Criterion-Based Method for Evaluating Material Strength Utilizing Reference Reflexes of Electrical Potentials

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Abstract: - Utilizing continuum medium models and the electron work function, a linear correlation was established connecting mechanical stresses to the electric potential difference in the surface layers of a metal. It was demonstrated that, given a metallic surface in the major stress coordinate system $\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_y}$, the orientations of the electric field intensities E_x and E_y coincide with the directions of $\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_y}$, respectively. An equation was derived for the sum of the diagonal components of the mechanical stress tensor as a function of the electric potential difference. The proportionality coefficient in this expression denotes the electric charge density associated with the kinetic process of electron transport between metallic surfaces.

Key-Words: - contact potential difference, stress tensor, electric potential gradient, stress-strain state, stress intensity, supporting reflex.

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1 Introduction

In electrophysical chromatography, there are two fundamentally different techniques to measuring material strength based on electrical potentials: the criterion-based approach and the wave-based approach. Both approaches deliver practical and reliable results. In the field of non-destructive electrical testing, this is especially valuable because it opens the way for the development of an efficient computational tool that can be applied in a wide range of applications.

This paper provides a rationale for applying a criterion-based approach to evaluating the strength and statistical fracture behavior of metals. The approach relies on the classical concept of a continuous and homogeneous medium, where structural inhomogeneities are regarded as local disturbances governed by Saint-Venant's principle of locality.

In the criterion-based approach, the electric potential within an elementary volume inside the specimen or near its surface is considered as a scalar quantity characterizing the energy state of the field or the potential energy of a charge within the elementary volume.

At the same time, under a complex stress state, according to the criteria of stress intensity and maximum shear stress within the framework of the continuum model, the stress at each point of the specimen must remain below the limiting permissible value. This condition ensures the continuity and integrity of the elastic matrix in the criterion-based approach. However, it should be noted that the formation of discontinuities and even cracks in a continuous medium does not contradict the concept of the proposed model.

In the general case, the local region in which the sought solution exists is bounded, in the (x,y) coordinate plane, by an ellipse or an irregular hexagon.

The question of the shape of the reference reflexes in potentiograms remains open. To date, the reflex is considered a representation of electrical potentials providing information about an existing structural defect. As a starting point in the present work, the following validated assumptions are adopted:

- The reflex features observed on the potentiograms are characterized by the geometry of an irregular hexagon in the two-dimensional (x,y) coordinate system, while in the three-dimensional (x,y,z) space they take the form of a hexagonal pyramid.

- The major, or elongated, diagonal of the hexagonal structure is oriented at an angle of approximately 45 ° with respect to the directions of the principal normal stresses.
- A discrete and spatially non-uniform distribution of electric potential is identified within the boundaries of the hexagonal region.

An alternative interpretation may be provided through a wave-based approach, which postulates the presence of an inhomogeneous distribution of electron density and deformation potential. Such a framework creates the possibility for applying electrophysical chromatography techniques in the analysis of the investigated phenomena.

2 Criteria for Evaluating Mechanical Stress in the Continuum Medium Model

Utilizing models of static fracture in metals [1], [2], we will examine a metal plate under tensile strain. The mechanical stress at various positions within the plate material will be defined by the equivalent stress σ_{eq}, expressed as a function:

$$\sigma_{\text{экв}} = f(\sigma_1, \sigma_2, \sigma_3; \alpha, \beta, \gamma, \dots), \tag{1}$$

where σ_1, σ_2, σ_3 represent the major stresses, with $\sigma_1 > \sigma_2 > \sigma_3$; and α, β, γ, … are parameters contingent upon the material's mechanical properties.

We employ the established expression for the stress intensity criterion:

$$\sigma_{\text{экв}} = \sigma_i, \tag{2}$$

where

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}{}^2+\tau_{xy}{}^2+\tau_{xy}{}^2)} \tag{3}$$

The stress intensity. When the x, y, and z axes coincide with the principal axes of the stress state, the shear stresses τ_{ij} become zero. Under these conditions, expression (3) can be written in the following form:

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2}, \tag{4}$$

For the case of a plane stress state($\sigma_z = 0$), the expression reduces to the following form:

$$\sigma_i = \sqrt{\sigma_x{}^2+\sigma_y{}^2-\sigma_x\sigma_y}. \tag{5}$$

In the principal stress plane, expression (5) describes an elliptical curve. As the values of σ_x and σ_y, vary successively, a family of concentric ellipses is formed (Figure 1).

In the principal stress plane, expression (5) corresponds to an elliptical curve. As the values of σ_x and σ_y, vary incrementally, a family of concentric ellipses is generated (Figure 1).

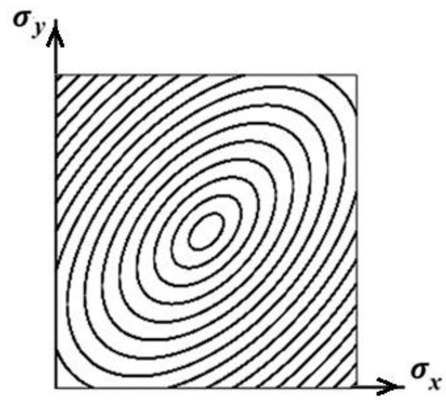

Fig. 1: Graphical representation of the stress intensity function in the principal stress coordinate plane for different increments of σ_{xi} *and* σ_{yi}.
Source: created by the authors

According to the above-mentioned strength criterion, the integrity of the specimen is preserved only if the following inequality is satisfied at every point within the material:

$$\sigma_i \le \sigma_{\text{B}}, \tag{6}$$

where σ_{B} denotes the ultimate tensile strength.

Using the classical formulation of the maximum shear stress criterion, the equivalent stress expression (2) may be written as:

$$\sigma_{\text{экв}} = \sigma_1 - \sigma_3 \le \sigma_{\text{B}}. \tag{7}$$

To evaluate σ_{eq}, one of the three corresponding expressions is selected according to the following condition:

$$\sigma_{\text{экв}} = max\{|\sigma_x-\sigma_y|, |\sigma_y-\sigma_z|, |\sigma_z-\sigma_x|\} \le \sigma_{\text{B}}. \tag{8}$$

It is then necessary to reconsider the geometry of the irregular hexagon and apply condition (7) while taking into account the discrete distribution of the electric potential within the hexagonal region, [3]. This approach leads to the formation of a family of concentric hexagons, as illustrated in Figure 2. It should be emphasized that such a potential distribution is observed only when a linear relationship exists between the electric potential and the mechanical stress.

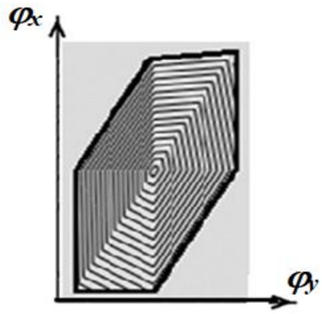

Fig. 2: Representation of concentric hexagons of electric potential distribution in the reference reflexes corresponding to the maximum shear stress criterion.
Source: created by the authors

Thus, in both approaches, for a continuous medium containing distributed local inhomogeneities, the governing conditions impose limits on the principal stresses σ_x and σ_y, which in turn correspond to bounded values of the electric potentials φ_x and φ_y .

2.1 Application of the Criterion-Based Approach to the Distribution of Electric Potentials within Reference Reflexes

When the conditions corresponding to the stress intensity criterion and the maximum shear stress criterion are satisfied, the distribution of electric potentials on metallic surfaces is governed by the boundary conditions and the electron work function.

In the present analysis, averaged electronic characteristics are employed under the assumption that the medium contains an additional component represented by a gas of nearly free electrons, [4]. This assumption is consistent with the fundamental postulates and properties of a continuous medium. Indeed, such an approach is particularly appropriate for metals, where the dominant contribution to interatomic interactions is provided by conduction electrons.

By definition, the electric potential difference $\Delta\varphi$ between points (1) and (2) on the surface is expressed as:

$$\Delta\varphi = \varphi_1 - \varphi_2. \tag{9}$$

Based on the above considerations, an electric potential gradient exists within the reference reflex:

$$grad\varphi(x,y) = \frac{\partial\varphi}{\partial x}\mathbf{i} + \frac{\partial\varphi}{\partial y}\mathbf{j}, \tag{10}$$

where **i** and **j** denote unit vectors directed along the x and y axes, respectively. The magnitude of the gradient, as well as the electric field intensity **E** at each point on the specimen surface, it can be determined from the following expressions:

$$|grad\varphi| = \sqrt{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2}, \tag{11}$$

$$\mathbf{E} = -grad\varphi. \tag{12}$$

Figure 3 presents an example of the electric potential distribution associated with a reference reflex generated by mechanical grinding of the metallic surface of the specimen.

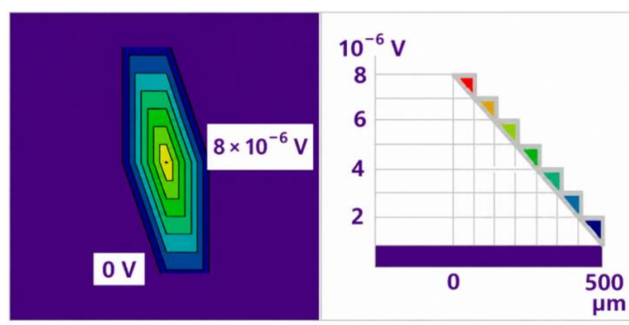

Fig. 3: Reference reflex produced by grinding a friction-welded joint (left) and the corresponding color map of the electric potential gradient (right).
Source: [3]

It should be emphasized that the potential gradient is independent of the geometrical shape of the reference reflex and appears in both the hexagonal and ellipsoidal configurations.

The specific method of surface treatment is not of fundamental importance, since reference reflexes obtained under different treatment conditions

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likewise exhibit the geometry of an irregular hexagon.

To date, approximately twenty reference reflexes have been identified for different specimen geometries, material types, and non-destructive testing conditions.

If the dependence of the direction of the hexagon axis *BE*, shown in Figure 2, on the orientation of the x -axis is taken into account (in the left part of Figure 3, the x axis is directed oppositely), then expressions (11) and (12) indicate that the directions of the electric field components E_x and E_y coincide with the directions of the principal stresses σ_x and σ_y, respectively. In other words, tensile loading of the plate results in the formation of a localized electric field within the hexagonal region, oriented along the principal stress axes.

Measurements of $\Delta\varphi_j$ are always carried out with respect to a fixed reference potential,
$\varphi_2 = const$, for example, the system ground. Accordingly, expression (9) may be rewritten as:

$$\Delta\varphi_j = \varphi_j + const, \tag{13}$$

Thus, the potential difference at point j can be considered as being determined up to an additive constant.
For the work function of nearly free electrons under conditions of bimetallic contact [5], [6], the shift of the Fermi level γ at point **r**, caused by the relative volume change $\delta V = (V - V_0)/V_0$,
is described by the following expression, [7], [8]:

$$\gamma(\mathbf{r}) = \frac{2\epsilon_F}{3}\delta\mathrm{V}(\mathbf{r}), \tag{14}$$

where ϵ_F denotes the Fermi energy.

In tensor notation, the relative volume variation associated with deformation is written as:

$$\delta\mathrm{V} = \varepsilon_{ii}, \tag{15}$$

where
$\varepsilon_{ii} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ represents the sum of the diagonal components of the strain tensor.

Transitioning to the stress tensor representation yields, [9]:

$$\delta\mathrm{V} = \frac{(1-2\mu)\sigma_{ii}}{E}, \tag{16}$$

where E is Young's modulus, μ is Poisson's ratio, and $\sigma_{ii} = \sigma_{xx} + \sigma_{yy} + \sigma_{zz,}$. Substituting expression (3) into (1), we obtain:

$$\gamma = \frac{2\epsilon_F(1-2\mu)}{3E}\sigma_{ii}. \tag{17}$$

Let us now write the corresponding expressions for the work function at a point r, where a local relative volume increase δV(**r**), exists, and at a reference point **r**$_0$, where no volume change occurs (δV = 0):

$$\mathrm{W}(\mathbf{r}) = \mathrm{e}\varphi(\mathbf{r}) \text{ и } \mathrm{W}(\mathbf{r_0}) = \mathrm{e}\varphi(\mathbf{r_0}), \tag{18}$$

where e denotes the electron charge, and φ is the electric potential.

The difference between these expressions, taking relation (16) into account, can then be written as:

$$\mathrm{W}(\mathbf{r}) - \mathrm{W}(\boldsymbol{r}_0) = \mathrm{e}\varphi(\mathbf{r}) - \mathrm{e}\varphi(\mathbf{r}_0) = \frac{2\epsilon_F(1-2\mu)}{3E}\sigma_{ii}, \tag{19}$$

By substituting
$\Delta\varphi = \mathrm{e}\varphi(\mathbf{r}) - \mathrm{e}\varphi(\mathbf{r}_0)$,

We obtain:

$$\Delta\varphi = \frac{2\epsilon_F(1-2\mu)\sigma_{ii}}{3\mathrm{e}E}, \tag{20}$$

Expression (20) demonstrates that the electric potential difference on metallic surfaces is directly related to the non-uniform distribution of strains and stresses, and that this dependence is linear in nature.

As a consequence of axial loading or shear deformation, the constant-energy surfaces in **k** -space become distorted. For instance, a spherical Fermi surface transforms into an ellipsoidal one because the wave-vector **k** coordinates in the Brillouin zone after deformation are given by
$\mathbf{k}_{def} = (1-\varepsilon)\,\mathbf{k}$, [5], [10].

Let the electron energy in the ν-th energy band of the deformed and undeformed crystal lattice be denoted by $E_\nu(\mathbf{k}_{def})$ and $E_\nu(\mathbf{k})$, respectively. Then:

$$E_\nu(\mathbf{k}_{def}) - E_\nu(\mathbf{k}) = \Delta E_\nu(\mathbf{k}) = \varepsilon_{ij}D_{ij}(\nu, \mathbf{k}), \tag{21}$$

where D_{ij} is the deformation potential tensor describing the variation of electron energy per unit strain.

For nearly free electrons:

$$D_{ij}(\nu, \mathbf{k}) = \frac{2\epsilon_F}{3}\delta_{ij}, \quad (22)$$

where $\delta_{ij} = 1$ for $i=j$ and $\delta_{ij} = 0$ for $i\neq j$. By expressing the effect of the deformation potential on the Fermi level and applying the considerations discussed above, expressions (17) and subsequently (20) are recovered once again:

$$\sigma_{ii} = \frac{3eE\Delta\varphi}{2\epsilon_F(1-2\mu)}. \quad (23)$$

introducing the constant Q_σ:

$$Q_\sigma = \frac{3eE}{2\epsilon_F(1-2\mu)}, \quad (24)$$

We finally obtain:

$$\sigma_{ii} = Q_\sigma\Delta\varphi. \quad (25)$$

The constant Q_σ has the dimension of charge density [Кл/м^3] and characterizes the electrical elasticity of metals.

For the sum of the diagonal strain components, we may write:

$$\varepsilon_{jj} = \frac{3e\Delta\varphi}{2\epsilon_F(1-2\mu)}, \quad (26)$$

or equivalently:

$$\varepsilon_{jj} = Q_\varepsilon\Delta\varphi, \quad (27)$$

where

$$Q_\varepsilon = \frac{3e}{2\epsilon_F(1-2\mu)}. \quad (28)$$

The constant of Q_ε has the dimension of inverse voltage [1/V].

3 Conclusions

Within the framework of the continuum medium model, an analytical expression has been derived for evaluating material strength based on the distribution of electric potential. In the principal stress coordinate system defined by σ_x and σ_y, the directions of the electric field components E_x and E_y are found to coincide with the directions of the corresponding principal stresses.

Furthermore, the sum of the diagonal components of the mechanical stress tensor, σ_{ii}, exhibits a linear dependence on the electric potential at the material surface. The proportionality coefficient Q_σ possesses the dimension of charge density [Кл/м^3]. and characterizes the electromechanical response of the metallic medium.

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The authors declare that they have no competing interests.