00 | 96.06 |
| | 10 | 2.55 | 0.25 | 0.25 | 64807.40 | 95.46 |
| | 15 | 3.37 | 0.25 | 0.26 | 65160.61 | 94.84 |
| | 20 | 3.90 | 0.25 | 0.25 | 65828.52 | 94.06 |
| | 25 | 4.47 | 0.25 | 0.25 | 66671.11 | 93.48 |
| | 30 | 5.01 | 0.25 | 0.25 | 67528.18 | 92.93 |
| | 35 | 5.50 | 0.25 | 0.25 | 68367.64 | 92.38 |
| | 40 | 5.92 | 0.25 | 0.25 | 69189.11 | 91.85 |

*Source: created by author*

Table 2. Regression parameters and goodness-of-fit indicators for different MCDM methods

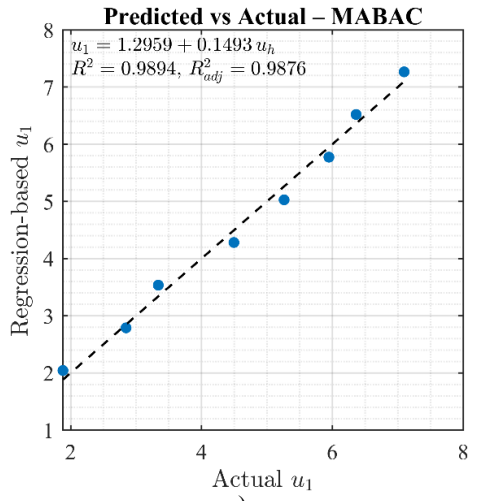
| Method | a | b | $R^2$ | $R^2$adj | RMSE | MAE |
|---|---|---|---|---|---|---|
| MABAC | 1.2959 | 0.1493 | 0.9894 | 0.9876 | 0.1774 | 0.1703 |
| TOPSIS | 1.4005 | 0.1179 | 0.9791 | 0.9757 | 0.1973 | 0.1672 |
| MARCOS | 1.3703 | 0.1190 | 0.9858 | 0.9834 | 0.1639 | 0.1366 |

*Source: created by the authors*

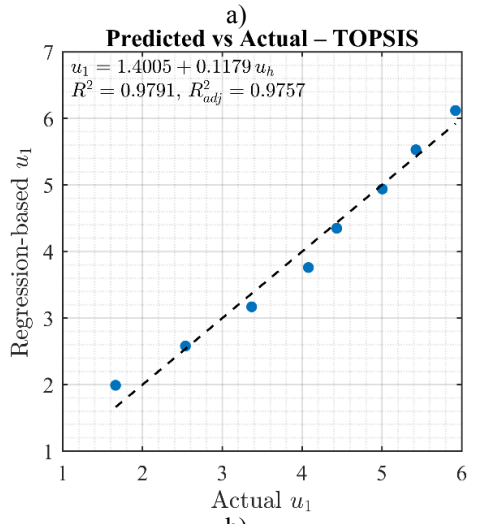
In addition to the conventional coefficient of determination ($R^2$), the adjusted coefficient of determination ($R^2_{\mathrm{adj}}$) is also reported in Table 2 to account for the limited sample size and to provide a more conservative measure of model adequacy. The values of $R^2_{\mathrm{adj}}$ remain very close to the corresponding $R^2$ values for all three methods, indicating that the high goodness of fit is not a result of overestimation due to sample size effects.

Among the considered methods, the MABAC-based regression achieves the highest $R^2$ and $R^2_{\mathrm{adj}}$, suggesting the strongest linear dependence between $u_h$ and $u_1$. In contrast, the MARCOS-based regression yields the lowest prediction errors, as reflected by the minimum RMSE and MAE values,

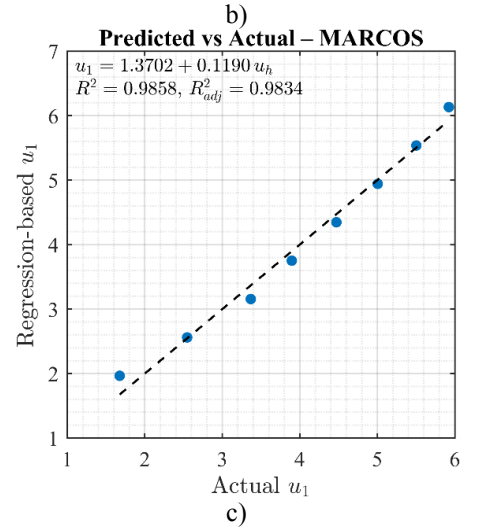
indicating superior accuracy in terms of absolute deviations. The TOPSIS-based regression also demonstrates a high level of fit, although its error metrics are slightly higher than those of MARCOS.




a)




b)




c)

Fig. 2: Predicted versus actual first-stage transmission ratio u1based on linear regression for (a) MABAC, (b) TOPSIS, and (c) MARCOS methods

*Source: created by the authors*

Overall, the regression analysis demonstrates that while all three MCDM methods produce structurally consistent linear relationships between the overall and first-stage transmission ratios, noticeable differences exist in terms of fit quality and error characteristics. These differences reflect the distinct decision-making mechanisms embedded in each MCDM method.

## 4.2 Mean Trends of Bottom Area and Efficiency

Figure 3 presents the mean trends of the Ab and $\eta_{gb}$ across different $u_h$. These trends were obtained by averaging the results of Pareto-optimal solutions for each $u_h$, thereby highlighting the global tendency of the two objectives.

A clear trade-off pattern emerges:

- At low transmission ratios ($u_h$=5–15), the gearbox bottom area remains relatively small, ranging from approximately $6.5 \times 10^4 mm^2$ to $6.6 \times 10^4 mm^2$, while the efficiency stays at its highest level (95.5–96.1%). These conditions represent the most favorable region where both compactness and performance are satisfactorily achieved.
- As the transmission ratio increases ($u_h$=20–25), the bottom area begins to expand moderately (up to around $6.7 \times 10^4 mm^2$), and the efficiency exhibits a steady decline to about 94%. This region reflects the increasing compromise required between structural compactness and power transmission quality.
- At high transmission ratios ($u_h$>30), the bottom area grows significantly, surpassing $6.9 \times 10^4 mm^2$, while efficiency reduces further to nearly 92%. Such conditions are less desirable for most engineering applications, as they indicate that achieving high ratios necessitates a substantial sacrifice in both compactness and performance.

These results underline the inherent conflict between minimizing Ab and maximizing ηgb. Smaller and more compact designs tend to involve narrower facewidths and higher mesh loads, which reduce efficiency due to elevated sliding and load-dependent losses. Conversely, designs targeting higher efficiency often employ larger gear dimensions and greater contact areas, which increase the overall gearbox footprint.

In summary, the mean trend analysis provides a macroscopic view of the design trade-off, clarifying that low to moderate transmission ratios yield the most balanced solutions, while higher ratios impose increasingly restrictive compromises.

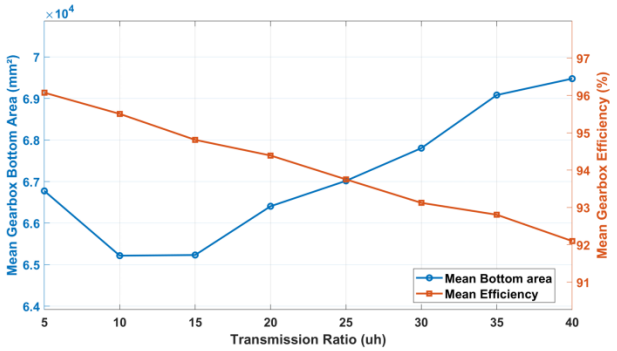



Fig. 3: Mean trends of gearbox bottom area and mean gearbox efficiency with respect to uh
*Source: created by the authors*

### 4.3 Pareto Fronts across Transmission Ratios

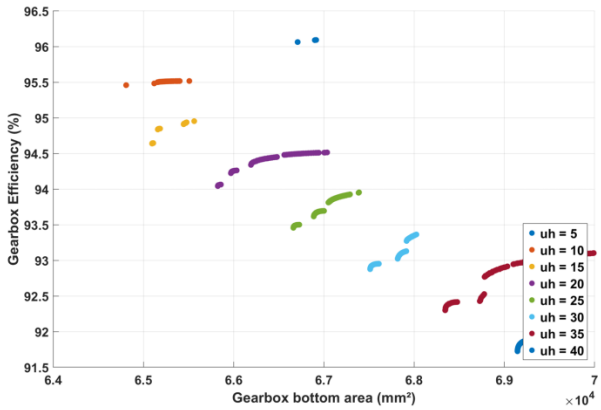



Fig. 4: Pareto-optimal fronts of a b versus ηgb across different uh
*Source: created by the authors*

Figure 4 illustrates the Pareto-optimal fronts generated by NSGA-II for different uh. Each front represents the trade-off between minimizing the Ab and maximizing ηgb under a specific transmission requirement. Several observations can be made:

- At small transmission ratios (uh=5–15), Pareto fronts are positioned toward the upper-left region, corresponding to small bottom areas ($\approx 6.5 \times 10^4 mm^2$) and high efficiencies (>95%). This zone indicates that compact and efficient designs are simultaneously attainable.

- For moderate ratios (uh=20–25), the Pareto fronts shift downward, showing expanded bottom areas ($\approx 6.6 - 6.7 \times 10^4 mm^2$) and reduced efficiencies (≈94%). Here, the conflict between compactness and efficiency becomes more pronounced, as improving one objective increasingly deteriorates the other.

- At large transmission ratios ($u_h$=30–40), the Pareto fronts move to the lower-right region, where designs are characterized by larger bottom areas (> $6.9 \times 10^4 mm^2$) and lower efficiencies (≈92–93%). Such solutions are less attractive in practice, since they fail to deliver a favorable balance between housing compactness and transmission performance.

The comparative layout of these Pareto fronts clearly demonstrates that the overall transmission ratio is the dominant factor shaping the trade-off surface. Smaller uh values create a design space with ample high-quality solutions, while larger uh restrict the feasible region to less desirable compromises.
This analysis reinforces the necessity of employing a multi-objective optimization framework: instead of converging to a single optimum, the Pareto approach reveals the full spectrum of feasible designs, empowering decision-makers to select solutions most appropriate for specific applications.

### 4.4 Spearman Rank Correlation Analysis of MCDM Methods

To further examine the consistency of the ranking results obtained by different MCDM methods, a Spearman rank correlation analysis was conducted between MABAC, TOPSIS, and MARCOS. Unlike metric-based comparisons of objective values, Spearman's rank correlation coefficient ($\rho$) evaluates the degree of agreement in the ordinal ranking of alternatives. This analysis therefore provides additional insight into the structural similarity or divergence among the decision-making strategies when selecting optimal gearbox designs from the same Pareto-optimal set.

#### 4.4.1 Rank Consistency across Transmission Ratios

Figure 5 presents a heatmap of pairwise Spearman correlation coefficients as a function of the overall transmission ratio $u_h$. A clear contrast can be observed among the three method pairs. The correlation between TOPSIS and MARCOS remains consistently high across the entire investigated range of $u_h$, with $\rho$ values close to unity for most transmission ratios. This indicates that TOPSIS and MARCOS produce highly consistent ranking results and tend to prioritize design alternatives in a very similar manner.

In contrast, the correlations involving the MABAC method exhibit markedly different behavior. For lower values of $u_h$, negative or weak correlations are observed between MABAC-TOPSIS and MABAC-MARCOS, indicating substantial disagreement in ranking outcomes. As $u_h$increases, the correlations fluctuate around zero and occasionally become moderately positive; however, they do not reach the consistently high levels observed for the TOPSIS–MARCOS pair. This

pattern suggests that MABAC follows a distinct ranking logic that responds differently to changes in the transmission ratio.

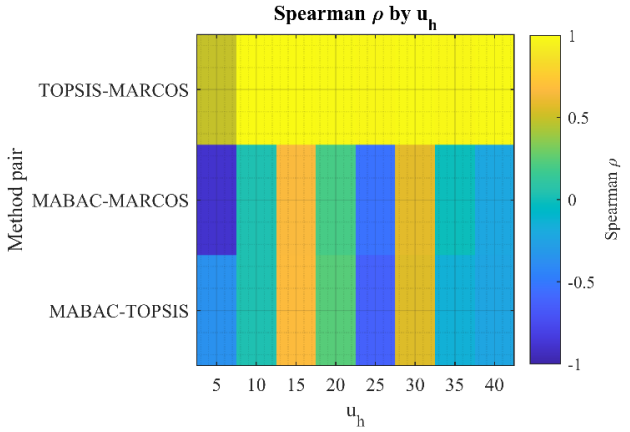



Fig. 5: Heatmap of pairwise Spearman rank correlation coefficients between MABAC, TOPSIS, and MARCOS as a function of uh

*Source: created by the authors*

### 4.4.2 Evolution of Pairwise Rank Correlations with uh

The detailed evolution of Spearman's $\rho$with respect to $u_h$is illustrated in Figure 6, where the pairwise correlation coefficients are plotted as continuous functions of the overall transmission ratio. The TOPSIS–MARCOS curve remains close to $\rho =$ 1over nearly the entire domain, confirming the strong agreement already observed in the heatmap representation. This stability implies that, from a ranking perspective, TOPSIS and MARCOS can be considered largely interchangeable for the gearbox design problem addressed in this study.

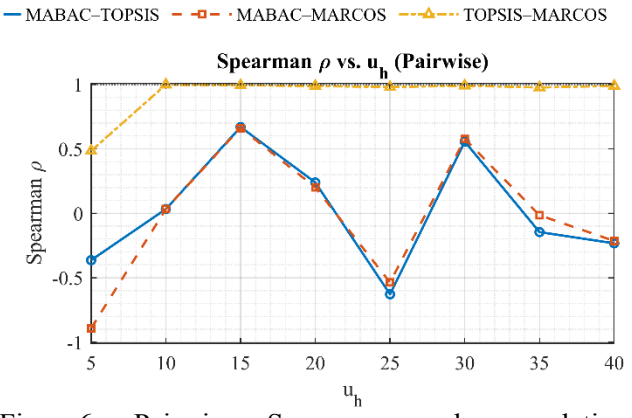



Fig. 6: Pairwise Spearman rank correlation coefficients versus uh for MABAC–TOPSIS, MABAC–MARCOS, and TOPSIS–MARCOS

*Source: created by the authors*

On the other hand, the MABAC–TOPSIS and MABAC–MARCOS curves display pronounced oscillations, including sign changes at specific values of $u_h$. Such behavior indicates that the relative ordering of alternatives produced by MABAC may differ substantially from that of TOPSIS and MARCOS depending on the transmission ratio. It reflects a fundamentally different decision-making philosophy embedded in the MABAC method, which emphasizes alternative trade-offs among the considered criteria.

### 4.4.3 Discussion on Decision Consistency and Methodological Implications

The Spearman rank correlation analysis highlights important methodological implications for multi-objective gearbox design. The consistently high correlation between TOPSIS and MARCOS demonstrates that these two methods provide robust and mutually consistent ranking results across different transmission ratios. In contrast, the lower and sometimes negative correlations involving MABAC reveal that this method offers an alternative and complementary perspective on the Pareto-optimal solutions.

From a practical standpoint, these findings suggest that designers seeking stable and reproducible ranking outcomes may expect similar decisions when employing TOPSIS or MARCOS. Conversely, the use of MABAC can be beneficial when the objective is to explore alternative ranking structures and to gain additional insight into the influence of trade-offs between compactness and efficiency. Overall, the comparative analysis confirms that the proposed NSGA-II–MCDM framework not only facilitates the selection of optimal gearbox designs but also enables a systematic evaluation of decision consistency among different MCDM methods.

## 5 Conclusions

This study presented a comparative NSGA-II–MCDM framework for the bi-objective optimization of two-stage helical gearboxes with a dual-gear first stage, aiming to minimize the gearbox bottom area while maximizing transmission efficiency. The NSGA-II algorithm was first employed to generate a diverse and well-distributed Pareto-optimal set, capturing the inherent trade-off between compactness and efficiency across different transmission ratios.

To select representative optimal designs from the Pareto set, three multi-criteria decision-making methods—MABAC, TOPSIS, and MARCOS—were applied and comparatively evaluated. The results demonstrated that, although all three methods operate on the same Pareto-optimal solutions, they

can lead to different optimal selections and design tendencies. The regression-based characterization further revealed that each method induces a distinct linear relationship between the overall transmission ratio and the first-stage transmission ratio, reflecting different preferences in balancing geometric compactness and efficiency.

A Spearman rank correlation analysis was subsequently conducted to assess the consistency of ranking results among the considered MCDM methods. The analysis showed that TOPSIS and MARCOS exhibit consistently high rank correlation across the investigated transmission ratio domain, indicating strong agreement and robustness in their ranking behavior. In contrast, the correlations involving MABAC were found to be weaker. This divergence suggests that MABAC provides a complementary decision-making perspective that may uncover alternative trade-off structures within the Pareto-optimal set.

Overall, the findings confirm that the proposed comparative NSGA-II–MCDM framework not only enables effective identification of optimal gearbox designs but also facilitates a deeper understanding of decision consistency and methodological differences among MCDM approaches. From an engineering design standpoint, TOPSIS and MARCOS can be regarded as reliable tools when stable and reproducible ranking outcomes are desired, whereas MABAC can serve as a valuable alternative for exploring diverse design preferences. The framework developed in this study therefore offers a systematic and transparent basis for robust decision-making in multi-objective gearbox optimization.

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**Declaration of Generative AI and AI-assisted Technologies in the Writing Process**
During the preparation of this work, the author(s) utilized artificial intelligence (AI) technologies solely for the purpose of linguistic refinement and grammatical correction. Following the application of these tools, the author(s) comprehensively reviewed, verified, and edited the resulting text. The author(s) maintain full accountability and responsibility for the accuracy, integrity, and final content of the publication.

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**Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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**Conflict of Interest**

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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