

Volume-Efficiency Trade-Off Optimization of Two-Stage Helical Gearboxes via MOEA/D-DE Algorithm

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Abstract: - The design of two-stage helical gearboxes inherently involves a trade-off between structural compactness and transmission efficiency. In this study, a multi-objective optimization framework based on the MOEA/D-DE (Multi-Objective Evolutionary Algorithm based on Decomposition with Differential Evolution) is proposed to simultaneously minimize gearbox volume and maximize power transmission efficiency. The optimization formulation integrates essential geometric and performance-related constraints to ensure feasible and manufacturable designs. By decomposing the bi-objective optimization problem into a set of scalar subproblems, the MOEA/D-DE algorithm effectively preserves solution diversity while achieving reliable convergence toward the Pareto-optimal front. Numerical investigations demonstrate that the proposed approach yields a well-distributed set of trade-off solutions, providing clear design insights into the balance between size reduction and efficiency enhancement. Comparative results indicate that the proposed framework achieves superior solution quality and computational performance compared to conventional optimization methods.

Key-Words: - Two-stage helical gearbox, Multi-objective optimization, MOEA/D-DE, Gearbox volume, Transmission efficiency, Evolutionary algorithm, Design trade-off.

Received: March 12, 2026. Revised: July 11, 2026. Accepted: August 7, 2026. Available online: September 25, 2026.

1 Introduction

Helical gearboxes constitute a fundamental component of modern mechanical power transmission systems, owing to their superior load-carrying capability, enhanced meshing continuity, and high operational reliability. Nevertheless, the design of multi-stage configurations - particularly two-stage helical gearboxes - poses a challenging multi-criteria decision problem, as improvements in structural compactness (i.e., reduced gearbox volume) are inherently antagonistic to achieving high power transmission efficiency. Traditional design approaches, which focus on single-objective or heuristic-based tuning, are insufficient to capture the complexity of this design space.

Multi-objective optimization (MOO) methods have become increasingly relevant in gearbox design as they enable the simultaneous consideration of multiple conflicting objectives. Early studies employed conventional optimization techniques for basic spur gear sets, [1], yet these methods lacked scalability and adaptability to more complex gear architectures. Evolutionary algorithms (EAs), as introduced comprehensively by [2], have proven to be more effective in solving complex multi-objective problems due to their population-based nature and global search capabilities.

Several researchers have applied EAs to gearbox optimization with promising outcomes. The study [3], developed a genetic algorithm (GA)-based approach to automate the preliminary design

of gear drives, achieving significant reductions in development time. Similarly, in [4], applied a genetic algorithm to automate the preliminary design of gear drives, demonstrating the effectiveness of GA in determining optimal gear parameters while satisfying multiple design constraints. However, these approaches were primarily mono-objective or lacked decomposition strategies for better solution diversity.

To enhance solution quality, recent studies have integrated advanced metaheuristics. The study [5], combined Particle Swarm Optimization (PSO) with Simulated Annealing (SA) to minimize the weight of gear trains, showing improved convergence over individual methods. The study [6], introduced tribological aspects into MOO of spur gearboxes, emphasizing real-world operational concerns.

A more direct comparison of evolutionary algorithms was provided by [7], who benchmarked different EAs in gear system design. They concluded that decomposition-based methods offer superior diversity preservation. This observation is consistent with the findings of [8], who employed the Non-dominated Sorting Genetic Algorithm II (NSGA-II) for the multi-objective optimization of two-stage helical gear trains with respect to strength and structural compactness. Despite its widespread adoption, NSGA-II is known to exhibit limitations in densely populated objective spaces, where the accuracy of the crowding-distance mechanism may deteriorate, potentially impairing solution diversity and Pareto-front resolution.

To overcome these limitations, the MOEA/D-DE algorithm, integrating the decomposition-based optimization framework of MOEA/D with the evolutionary search capability of differential evolution, has emerged as a robust and scalable alternative for multi-objective optimization. Recent advances in gearbox optimization have increasingly focused on balancing multiple conflicting objectives through evolutionary and hybrid optimization frameworks. Maputi and Arora [9] applied NSGA-II combined with decision-making techniques to improve the performance of multi-stage spur gearboxes under competing design criteria. In [10] investigated the multi-objective optimization of gear units with emphasis on transmission efficiency and transmission error reduction, highlighting the importance of integrating performance-oriented objectives into gearbox design. Similarly, in [11] conducted a multi-objective optimization study focused on simultaneously minimizing gearbox volume and maximizing efficiency, demonstrating the practical relevance of trade-off-based optimization strategies in modern gear transmission

systems. Although effective, these methods rely heavily on preference-based decision-making post-optimization.

In this paper, we propose a novel MOEA/D-DE based framework for the dual-objective optimization of two-stage helical gearboxes, targeting the minimization of gearbox volume and the maximization of transmission efficiency. Our contribution lies in integrating geometric, physical, and performance constraints into a unified bi-objective formulation while leveraging the decomposition-based search mechanism of MOEA/D-DE to maintain a diverse and convergent Pareto front. Numerical experiments demonstrate the superiority of our method over traditional approaches in terms of both solution quality and computational efficiency.

2 Optimization Problem

2.1 Gearbox Volume Calculation

The volume of a two-stage helical gearbox can be calculated by (1) (Figure 1):

$$V_{gb} = L \cdot B \cdot H \quad (1)$$

where L , B , and H are the length, width, and height of the gearbox, respectively, and are determined by (2) to (4):

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta \quad (2)$$

$$B = b_{w1} + b_{w2} + 6 \cdot \delta \quad (3)$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot \delta \quad (4)$$

In which, δ is a design clearance parameter, typically ranging from 7 to 10 mm, [12]. b_{wi} , d_{w1i} , and d_{w2i} (for $i=1$ to 2) are the gear face width, pitch diameter of the pinion, and pitch diameter of the gear in stage i , respectively. These elements are defined by (5) to (7):

$$b_{wi} = X_{bai} \cdot a_{wi} \quad (5)$$

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (6)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (7)$$

In which, X_{bai} and a_{wi} ($i=1 \div 2$) denote the face width coefficient and the center distance of stage i ; a_{wi} is calculated by (8), [12]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (8)$$

where T_{1i} ($i=1 \div 2$) is the pinion torque of stage i which is computed by (9):

$$T_{1i} = \frac{T_r}{\prod_{j=i}^3 (u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \quad (9)$$

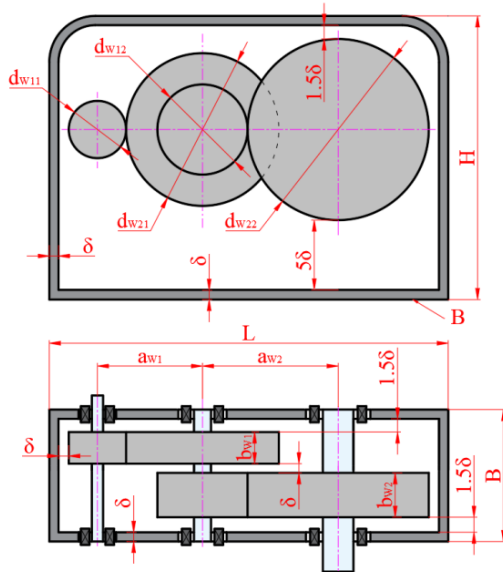


Fig. 1: For the determination of the gearbox volume
Source: created by the authors

2.2 Gearbox Efficiency Calculation

The overall efficiency of the gearbox η_{gb} can be defined as (10):

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (10)$$

In which, P_l is the total power loss in the system, and P_{in} is the input power. The total loss P_l can be determined as (11), [13]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{zo} \quad (11)$$

where P_{lg} , P_{lb} , P_{ls} , and P_{zo} denote gear meshing loss, bearing friction loss, seal resistance loss, and loss due to idle motion, respectively. The detailed determination of these loss elements can be found in [14].

2.3 Objective Functions

The design optimization of the two-stage helical gearbox is formulated as a bi-objective problem, seeking to simultaneously enhance structural compactness and transmission efficiency. These objectives are inherently antagonistic, as improvements in one criterion typically entail trade-offs that adversely affect the other. Accordingly, the objective functions are formulated as follows:

- Minimization of gearbox volume as (12):

$$\min f_1(X) = V_{gb} \quad (12)$$

- Maximization of gearbox efficiency as (13):

$$\max f_2(X) = \eta_{gb} \quad (13)$$

In which, η_{gb} is the overall gearbox efficiency as defined in Equation (10), and P_l includes the

cumulative power losses from gears, bearings, seals, and idle operation.

As most multi-objective evolutionary algorithms, including MOEA/D-DE, are inherently formulated to handle minimization problems, the second objective is therefore reformulated into an equivalent minimization form, as (14):

$$\min f_2(X) = 100 - \eta_{gb} \quad (14)$$

2.4 Constrains

In the optimization of two-stage helical gearboxes, it is essential to incorporate design constraints that ensure both geometric feasibility and performance integrity. Such constraints guarantee that the resulting gearbox configurations are not only manufacturable but also capable of reliable and efficient operation under practical working conditions.

One of the primary parameters subject to constraint is the gear ratio u_i for each stage ($i=1, 2$). This ratio governs the speed reduction and torque amplification characteristics of the gearbox. From a practical engineering perspective, accounting for mechanical reliability, load-sharing behavior, and packaging limitations, the gear ratio of each stage is therefore constrained as (15) and (16):

$$1 \leq u_i \leq 9 \quad (15)$$

$$0.25 \leq Xba_i \leq 0.4 \quad (16)$$

3 Problem Formulation in MOEA/D-DE

The MOP for the design of a two-stage helical gearbox is addressed using the MOEA/D-DE. This advanced evolutionary framework transforms the original MOP into a collection of scalar subproblems, which are solved concurrently using a population of evolving candidate solutions. The approach leverages both problem decomposition and the exploration capabilities of differential evolution to efficiently search for optimal trade-off solutions.

The general multi-objective optimization problem is mathematically stated as (17):

$$\min F_x = [f_1(x), f_2(x)] \quad (17)$$

where:

- $x = [u_1, X_{ba1}, X_{ba2}]$ is the decision variable vector,
- $f_1(x) = V_{gb}(u_1, X_{ba1}, X_{ba2})$ represents the volume of the gearbox, which is to be minimized,

- $f_2(x) = -\eta_{gb}(u_1, X_{ba1}, X_{ba2})$ is the negative of the gearbox efficiency, reformulated for minimization purposes.

The decision variables are subject to practical engineering bounds to ensure manufacturability and operational feasibility, as defined in Equations (15)–(16), to ensure feasibility in terms of manufacturability, structural integrity, and operational functionality.

In the MOEA/D-DE framework, the original bi-objective problem is decomposed into NNN scalar optimization subproblems using a uniformly distributed set of weight vectors $\lambda^{(1)}, \lambda^{(2)}, \dots, \lambda^{(N)}$. Each subproblem is optimized independently but cooperatively through information sharing among neighboring solutions.

A commonly used decomposition method is the Tchebycheff approach, formulated as (18) [2]:

$$\min g^{te}(x|\lambda^{(i)}, z^*) = \max_{1 \leq j \leq m} \{\lambda_j^{(i)} \cdot |f_j(x) - z_j^*|\} \quad (18)$$

In which, $z^* = (z_1^*, z_2^*)$ is the ideal objective vector, and $m=2$ is the number of objectives.

The differential evolution (DE) operator embedded within MOEA/D-DE is employed to generate new solutions via mutation and crossover. The mutation operation follows the DE/rand/1 scheme, expressed as [15]:

$$V = x_r^{(1)} + F \cdot (x_r^{(2)} - x_r^{(3)}) \quad (19)$$

where $x_r^{(1)}, x_r^{(2)}, x_r^{(3)}$ are distinct and randomly selected individuals from the neighborhood or the overall population, and F is the mutation scaling factor, typically within the range $0.4 \leq F \leq 1.0$.

The resulting mutant vector is combined with the current vector using binomial crossover, producing a trial solution u . This trial solution replaces the original solution if it offers a better fitness value in the context of the corresponding scalar subproblem.

By integrating decomposition strategies with evolutionary operators, MOEA/D-DE enables effective exploration and exploitation of the search space. It yields a well-distributed set of Pareto-optimal solutions, offering designers a comprehensive view of the trade-off surface between gearbox compactness and efficiency.

4 Results and Discussions

This section presents and discusses the numerical results obtained from the MOEA/D-DE-based

optimization of the two-stage helical gearbox. The main focus lies in analyzing the effects of varying the total transmission ratio u_h on the Stage 1 gear ratio u_1 , gearbox volume, and efficiency, as well as evaluating the trade-off relationship between the two conflicting objectives.

4.1 Relationship Between u_1 and u_h

Figure 2 depicts the relationship between the first-stage gear ratio u_1 and the overall transmission ratio u_h . The data points represent the optimal values of u_1 obtained through the optimization process at different u_h values. A linear regression model was applied to the data, yielding Equation (20) with a high coefficient of determination ($R^2 = 0.9870$), which indicates a strong linear relationship between the variables.

$$u_1 = 0.208 \cdot u_h + 1.7819 \quad (20)$$

This trend confirms that as the required overall transmission ratio increases, the optimal first-stage gear ratio also increases almost proportionally. This result is consistent with the fundamental design principles of staged gearboxes, wherein higher values of u_h necessitate a more balanced contribution from both stages to preserve mechanical equilibrium and mitigate power losses.

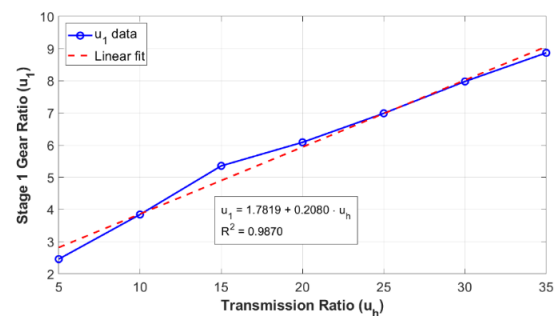


Fig. 2: Linear regression of u_1 and u_h

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4.2 Mean Gearbox Volume and Efficiency Trends

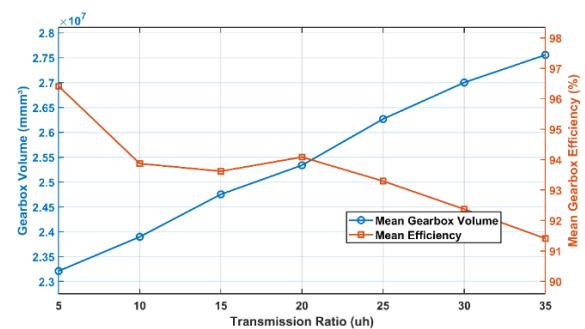


Fig. 3: Variation of mean gearbox volume and mean gearbox efficiency with respect to u_h

Source: created by the authors

Figure 3 presents the variation of mean gearbox volume and efficiency as functions of the overall transmission ratio u_h . The results indicate a monotonic increase in gearbox volume with increasing u_h , particularly within the range of 20-35. This behavior is anticipated, as higher transmission ratios generally necessitate larger gear diameters and increased center distances, which consequently lead to enlarged housing dimensions.

In contrast, gearbox efficiency exhibits a decreasing trend as u_h increases. Initially, for lower u_h values (5 to 15), the efficiency remains relatively high and stable. However, beyond $u_h=20$, a more pronounced drop in efficiency is observed. This phenomenon is attributed to increased gear meshing losses, bearing friction, and idle power consumption in high-ratio configurations. Therefore, a balance between acceptable volume and desirable efficiency becomes critical for selecting an optimal design point.

4.3 Volume-Efficiency Trade-Off Analysis

The Pareto front of optimal solutions across different u_h values is presented in Figure 4. Each point on the scatter plot corresponds to a non-dominated solution for a given value of u_h , with gearbox volume represented on the x-axis and transmission efficiency on the y-axis. As anticipated, the figure reveals a pronounced trade-off between the two objectives: configurations achieving higher efficiency are generally associated with larger gearbox volumes, whereas more compact designs tend to exhibit reduced efficiency.

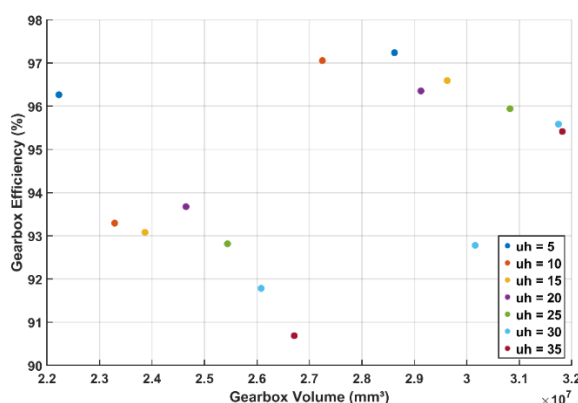


Fig. 4: Pareto front showing the trade-off between gearbox volume and efficiency for various transmission ratios u_h

Source: created by the authors

Among the obtained solutions, the configuration corresponding to $u_h = 5$ attains the highest transmission efficiency (approximately 96.5%)

while simultaneously exhibiting the smallest gearbox volume (approximately $2.3 \times 10^7 \text{ mm}^3$), rendering it particularly attractive for applications requiring both compactness and high performance. In contrast, at $u_h = 35$, the gearbox demonstrates a marked reduction in efficiency (approximately 91%) accompanied by a substantial increase in volume (approximately $3.2 \times 10^7 \text{ mm}^3$), thereby highlighting the performance penalties associated with excessively high transmission ratios.

The resulting Pareto front provides a systematic basis for decision-making, enabling designers to select gearbox configurations in accordance with specific application priorities, such as maximizing efficiency for continuous-duty operation or minimizing size for space-constrained installations. Furthermore, the capability of the MOEA/D-DE algorithm to generate a well-distributed and high-quality set of Pareto-optimal solutions across the entire design space confirms its effectiveness and suitability for multi-objective optimization of two-stage helical gearboxes.

4.4 Statistical Stability Analysis Based on Normalized Hypervolume

To strengthen the quantitative evaluation of the proposed MOEA/D-DE framework and to address the reviewer's comment regarding the lack of deeper statistical analysis, a hypervolume (HV)-based stability assessment was conducted over 30 independent runs for each transmission ratio u_h .

The distributions of the normalized hypervolume (HV) values are presented in Figure 5. While Figure 4 illustrates the trade-off characteristics for representative solutions, Figure 5 statistically confirms the stability and repeatability of these results across multiple independent runs. In this analysis, a shared normalization scheme was applied across all runs for each u_h , thereby eliminating the scale dependency of the objective space and enabling a fair statistical comparison of solution quality. As the hypervolume simultaneously reflects both convergence and diversity of Pareto-optimal solutions, it serves as a comprehensive performance indicator for multi-objective optimization.

As observed in Figure 5, the median HV values increase from $u_h = 5$ to $u_h = 20$, indicating a progressive improvement in the convergence-diversity balance of the obtained solution sets. Notably, the case $u_h = 20$ achieves the highest median HV with a relatively narrow interquartile range, demonstrating not only superior solution quality but also strong algorithmic stability across

repeated runs. For higher transmission ratios ($u_h = 25$ and $u_h = 30$), the median HV remains relatively high; however, a wider dispersion is observed, reflecting the increased complexity and constraint intensity of the design space. When u_h reaches 35, the normalized HV decreases significantly, suggesting that stringent geometric and performance constraints limit the availability of high-quality trade-off solutions.

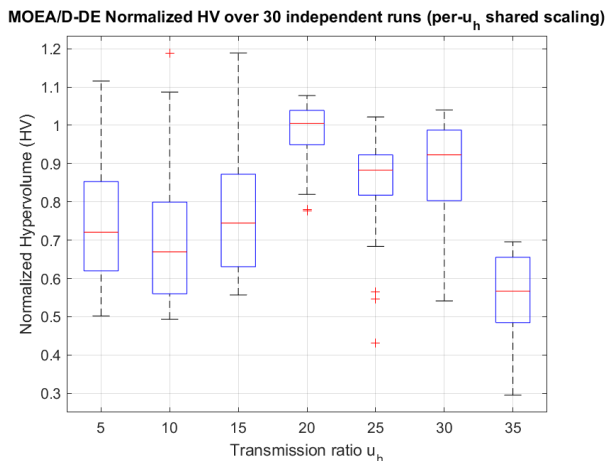


Fig. 5: Boxplot of normalized hypervolume (HV) over 30 independent runs for different transmission ratios u_h using shared per- u_h scaling

Source: created by the authors

Overall, the normalized hypervolume analysis in Figure 5 provides robust statistical evidence that the proposed MOEA/D-DE algorithm consistently generates high-quality and well-balanced Pareto solutions, particularly for moderate transmission ratios. This statistical validation substantially strengthens the reliability of the optimization results beyond single-run or purely visual Pareto-front analysis.

5 Conclusions

This study presents a comprehensive multi-objective optimization approach for the design of two-stage helical gearboxes, aiming to minimize structural volume while maximizing transmission efficiency. By employing the MOEA/D-DE algorithm, the complex trade-off between these two conflicting objectives was effectively explored, yielding a well-distributed Pareto front of optimal solutions.

The results reveal a strong linear relationship between the first-stage gear ratio u_1 and the total transmission ratio u_h , providing a practical guideline for initial gear train partitioning. Furthermore, analysis of the mean trends indicates that increasing u_h leads to a substantial rise in

gearbox volume and a simultaneous decline in efficiency—highlighting the necessity of balanced design choices. The generated Pareto-optimal solutions offer valuable insights for designers to select gearbox configurations that best meet specific application requirements in terms of space constraints and energy performance.

Furthermore, the normalized hypervolume (HV) analysis conducted over 30 independent runs confirms the statistical stability and repeatability of the proposed MOEA/D-DE framework. The results demonstrate that the obtained Pareto-optimal solutions exhibit consistent convergence–diversity characteristics across multiple executions, particularly for moderate transmission ratios. This statistical validation reinforces the reliability and practical applicability of the proposed approach for multi-objective optimization of two-stage helical gearbox designs.

Acknowledgment:

This work has been supported by Thai Nguyen University of Technology.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed to the present research, at all stages from the formulation of the problem to the final findings and solution.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself

Thai Nguyen University of Technology, VIETNAM.

Conflict of Interest

The authors have no conflicts of interest to declare.

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