Influence of Workpiece Inclination Angle on Tool Life and Surface Integrity in Ball-End Milling of Hardened CR12MoV Steel

HOANG ANH TUAN, TRIEU QUY HUY*
University of Economics - Technology for Industries,
456 Minh Khai Street, Vinh Tuy ward, Ha Noi,
VIETNAM

*Corresponding Author

Abstract: - This study investigates how the workpiece inclination angle influences tool life and surface integrity during finish ball-end milling of hardened CR12MoV steel. Experiments were carried out at cutting speeds of 50-110 m/min and inclination angles of 10-75°, while feed per tooth and axial depth remained constant at 0.2 mm/tooth and 0.5 mm. Surface roughness Ra was monitored over the cutting time, and tool life was defined as the point at which the Ra-time curve reached a breakpoint. Increasing the inclination angle shifted the contact zone away from the tool tip, enlarged the effective cutting radius, improved chip formation, and reduced localized wear. As θy increased from 10° to 75°, tool life increased by approximately 60-70% under similar conditions. A second-order logarithmic model also predicted tool life with errors below 6%, offering practical guidance for finish machining of hardened die steels.

Key-Words: - Hard milling, Ball-end milling, Hardened steel, Workpiece inclination angle, Surface integrity, Hardened CR12MoV steel, Tool life, Surface roughness.

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1 Introduction

Finishing free-form surfaces on dies, molds, and other complex components is often performed using ball-end milling because of its high geometric accessibility and ability to create smooth surface profiles, [1], [2], [3]. However, the cutting mechanics of a ball-end mill are inherently complex because the engagement zone changes continuously with the contact position. This results in significant variations in effective cutting speed, instantaneous uncut chip thickness, cutting-force direction, and sliding–sticking conditions, which in turn influence tool wear, tool life, and surface quality, [4], [5], [6]. Simultaneously, surface integrity-including roughness, work hardening, residual stresses, and microstructural changes-is widely recognized as a key factor affecting in-service performance, such as fatigue, wear, and corrosion resistance, [7], [8], [9].

A characteristic limitation of ball-end milling is that the cutting speed approaches zero near the tool tip. This low-speed region promotes plowing, increases friction and localized heat generation, accelerates concentrated wear, and deteriorates surface integrity, [10], [11], [12], [13]. From a geometric–kinematic viewpoint, the effective cutting radius and contact condition depend strongly on the relative inclination between the tool and the workpiece surface, thereby affecting wear evolution and tool life. Earlier studies on high-speed milling for die and mold manufacture also emphasized the importance of cutting conditions and contact configuration in machining hardened steels, [14], [15].

To prevent unfavorable cutting conditions near the tool tip, many studies have focused on adjusting the tool-axis orientation and toolpath direction, especially in five-axis machining. Experiments on hard-to-machine alloys such as Inconel 718 have demonstrated that tool orientation can significantly influence cutting forces, surface roughness, and tool life, [16], [17], [18]. Similar results were found in the finish milling of inclined hardened-steel surfaces, where cutter-path orientation affected both tool life and roughness, [19], [20], [21], [22]. At the same time, modeling and optimization studies proposed criteria for choosing tool orientations that improve cutting conditions and reduce deformation, [23], [24], [25], [26], [27], [28]. Other recent research examined the relationship between tilt/lead angles and surface roughness, including lead-angle limits and topography simulation, and developed cutting-force models for inclined-surface ball-end milling, [29], [30], [31], [32].

Despite these advances, most previous research has concentrated on CAM-based toolpath optimization, tool orientation, cutting parameters, and roughness assessment. The direct, quantitative effect of the workpiece inclination angle-a simple, low-cost factor influencing tool life and surface integrity-remains insufficiently understood in finish milling of hardened tool steels. This issue applies to CR12MoV steel, which is widely used in stamping and cutting dies due to its high hardness and wear resistance. In these materials, rapid tool wear can occur, and surface roughness, residual stresses, and work hardening directly affect functional performance. Studies on similar hardened steels have shown that cutting speed can influence surface topography, microhardness, residual stresses, and fatigue behavior. At the same time, optimization research has confirmed the strong link between process parameters and surface integrity, [33], [34]. Additionally, reviews of hardened-steel machining have emphasized white-layer formation, residual stresses, and roughness as key factors when selecting finishing conditions, [14].

From a production standpoint, tool life in finish milling can be indicated by a sudden increase in Ra, as roughness directly reflects cutting-edge wear. This study examines how the workpiece inclination angle θy affects tool life and surface quality in ball-end milling of hardened CR12MoV steel using a TiAlN-coated ball-end mill. The main idea is that increasing θy moves the contact zone away from the tool tip, enlarges the effective cutting radius, reduces sliding and plowing effects, and thereby slows wear while extending the stable wear phase. Figure 1 illustrates this tool–workpiece engagement process.

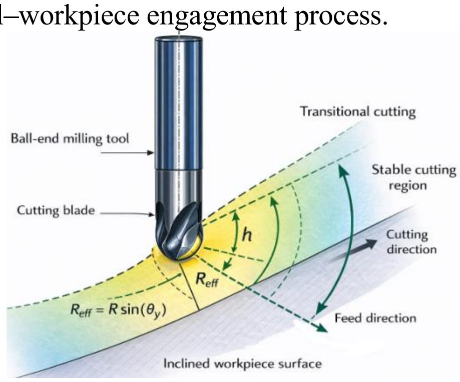

Fig. 1: Tool-workpiece engagement geometry in ball-end milling of an inclined surface
Source: created by the authors

This study's findings enhance understanding of the relationships among contact geometry, tool wear, and surface integrity, while also providing practical guidance on selecting the workpiece inclination angle to extend tool life and improve surface quality in die and mold machining.

2 Experimental Setup

2.1 Workpiece Material

In this study, heat-treated CR12MoV alloy steel was chosen as the workpiece material. CR12MoV is a high-chromium tool steel commonly used for stamping and cold-forming dies because of its high wear resistance, high hardness after quenching, and good dimensional stability. These features also make it difficult to machine; in finish ball-end milling, rapid tool wear and deterioration of surface quality often occur. The specimens were hardened before machining, maintaining a steady hardness of 40-45 HRC during testing, which reflects typical finishing conditions in die and mold manufacturing.

Table 1. Chemical composition and mechanical properties of CR12MoV steel

Category	Property / Element	Value	Unit
Chemical composition	C	1.57	%
	Si	0.29	%
	Mn	0.25	%
	Cr	11.46	%
	Mo	0.44	%
	V	0.19	%
Material properties	Hardness	40-45	HRC
	Heat treatment condition	Quenched and tempered	-
	Material standard	CR12MoV cold-work tool steel	-
	Typical application	Stamping and cold-forming dies.	-
	Workpiece dimensions	300 × 150 × 40	mm

Source: created by the authors based on the material certificate and experimental data.

Table 1 summarizes the chemical composition and main properties of the CR12MoV steel used in this study. The high chromium content (11-12%), along with Mo and V additions, provides high wear resistance and improves carbide stability after heat treatment. These alloying elements also increase machining difficulty and promote abrasive and adhesive wear during ball-end milling. The workpiece specimens measure 300 × 150 × 40 mm, offering sufficient size to perform multiple cutting passes at different inclination angles without edge effects or excessive heat buildup. Choosing CR12MoV allows a clear evaluation of how the workpiece inclination angle influences tool wear mechanisms and tool life under demanding machining conditions typical of die and mold finishing operations.

2.2 Cutting Tool

This study used a two-flute TiAlN-coated ball-end milling tool for finish machining of hardened CR12MoV steel. The cutting tool (Figure 2) was selected to meet the requirements of finishing free-form surfaces under hard-milling conditions, where cutting mechanics are complex, and tool wear is significant near the tool tip. Using a ball-end mill aligns with practical die and mold manufacturing. It allows a clear assessment of how the engagement location along the ball profile changes with the workpiece's inclination angle. The geometry of the ball-end tool influences the engagement conditions during the milling of inclined surfaces. As the workpiece's inclination angle varies, the contact zone shifts along the spherical cutting edge, changing the effective cutting speed and affecting tool wear behavior.

Fig. 2: Geometry of the TiAlN-coated ball-end milling tool used in the experiments
Source: created by the authors

The cutting tool used in this study was a TiAlN-coated ball-end insert (VP15TF, Mitsubishi Materials, Japan) mounted on an SRFH10S12M cutter body. The tool has a nominal diameter of 10 mm, two flutes, a cutting length of 8.5 mm, a rake angle $\gamma = 0°$, and a clearance angle $\alpha = 5°$. The insert hardness is 91.5 HRA with a compressive strength of about 2.5 GPa, making it suitable for machining hardened steels. The TiAlN coating provides high thermal stability and wear resistance under flood cooling conditions. During the experiments, tool wear was mainly observed on the flank face due to combined abrasive and adhesive wear mechanisms typical of ball-end milling of hardened steels.

2.3 Machine Tool and Cooling Strategy

The milling experiments were conducted on a vertical CNC machining center (Figure 3), VMC-85S (Maximart, Taiwan), equipped with a Fanuc OMD controller. The machine offers high structural rigidity and a spindle speed range of 60-8000 rpm, suitable for finishing hardened steels. A BT40 spindle paired with a cutter body with a 12 mm shank diameter ensured high tool-spindle stiffness, which is crucial for stable ball-end milling with a small depth of cut (t = 0.5 mm). To ensure consistency, the machine setup, fixture, and tool-clamping conditions were kept unchanged throughout the tests.

Flood cooling with a 5% Emusil Mira EM40 water-based emulsion was employed to manage the cutting temperature and minimize adhesive and oxidation wear. This cooling method ensured that variations in tool life were primarily due to changes in cutting parameters and workpiece inclination, rather than thermal effects.

Fig. 3: CNC vertical machining center VMC-85S used for the milling experiments
Source: created by the authors

2.4 Machining Conditions

The ball-end milling experiments were performed to finish-machine an inclined planar surface and assess how cutting speed and workpiece inclination angle affect tool life and surface quality. To isolate these factors, the feed per tooth and axial depth of cut were held constant. The cutting speed v ranged from 50 to 110 m/min, representing typical conditions for hard milling of high-chromium tool steels with TiAlN-coated tools. The workpiece inclination angle θy varied from 10° to 75° to change the engagement position along the spherical cutting edge, covering both near-tip and off-tip cutting regions. The feed per tooth and axial depth of cut were fixed at s = 0.2 $mm/toot$ and t = 0.5 mm, respectively, to ensure stable finishing conditions. The combinations of parameters were arranged in an experimental matrix with center-point replications to estimate repeatability and support regression modeling of tool life (Table 2).

Table 2. Machining parameters and their levels

Parameter	Symbol	Value
Cutting speed	v (m/min)	50 - 110
Workpiece inclination angle	θy (°)	10 - 75
Feed per tooth	s (mm/tooth)	0.2
Axial depth of cut	t (mm)	0.5

Source: created by the authors.

2.5 Geometric-mechanistic Analysis of Tool-Workpiece Engagement

In ball-end milling, the position of the contact zone on the spherical cutting edge governs the effective cutting speed, chip formation mechanism, and load distribution. When machining an inclined surface, the workpiece inclination angle θ_y directly determines this engagement geometry.

For a tool with radius R, the effective cutting radius is:

$$R_{eff} = R\sin(\theta_y) \quad (1)$$

and the corresponding effective cutting speed is:

$$v_{eff} = \omega R_{eff} = \omega R\sin(\theta_y) \quad (2)$$

where: ω is the spindle angular velocity.

when θ_y is small, contact occurs near the tool tip where $R_{eff} \to 0$. The cutting speed drops significantly, and the removal mechanism shifts from shearing to plowing, leading to increased friction, heat accumulation, and accelerated local tool wear.
The instantaneous uncut chip thickness can be written as:

$$h = f_z\sin(\phi)\sin(\theta_y) \quad (3)$$

where: f_z is the feed per tooth and ϕ is the immersion angle.

The instantaneous chip thickness directly affects the cutting load acting on the tool edge. The cutting force component can be approximately expressed as:

$$F_c = K_c \cdot h \cdot b \quad (4)$$

where: K_c is the specific cutting force; h is the chip thickness; b is the engagement width.

These relations indicate that increasing θy enlarges the effective cutting radius and cutting speed, which improves chip formation stability and reduces contact stress. This behavior aligns with the experimental results in Figure 4, where larger θy extends the stable wear stage and enhances tool life.

3 Tool Life Criterion and Surface Roughness Evaluation

In finish ball-end milling of hardened steels, surface quality serves as both a key functional requirement and a sensitive indicator of tool wear. Surface roughness was used as the main criterion for assessing degradation in cutting performance and for determining tool life.

Surface roughness was characterized using the parameters Ra and Rz, with Ra chosen as the primary evaluation metric because it captures average roughness and clearly indicates the transition from steady to accelerated wear. The parameter Rz was used as an additional indicator for surface profile analysis.

Fig. 4: Surface roughness measurement using a Mitutoyo SJ-201 contact profilometer
Source: Photos were taken by the authors

Surface roughness measurements were performed using a Mitutoyo SJ-201 contact stylus profilometer (Japan), as shown in Figure 4. For each cutting condition, measurements were taken at multiple locations along the machined toolpath and averaged to minimize random errors.

During the experiments, roughness was measured at set intervals of cutting time until a sudden rise in Ra was detected. Tool life T was defined as the cutting time at the break point on the Ra-time curve, marking the transition to accelerated wear. Optical microscopy and SEM observations were also conducted to analyze wear mechanisms and to confirm the link between tool wear and changes in roughness.

4 Results and Discussion

4.1 Evolution of Surface Roughness and Tool Life

At a small inclination angle (θy = 10°), surface roughness increases quickly with cutting time. At v = 50 m/min, Ra remains steady only in the early stage and then rises sharply after about 9 minutes, whereas at v = 110 m/min, this increase occurs sooner, after 6-7 minutes. This pattern indicates that cutting is concentrated near the tool tip, where the effective cutting speed is low and sliding contact speeds accelerate localized wear.

When the inclination angle increases to θy = 75°, the stable phase of the Ra curve is notably prolonged. Ra remains stable for about 15-16 minutes at v = 50 m/min and 14-15 minutes at v = 110 m/min, leading to a 60-70% increase in tool life compared to θy = 10°.

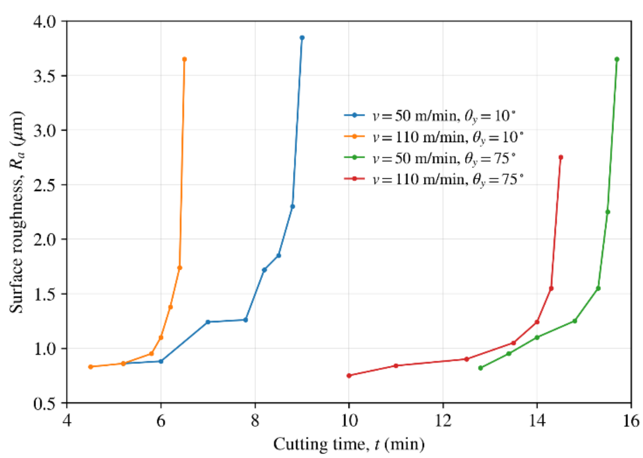

Fig. 5: Surface roughness value over cutting time

Source: Created by the authors based on experimental data

Figure 5 illustrates how surface roughness Ra changes over cutting time for representative combinations of cutting speed and workpiece inclination angle. In all tested conditions, the Ra-time curves display three typical stages: an initial stable phase, a progressive wear phase, and a rapid deterioration phase, showing the wear progression of the ball-end mill during hard milling.

These results show that the inclination angle of the workpiece has a greater effect than cutting speed on the stability of surface roughness. Increasing θy moves the engagement zone away from the tool tip, enlarges the effective cutting radius, improves chip formation, and distributes cutting loads more evenly along the spherical edge. Localized wear decreases, and tool life is significantly extended.

4.2 Statistical Modeling of Tool Life

4.2.1 Model Form and Rationale for Selection

Tool life typically exhibits a nonlinear dependence on process parameters and is often modeled using power-law or logarithmic forms. In this case, a second-order logarithmic model was utilized because it simplifies the regression process and effectively captures the primary, quadratic, and interaction effects within the studied domain. The model is expressed as:

$$\ln T = b_0 + b_1 \ln v + b_2 \ln \theta_y + b_{12}(\ln v)(\ln \theta_y) + b_{11}(\ln v)^2 + b_{22}(\ln \theta_y)^2 \quad (5)$$

where: T is the tool life (min), v is the cutting speed (m/min), and θ_y is the workpiece inclination angle.

4.2.2 Data Normalization and Processing

From the experimental results, each parameter combination (v, θ_y) yields one tool-life value T (determined at the break point on the Ra time curve). The variables are transformed using the natural logarithm as follows: $x_1 = \ln v$; $x_2 = \ln \theta_y$; $y = \ln T$.

After transformation, the model can be written in a linear form with respect to the coefficients:

$$y = Xb + \varepsilon \quad (6)$$

where: ε: Denotes the random error term;

$$X = \begin{bmatrix} 1 & x_{1,1} & x_{2,1} & x_{1,1}x_{2,1} & x_{1,1}^2 & x_{2,1}^2 \\ 1 & x_{2,1} & x_{2,2} & x_{1,2}x_{2,2} & x_{1,2}^2 & x_{2,2}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1,n} & x_{2,n} & x_{1,n}x_{2,n} & x_{1,n}^2 & x_{2,n}^2 \end{bmatrix}$$

$$b = [b_0 \quad b_1 \quad b_2 \quad b_{12} \quad b_{11} \quad b_{22}]^T$$

The regression coefficients are estimated using Ordinary Least Squares (OLS), with the closed-form solution:

$$\mathrm{b} = \left(\mathrm{X}^{\mathrm{T}}X\right)^{-1}\mathrm{X}^{\mathrm{T}}y \tag{7}$$

This method determines b such that the sum of squared errors between the predicted and experimental values of $\ln T$ is minimized:

$$\min \sum_{i=1}^{n} \left(\ln \mathrm{T}_i^{\mathrm{exp}} - \ln \mathrm{T}_i^{\mathrm{pred}}\right)^2 \tag{8}$$

In the experimental design, center point replications were included to estimate pure error, evaluate model adequacy (goodness of fit), and reduce the risk of overfitting when degrees of freedom are limited.

This repeatability error is utilized directly in the analysis of variance (ANOVA) and in the overall F-test for the regression model.

4.2.3 Statistical Significance Testing of Regression Coefficients

After estimation, each coefficient bi was tested using Student's t-test:

$$\mathrm{t_i} = \frac{\mathrm{b_i}}{\mathrm{SE(b_i)}} \tag{9}$$

where: $\mathrm{SE}(b_i)$ is the standard error of the coefficient.
If (p-value < 0.05): The coefficient is statistically significant. If (p-value ≥ 0.05): The coefficient may not have a significant effect. The results indicate that the main effect coefficients b_1 and b_2 are statistically significant, confirming the dominant roles of cutting speed and workpiece inclination angle in determining tool life.

4.2.4 Overall Model Adequacy Test

The overall model adequacy was evaluated using:

$$\mathrm{F} = \frac{\mathrm{M}S_{\mathrm{model}}}{\mathrm{M}S_{\mathrm{error}}} \tag{10}$$

If $\mathrm{F_{calc}} > \mathrm{F_{crit}}$ at a significance level of 0,05, the regression model is statistically significant and can be used to predict tool life within the investigated domain. The estimated regression coefficients are as follows: $\mathrm{b_0} = 14.8$; $\mathrm{b_1} = -1.2$; $\mathrm{b_2} = 4.35$; $\mathrm{b_{12}} = 0.18$; $\mathrm{b_{11}} = -1.35$; $\mathrm{b_{22}} = -7.7$.

The negative coefficient $b1$ shows that tool life decreases as cutting speed increases. Conversely, the positive coefficient $b2$ indicates that a larger inclination angle enhances tool life by shifting engagement away from the tool tip. The quadratic terms suggest an optimal range. Student's t-test (α=0.05) reveals that $\ln v$ and $\ln\theta y$ are significant (p<0.05), with $b12$ representing their interaction effect.

The adequacy of the regression model was assessed using Fisher's F-test by comparing the lack-of-fit variance with the pure error variance estimated from the center-point replications. The two center-point experiments produced tool-life values of 20.0 and 20.8 minutes, resulting in an average of 20.4 minutes. The corresponding pure error sum of squares was $SSts$=0.32 with one degree of freedom fts=1, leading to a variance of $sts2$=0.32. The total residual sum of squares was SSE=23.7. With N=9 experiments and m=6 regression coefficients, the remaining error degrees of freedom were fE=3, from which the lack-of-fit sum of squares was calculated.

$$\mathrm{SS_{ad}} = \mathrm{SS_E} - \mathrm{SS_{ts}} = 23.38$$

with: $\mathrm{f_{ad}} = 2$, corresponding to a lack of fit variance of $\mathrm{s_{ad}^2} = 11.69$.

The Fisher statistic was calculated as:

$$\mathrm{F_{calc}} = \frac{\mathrm{s_{ad}^2}}{\mathrm{s_{ts}^2}} = 36.53$$

At a significance level of $\alpha = 0.05$, the critical value is $F_{crit} = 98.5$. Since $F_{calc} < F_{crit}$, the lack-of-fit is not statistically significant, indicating that the regression model is adequate for the experimental data within the investigated domain. This confirms the model's reliability in describing and predicting the influence of cutting speed and workpiece inclination angle on tool life in ball-end milling of hardened CR12MoV steel.

Table 3. Comparison between predicted and experimental tool life

V (m/min)	θy (°)	Experimental T (min)	Model T (min)	Error (%)
50	10	9.2	8.7	5.4
80	45	20.4	21.1	3.4
110	75	15.1	14.6	3.3

Source: created by the authors based on experimental.

Table 3 compares the predicted tool life values from the regression model with the experimental measurements. The prediction errors are mostly below

6%, indicating that the model offers good predictive accuracy under the tested cutting conditions.

In Figure 6, data points lie close to the parity line (y = x), indicating good agreement between the model predictions and experimental measurements, with prediction errors below 6%.

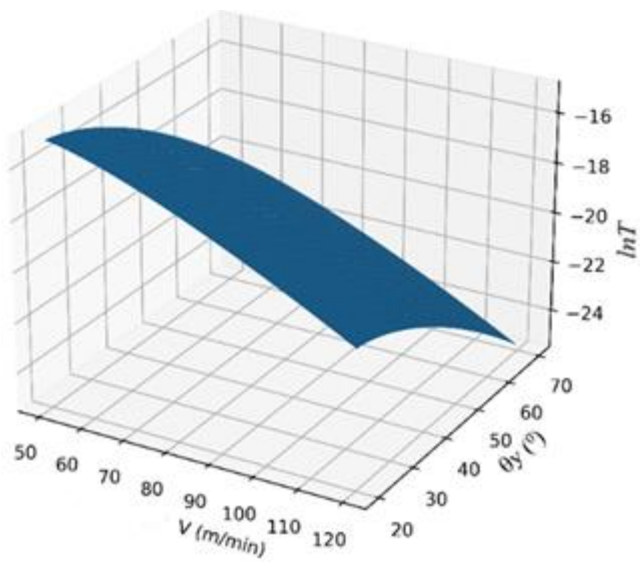

Fig. 6: Comparison between predicted and experimental tool life
Source: Created by the authors based on experimental data

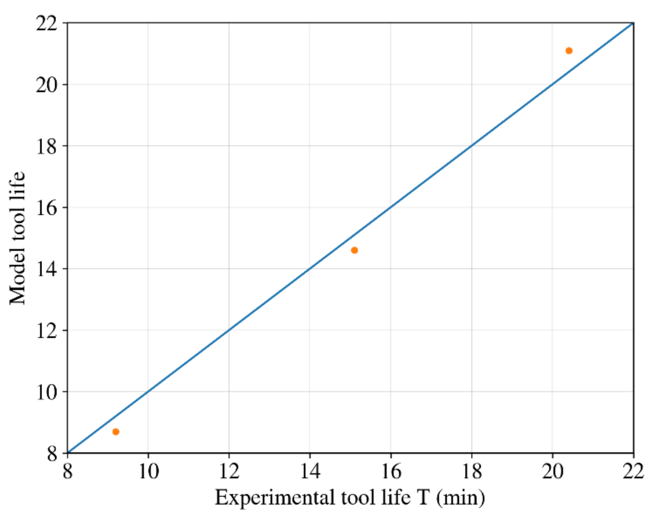

Fig. 7: Response surface of $ln\,T$ as function of v and θ_y
Source: Created by the authors based on experimental data

Figure 7 and Figure 8 present the response surface and contour plots of tool life as functions of cutting speed and workpiece inclination angle. It can be clearly observed that tool life increases markedly with increasing θy, particularly in the medium cutting-speed range, whereas increasing cutting speed results in a decreasing trend in tool life.

A statistically and physically meaningful quadratic-logarithmic regression model was developed to predict tool life and to determine the optimal range of workpiece inclination angles for ball-end milling of hardened CR12MoV steel under the studied conditions.

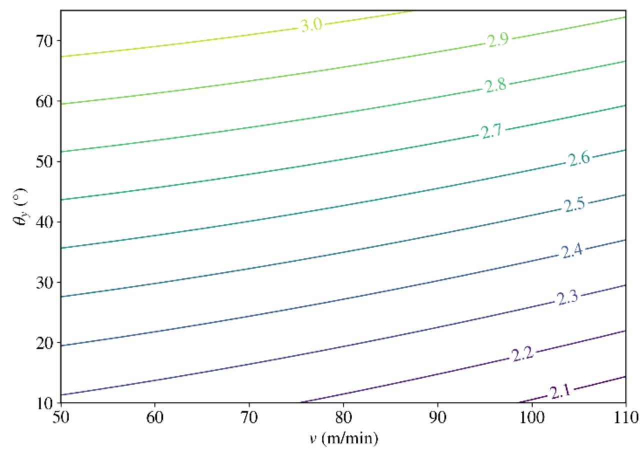

Fig. 8: Contour plots of lnT as function of v and θ_y
Source: Created by the authors based on experimental data

4.3 Physical Interpretation of Inclination Angle Effect

The results indicate that the workpiece inclination angle θ_y significantly influences tool life and surface quality during ball-end milling of hardened CR12MoV steel. At a small angle (θ_y = 10°), the engagement zone stays near the tool tip, where the effective cutting radius is nearly zero. As a result, the effective cutting speed is low, material removal mainly happens through plowing, and friction and localized heat increase. These conditions lead to faster tool wear, shorten the stable wear period, and decrease the tool's lifespan.

As the inclination angle increases, the contact point moves gradually toward the flank region of the ball-end tool, leading to a higher effective cutting radius and speed. This geometric change improves chip formation stability and distributes the cutting load more evenly along the spherical cutting edge, reducing localized contact stresses and preventing severe wear in specific areas. SEM observations in Figure 9 clearly show this behavior. At small inclination angles, the wear zone near the tool tip features prominent abrasive grooves, adhered material layers, and micro-chipping along the cutting edge. In contrast, at larger inclination angles (θy=75∘), the wear pattern becomes more uniform and is mainly marked by mild flank wear. These microstructural findings match the surface roughness development; a larger inclination angle significantly lengthens the stable phase of the Ra-time curve and enhances tool life.

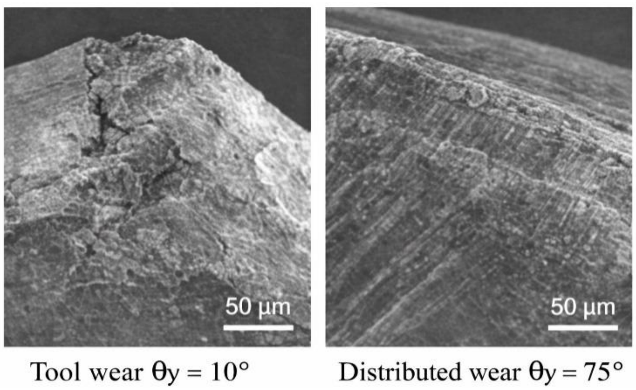

Fig. 9: SEM images of tool wear at different workpiece inclination angles

Source: Created by the authors based on experimental data

5 Conclusion

This study examined the influence of the workpiece inclination angle on tool life and surface integrity in ball-end milling of hardened CR12MoV steel using a TiAlN-coated tool. The results demonstrate that the inclination angle is a key geometric parameter governing tool-workpiece engagement and wear evolution.

Increasing the inclination angle from 10° to 75° significantly extends the stable machining stage, resulting in a 60-70% increase in tool life. The evolution of Ra with cutting time proved to be an effective indicator of tool degradation; the break point on the Ra-time curve corresponds to the transition from steady wear to accelerated failure.

A second-order logarithmic regression model relating tool life to cutting speed and inclination angle was developed and statistically validated. The model shows good predictive capability with prediction errors below 6%, confirming its reliability within the investigated cutting domain.

Mechanistically, the improvement in tool life is attributed to the shift of the engagement zone away from the tool tip, which increases the effective cutting radius, improves chip formation conditions, and distributes cutting loads more uniformly along the cutting edge.

From a manufacturing standpoint, adjusting the workpiece inclination angle provides a simple and effective strategy to enhance tool life and maintain surface integrity in finish ball-end milling of hardened steels used in die and mold applications.

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Hoang Anh Tuan conceived the study, designed the experiments, and conducted the machining tests. Trieu Quy Huy performed the statistical analysis, interpreted the results, and contributed to developing the regression model. Both authors participated in writing, reviewing, and approving the final manuscript.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.