Modeling of Drill String Dynamics Under Seismic Activity of a Well

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Abstract: – The study analyzes the nonlinear dynamics of shallow drill strings under seismic activity in a geological environment. The main objective is to determine the impact of seismic wave influence on the oscillatory process by identifying the dynamic response to types and intensities of seismic excitation, considering the presence of finite deformations. Based on Hamilton principle, a nonlinear mathematical model for the spatial motion of the drill string was developed. A key point was the inclusion of seismic effects in the kinetic energy of the system, which is achieved by accounting for the displacement of the wellbore relative to the global coordinate system. This methodological approach provided description of the influence of dynamic ground mass displacements on the inertial parameters of the system. The research results indicate the transition of the drill string to quasi-periodic oscillation modes, which increase the unpredictability of its dynamics, the risks of resonant vibrations.

Key-words: - Drill string, deformations, dynamics, nonlinearity, seismic waves, vibrations, seismic impact.

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1 Introduction

The threat of seismic impact is one of the key factors determining the reliability and safety of underground infrastructure, including oil and gas and mining facilities. Drill strings, which lie at relatively shallow depths, form the basis of these industrial systems. An accident at such enterprises as a result of seismic activity entails significant economic losses associated with well plugging and abandonment and production downtime. Furthermore, potential man-made and ecological disasters caused by the uncontrolled release of fluids are possible. Therefore, the urgency of ensuring the seismic resistance of drill strings in geodynamically active regions is strategically significant.

Traditional methods for calculating seismic resistance, developed for above-ground structures, rely primarily on accounting for inertial forces. However, this paradigm is inappropriate for underground structures, including drill strings. The main danger for flexible underground structures is represented by imposed deformations, such as axial elongations, shears, and bends, created by a seismic wave passing through a non-uniform soil mass.

Considering the dynamic nature of drill strings, the mechanisms of the nonlinear interaction of the "string–soil" system under conditions of strong seismic impacts remain insufficiently studied, as does the periodic growth of stress amplitude caused by the interference of close oscillation frequencies of the string in a non-uniform medium. To achieve the closest approximation to reality when studying such effects, it is required to account for the nonlinear dynamics of the structure, which necessitates the application of the fundamental principles of nonlinear elasticity theory.

Analysis of works concerning the influence of seismic load on underground structures indicates researchers' interest in this problem. It has been established that underground structures in soil liquefaction zones are areas of increased danger, as low-frequency seismic waves significantly amplify their deformations. Critical damage zones have been identified both in connecting joints and in supporting elements. Moreover, the use of expansion joints, [1], can reduce stress, and accounting for vertical inertia and the specifics of contact pressure, [2], [3], is mandatory for design to counteract compressive damage and lateral structural deformation. For the seismic analysis of complex metro station systems in soft soils, comprehensive approaches are applied, including theoretical and quasi-static methods, [4], [5], [6], as well as physical and numerical modeling using the total and effective stress approach, [7], [8], [9], [10], [11].

Before the earthquake in Japan in 1995, it was believed that underground structures possessed seismic resistance significantly greater than above-ground structures and, consequently, did not require specialized seismic design. However, a series of major earthquakes, [12], demonstrated severe damage to subway stations, which disproved this hypothesis and highlighted the problem of the seismic response of underground structures in the fields of geotechnical engineering and disaster protection. Specifically, during one earthquake, the station suffered the collapse and bending of its central columns, which led to the roof collapsing, causing massive ground subsidence of up to four meters above the station, [13], [14], [15].

The objective of the present study is to develop and numerically analyze a nonlinear mathematical model for the seismic response of drill strings used in shallow drilling.

2 Mathematical model

In this work, the modeling of drill string dynamics under conditions of well seismic activity is based on the fundamental principles of nonlinear mechanics of deformable bodies. As part of the research, a nonlinear mathematical model was developed for the spatial vibrations of a rotating rod element subjected to the influence of seismic waves. The seismogenic impact in the model is reflected through induced vibrations of the wellbore walls, allowing for the dynamic displacement of its profile relative to a fixed reference frame. The drill string is represented as a rod structure, pin-supported at the ends and rotating around the *Oz* axis with a constant angular velocity ω. The equations of motion for the string are derived based on the Ostrogradsky–Hamilton variational method, similar to works, [16], [17]. This method guarantees a rigorous derivation of the dynamic relationships, considering nonlinear deformations and external influences.

$$\int_{t_1}^{t_2} \delta\left(T_{kin} - U_p + \Pi\right) dt = 0, \tag{1}$$

where T_{kin} is the kinetic energy, U_p is the potential energy of elastic deformation, Π is the potential energy of the compressive load, and is the torque of external forces acting on the string. They are defined as:

$$U_p = \frac{1}{2}\rho\int_0^l \int_{A(z)} \sigma_{ij}\varepsilon_{ij} dA(z)dz, \tag{2}$$

$$T_{kin} = \frac{1}{2}\rho\int_0^l \int_{A(z)} \left(\vec{R}' \cdot \vec{R}'\right) dA(z)dz, \tag{3}$$

$$\Pi = \Pi_N + \Pi_M. \tag{4}$$

Here, σ_{ij} is the stress tensor, ε_{ij} is the strain tensor, and $\vec{R}$ is the radius vector defining the position of the center of the drill string's cross-section, A is the cross-sectional area of the drill string.

The strain potential energy function (2) is based on the fundamental nonlinear elasticity theory, [18]. The model utilizes the second system of simplifications, which establishes the smallness of relative elongations, shears, and rotation angles compared to unity. Despite neglecting most components of the second and higher-order strains, the inclusion of second-order rotation angles ensures the nonlinear form of the strain tensor component.

$$\begin{aligned}
&\varepsilon_{xx} \approx e_{xx} + \frac{1}{2}\left(\omega_y^2 + \omega_z^2\right), && \varepsilon_{xy} \approx e_{xy} - \omega_x\omega_y, \\
&\varepsilon_{yy} \approx e_{yy} + \frac{1}{2}\left(\omega_x^2 + \omega_z^2\right), && \varepsilon_{yz} \approx e_{yz} - \omega_y\omega_z, \\
&\varepsilon_{zz} \approx e_{zz} + \frac{1}{2}\left(\omega_x^2 + \omega_y^2\right), && \varepsilon_{zx} \approx e_{zx} - \omega_z\omega_x,
\end{aligned} \tag{5}$$

ε_{ii} are relative elongations along the coordinate axes, ω_i and are rotation angles around the axes.

To construct the vibration model of the well, considering the seismic load, the transition from the local coordinate system to the global coordinate system was modified, defined by the following relation:

$$
\begin{aligned}
X &= X_0 + \left(x+U\right)\cos\omega t + \left(y+V\right)\sin\omega t,\\
Y &= Y_0 - \left(x+U\right)\sin\omega t + \left(y+V\right)\cos\omega t,\\
Z &= Z_0 + z + W,
\end{aligned} \tag{6}
$$

applied to determine the kinetic energy (3) of the drill string.

The parameters X_0, Y_0, Z_0 define the kinematics of the wellbore within the global reference frame. Within the framework of model (6), they represent the seismic input transferred from the formation to the drill string, effectively capturing the rigid-body translation of the borehole walls during a seismic event.

Here ω is the angular velocity of the drill string rotation; X, Y, Z are the global coordinate system, defining the radius vector $\vec{R}$ of the drill string's final position; x, y, z are the local coordinate system, associated with the deformed state of the string resulting from bending; U, V, W are the components of the drill string displacement vector.

According to the theory of elastic deformation of rod elements, their elastic displacements are defined as:

$$
\begin{aligned}
U &= u_1(z,t) + \theta(z,t)y,\\
V &= v_1(z,t) - \theta(z,t)x,\\
W &= w(z,t) - \frac{\partial u_1(z,t)}{\partial z}x - \frac{\partial v_1(z,t)}{\partial z}y,
\end{aligned} \tag{7}
$$

where $u_1(x,t), v_1(x,t)$ are the displacements of the cross-section bending center along the axes, w is the longitudinal displacement, and θ is the displacement due to the drill string torsion.

Based on (6) and (7), the kinetic energy of the system "drill string – vibrating well" is presented as:

$$
\begin{aligned}
T_{kin} &= \frac{1}{2}\rho\int_0^l \Bigg[A\left(\frac{\partial u_1}{\partial t}\right)^2 + A\left(\frac{\partial v_1}{\partial t}\right)^2 + \\
&+ A\left(\frac{\partial w}{\partial t}\right)^2 + I_x\left(\frac{\partial \theta}{\partial t}\right)^2 + I_y\left(\frac{\partial \theta}{\partial t}\right)^2 + \\
&+ I_y\left(\frac{\partial^2 u_1}{\partial z\partial t}\right)^2 + I_x\left(\frac{\partial^2 v_1}{\partial z\partial t}\right)^2 + 2I_{xy}\frac{\partial^2 u_1}{\partial z\partial t}\frac{\partial^2 v_1}{\partial z\partial t} + \\
&+ \omega^2\{I_p + A\left(u_1{}^2 + v_1{}^2\right) + I_p\theta^2\} + \\
&+ 2\omega\{I_p\frac{\partial \theta}{\partial t} + Av_1\frac{\partial u_1}{\partial t} - -Au_1\frac{\partial v_1}{\partial t}\} + \\
&+ 2A\Bigg[\left(\frac{\partial X_0}{\partial t}\right)^2 + \left(\frac{\partial Y_0}{\partial t}\right)^2 + \left(\frac{\partial Z_0}{\partial t}\right)^2 + \\
&+ \frac{\partial Z_0}{\partial t}\frac{\partial w}{\partial t} + \cos\omega t(\frac{\partial X_0}{\partial t}(\frac{\partial u_1}{\partial t} + \omega v_1) + \\
&+ \frac{\partial Y_0}{\partial t}(\frac{\partial v_1}{\partial t} - \omega u_1)) + \sin\omega t(\frac{\partial X_0}{\partial t}(\frac{\partial v_1}{\partial t} - \omega u_1) - \\
&- \frac{\partial Y_0}{\partial t}(\frac{\partial u_1}{\partial t} + \omega v_1))\Bigg]\Bigg] dz.
\end{aligned} \tag{8}
$$

Similarly to the kinetic energy(8), the potential energy U_p is expressed via displacements using the finite strain relation (5) subject to (7).

The potential energies of external forces (4) are expressed as follows:

$$
\begin{aligned}
\Pi_N &= \frac{1}{2}\int_0^l N\left(z,t\right)\left[\left(\frac{\partial u_1}{\partial z}\right)^2 + \left(\frac{\partial v_1}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] dz,\\
\Pi_M &= \frac{1}{2}\int_0^l M\left(t\right)\left[\left(\frac{\partial^2 u_1}{\partial z^2}\right)\left(\frac{\partial v_1}{\partial z}\right) - \left(\frac{\partial^2 v_1}{\partial z^2}\right)\left(\frac{\partial u_1}{\partial z}\right)\right] dz.
\end{aligned} \tag{9}
$$

Based on relations (5)–(9) and the variational principle (1), a generalized nonlinear model of the well drilling dynamics under conditions of a seismically active environment was constructed:

$$
\begin{aligned}
&\rho\left\{-\frac{\partial}{\partial t}\left(A\frac{\partial u_1}{\partial t}\right) + \frac{\partial^2}{\partial z\partial t}\left(I_y\frac{\partial^2 u_1}{\partial z\partial t}\right) + \frac{\partial^2}{\partial z\partial t}\left(I_{xy}\frac{\partial^2 v_1}{\partial z\partial t}\right)\right\} - \\
&- \frac{\partial}{\partial z}\left(N\left(z,t\right)\frac{\partial u_1}{\partial z}\right) - \frac{G}{(1-2\nu)}\left\{-2\frac{\partial}{\partial z}\left(A\left(\frac{\partial u_1}{\partial z}\right)^3\right) - \right.
\end{aligned}
$$

$$
\begin{aligned}
&-2\frac{\partial}{\partial z}\left(A\frac{\partial w}{\partial z}\frac{\partial u_1}{\partial z}\right)-\frac{3}{4}\frac{\partial}{\partial z}\left(I_x\frac{\partial u_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)+\\
&+2(1-\nu)\left(\frac{\partial^2}{\partial z^2}\left(I_y\frac{\partial^2 u_1}{\partial z^2}\right)+\frac{\partial^2}{\partial z^2}\left(I_{xy}\frac{\partial^2 v_1}{\partial z^2}\right)\right)+\\
&+\frac{\partial^2}{\partial z^2}\left(I_y\frac{\partial v_1}{\partial z}\frac{\partial\theta}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_x\frac{\partial^2 v_1}{\partial z^2}\frac{\partial\theta}{\partial z}\right)+\\
&+\frac{\partial^2}{\partial z^2}\left(I_{xy}\frac{\partial u_1}{\partial z}\frac{\partial\theta}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial^2 u_1}{\partial z^2}\frac{\partial\theta}{\partial z}\right)\Bigg\}+\\
&+\frac{\partial^2}{\partial z^2}\left(M(t)\frac{\partial v_1}{\partial z}\right)+\frac{G(5-6\nu)}{(1-2\nu)}\Bigg\{\frac{\partial}{\partial z}\left(A\frac{\partial u_1}{\partial z}\left(\frac{\partial v_1}{\partial z}\right)^2\right)+\\
&+\frac{\partial}{\partial z}\left(A\theta^2\frac{\partial u_1}{\partial z}\right)+\frac{1}{4}\frac{\partial}{\partial z}\left(I_y\frac{\partial u_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)+\\
&+\frac{1}{2}\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial v_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)\Bigg\}+\rho A\omega^2u_1-2\rho A\omega\frac{\partial v_1}{\partial t}-\\
&-\cos\omega t\left(\frac{\partial^2X_0}{\partial t^2}+\omega\frac{\partial Y_0}{\partial t}\right)+\sin\omega t\left(\frac{\partial^2Y_0}{\partial t^2}-\omega\frac{\partial X_0}{\partial t}\right)=0,
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&\rho\left\{-\frac{\partial}{\partial t}\left(A\frac{\partial v_1}{\partial t}\right)+\frac{\partial^2}{\partial z\partial t}\left(I_x\frac{\partial^2 v_1}{\partial z\partial t}\right)+\frac{\partial^2}{\partial z\partial t}\left(I_{xy}\frac{\partial^2 u_1}{\partial z\partial t}\right)\right\}-\\
&-\frac{\partial}{\partial z}\left(N(z,t)\frac{\partial v_1}{\partial z}\right)-\frac{G}{(1-2\nu)}\Bigg\{-2\frac{\partial}{\partial z}\left(A\left(\frac{\partial v_1}{\partial z}\right)^3\right)-\\
&-2\frac{\partial}{\partial z}\left(A\frac{\partial w}{\partial z}\frac{\partial v_1}{\partial z}\right)-\frac{3}{4}\frac{\partial}{\partial z}\left(I_y\frac{\partial v_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)+\\
&+2(1-\nu)\left(\frac{\partial^2}{\partial z^2}\left(I_x\frac{\partial^2 v_1}{\partial z^2}\right)+\frac{\partial^2}{\partial z^2}\left(I_{xy}\frac{\partial^2 u_1}{\partial z^2}\right)\right)+\\
&+\frac{\partial^2}{\partial z^2}\left(I_x\frac{\partial u_1}{\partial z}\frac{\partial\theta}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_y\frac{\partial^2 u_1}{\partial z^2}\frac{\partial\theta}{\partial z}\right)+\\
&+\frac{\partial^2}{\partial z^2}\left(I_{xy}\frac{\partial v_1}{\partial z}\frac{\partial\theta}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial^2 v_1}{\partial z^2}\frac{\partial\theta}{\partial z}\right)\Bigg\}+\\
&+\frac{G(5-6\nu)}{(1-2\nu)}\Bigg\{\frac{\partial}{\partial z}\left(A\frac{\partial v_1}{\partial z}\left(\frac{\partial u_1}{\partial z}\right)^2\right)+\frac{\partial}{\partial z}\left(A\theta^2\frac{\partial v_1}{\partial z}\right)-\\
&-\frac{\partial^2}{\partial z^2}\left(M(t)\frac{\partial u_1}{\partial z}\right)+\frac{1}{4}\frac{\partial}{\partial z}\left(I_x\frac{\partial v_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)+\\
&+\frac{1}{2}\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial u_1}{\partial z}\left(\frac{\partial\theta}{\partial z}\right)^2\right)\Bigg\}+\rho A\omega^2v_1+2\rho A\omega\frac{\partial u_1}{\partial t}+\\
&+\cos\omega t\left(-\frac{\partial^2Y_0}{\partial t^2}+\omega\frac{\partial X_0}{\partial t}\right)-\sin\omega t\left(\frac{\partial^2X_0}{\partial t^2}+\omega\frac{\partial Y_0}{\partial t}\right)=0,
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&-\rho\frac{\partial}{\partial t}\left(I_p\frac{\partial\theta}{\partial t}\right)-\frac{G}{(1-2\nu)}\Bigg\{-\frac{1}{8}\frac{\partial}{\partial z}\left(I_r\left(\frac{\partial\theta}{\partial z}\right)^3\right)+2A\theta^3+\\
&+4\nu A\theta\frac{\partial w}{\partial z}-\frac{1}{2}\frac{\partial}{\partial z}\left(I_p\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}\right)-\frac{3}{2}\Bigg[\frac{\partial}{\partial z}\left(I_y\frac{\partial\theta}{\partial z}\left(\frac{\partial v_1}{\partial z}\right)^2\right)+\\
&+\frac{\partial}{\partial z}\left(I_x\frac{\partial\theta}{\partial z}\left(\frac{\partial u_1}{\partial z}\right)^2\right)\Bigg]-\frac{\partial}{\partial z}\left(I_y\frac{\partial^2 u_1}{\partial z^2}\frac{\partial v_1}{\partial z}\right)-\\
&-\frac{\partial}{\partial z}\left(I_x\frac{\partial^2 v_1}{\partial z^2}\frac{\partial u_1}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial^2 u_1}{\partial z^2}\frac{\partial u_1}{\partial z}\right)-\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial^2 v_1}{\partial z^2}\frac{\partial v_1}{\partial z}\right)\Bigg\}+\\
&+\frac{G(5-6\nu)}{2(1-2\nu)}\Bigg\{\frac{1}{2}\frac{\partial}{\partial z}\left(I_x\frac{\partial\theta}{\partial z}\left(\frac{\partial v_1}{\partial z}\right)^2\right)-\frac{1}{2}I_p\theta\left(\frac{\partial\theta}{\partial z}\right)^2+\\
&+\frac{1}{2}\frac{\partial}{\partial z}\left(I_y\frac{\partial\theta}{\partial z}\left(\frac{\partial u_1}{\partial z}\right)^2\right)+\frac{1}{2}\frac{\partial}{\partial z}\left(I_p\theta^2\frac{\partial\theta}{\partial z}\right)-\\
&-2A\theta\left(\frac{\partial u_1}{\partial z}\right)^2-2A\theta\left(\frac{\partial v_1}{\partial z}\right)^2+2\frac{\partial}{\partial z}\left(I_{xy}\frac{\partial u_1}{\partial z}\frac{\partial v_1}{\partial z}\frac{\partial\theta}{\partial z}\right)+\\
&+\frac{1}{4}\frac{\partial}{\partial z}\left(I_{x^2y^2}\left(\frac{\partial\theta}{\partial z}\right)^3\right)\Bigg\}+4G\frac{\partial}{\partial z}\left(I_p\frac{\partial\theta}{\partial z}\right)+\rho I_p\omega^2\theta=0,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&\rho\left\{-\frac{\partial}{\partial t}\left(A\frac{\partial w}{\partial t}\right)\right\}+\frac{2G}{(1-2\nu)}\Bigg\{\frac{1}{2}A\frac{\partial}{\partial z}\left[\left(\frac{\partial u_1}{\partial z}\right)^2+\left(\frac{\partial v_1}{\partial z}\right)^2\right]+\\
&+(1-\nu)\frac{\partial}{\partial z}\left(A\frac{\partial w}{\partial z}\right)+\nu\frac{\partial}{\partial z}\left(A\theta^2\right)+\\
&+\frac{1}{8}\frac{\partial}{\partial z}\left(I_p\left(\frac{\partial\theta}{\partial z}\right)^2\right)\Bigg\}-\frac{\partial}{\partial z}\left(N(z,t)\frac{\partial w}{\partial z}\right)-\frac{\partial^2Z_0}{\partial t^2}=0,
\end{aligned}
\tag{13}
$$

where E is the elastic modulus of the drill string material, G is the shear modulus, ν is Poisson's ratio, ρ is the density of the string material; $I_x = I_y = I$ is the axial moment of inertia of the string; I_p is the polar moment of inertia; I_{xy} is the centrifugal moment.

$$u_1(z,t)\big|_{t=0}=0,\ \frac{\partial u_1(z,t)}{\partial t}\bigg|_{t=0}=c_1,$$
$$v_1(z,t)\big|_{t=0}=0,\ \frac{\partial v_1(z,t)}{\partial t}\bigg|_{t=0}=c_2,$$
$$w(z,t)\big|_{t=0}=\frac{\partial w(z,t)}{\partial t}\bigg|_{t=0}=0, \tag{14}$$
$$\theta(z,t)\big|_{t=0}=\frac{\partial \theta(z,t)}{\partial t}\bigg|_{t=0}=0,$$

$$u_1(z,t)\big|_{\substack{z=0\\z=l}}=v_1(z,t)\big|_{\substack{z=0\\z=l}}=0,$$
$$\frac{\partial^2 u_1(z,t)}{\partial z^2}\bigg|_{\substack{z=0\\z=l}}=\frac{\partial^2 v_1(z,t)}{\partial z^2}\bigg|_{\substack{z=0\\z=l}}=0,$$
$$\frac{\partial w(z,t)}{\partial z}\bigg|_{\substack{z=0\\z=l}}=\frac{\partial \theta(z,t)}{\partial z}\bigg|_{\substack{z=0\\z=l}}=0. \tag{15}$$

3 Results and discussions

To address the nonlinear system of partial differential equations (10)–(13) under natural boundary (14) and initial (15) conditions, the Bubnov–Galerkin method is utilized. The boundary conditions describe the simply supported mounting of the drill string, which allows for free longitudinal displacement of its end. This method is one of the most effective variational approaches, combining a rigorous mathematical framework with a high convergence rate, provided that an optimal basis is selected. The essence of the method lies in selecting a set of basis functions that satisfy the system's boundary conditions:

$$u(z,t)=\sum_{i=1}^{n}\bar{u}_i(t)\phi_i(z),\quad v(z,t)=\sum_{i=1}^{n}\bar{v}_i(t)\phi_i(z),$$
$$w(z,t)=\sum_{i=1}^{n}\bar{w}_i(t)\phi_i(z),\quad \theta(z,t)=\sum_{i=1}^{n}\bar{\theta}_i(t)\phi_i(z). \tag{16}$$

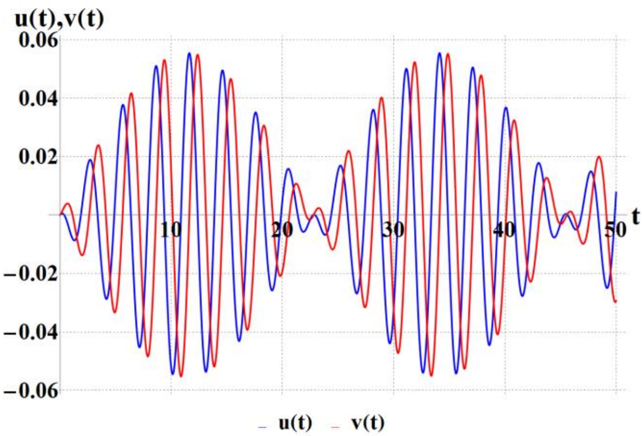
The expansion (16) is considered based on the fundamental mode. The behavior of the temporal components for displacements u,v,w,θ is investigated in the following sections.

The numerical solution was carried out using the Wolfram Mathematica symbolic computation package. The implemented approach is based on the use of adaptive numerical schemes, the selection of which is determined by the properties of the problem solved. The mathematical framework incorporates explicit Runge–Kutta methods for non-stiff systems and implicit Backward Differentiation Formulas (BDF) for stiff equations. Automatic variation of the integration step and order ensures a rational compromise between computational accuracy and speed. The application of the stiffness switching method for analyzing drill string displacements can be found in [19].

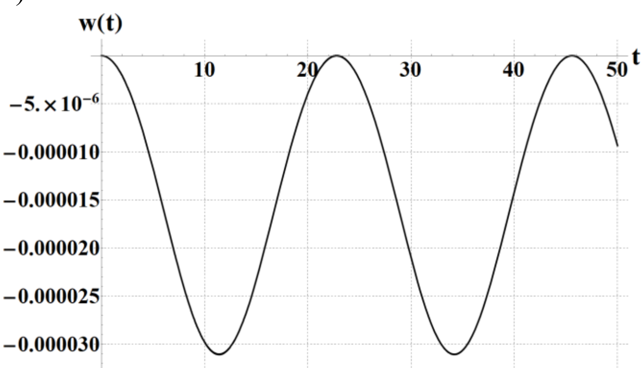
Calculations were performed based on data covering the physical and geometric parameters of the drill string, as well as the nature of the seismic load. Specifically, the following parameters were set for a steel drill string: $E=2.2\times10^5\,MPa$ – Young's modulus for steel; $\rho=7800\ kg/m^3$ – volumetric density of the material; $\nu=0.28$ – Poisson's ratio; $D_h=0.35\,m$ – external diameter of the drill string; $d=0.28\,m$ – internal diameter of the drill string; $l=200\ m$ – length of the drill string; $\omega=30$ angular velocity of the string rotation (revolutions per minute);

The definition of ground motion X_0,Y_0,Z_0 is based on the wave nature of seismic excitation. In most studies concerning seismic effects on underground structures, [20], the motion of the contacting soil is represented as a decaying traveling wave. This model of seismic impact on underground structures is the most widely used approach. In this work, the authors consider seismic action over a short time interval. The seismic waves were specified as simple periodic functions $X_0=x_0\sin\omega(t),Y_0=y_0\cos\omega(t),Z_0=z_0\sin\omega(t)$. The seismic data were sourced from official seismological centers, such as the United States Geological Survey (USGS), the European-Mediterranean Seismological Centre (EMSC), and the International Seismological Centre (ISC). The seismic wave magnitude was calculated using the formula proposed by Charles Richter: $\lambda=10^{M_l+\lg(\lambda_0(\delta))}$, where M_l is the local amplitude, λ is the maximum vibration amplitude on the Wood-Anderson seismogram, and $\lambda_0(\delta)$ is the calibration function depending on the epicentral distance to the station.

The figures below illustrate the simulation results of the drill string dynamics located at a distance of 100 km from the earthquake epicenter with a magnitude ranging from 5 to 9. Figure 1 shows the amplitudes of longitudinal and transverse vibrations of the drill string under operating conditions in the absence of seismic excitation.



a)

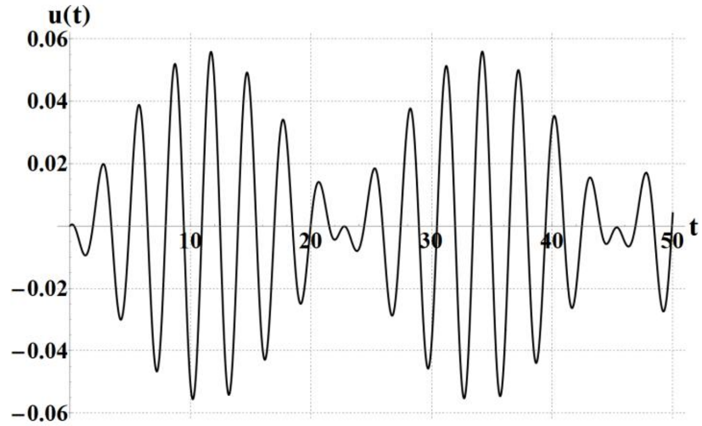


b)

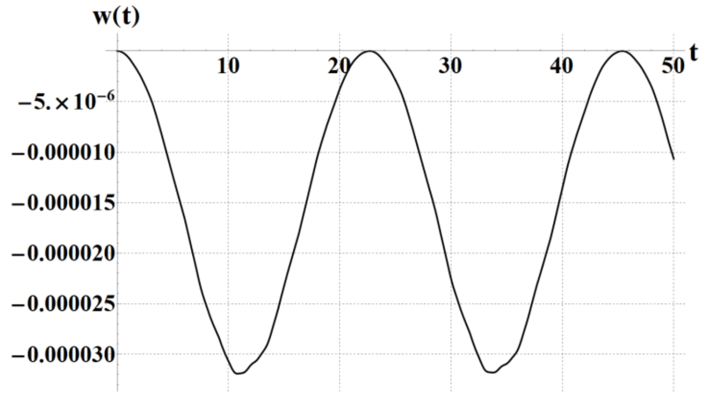
Fig.1: Drill String vibration without Seismic Load.
a) – the transverse displacement components u and v of the drillstring,b) – longitudinal displacement.
Source: created by the authors.

The results of the study on seismic wave effects on drill string dynamics are presented in Figure 2, Figure 3, Figure 4, and Figure 5. As shown in Figure 2, the vibration patterns of the transverse displacement components u and v are similar. Therefore, only one of them will be illustrated in the subsequent plots.

From the comparison of the numerical results of the drill string dynamics (Figure 1 and Figure 2), the influence of seismic waves with a magnitude of up to 5 on the amplitudes of the longitudinal and transverse vibrations of the drill string is insignificant (Figure 2). This allows for the conclusion that the safe operation of the drill string is possible under the specified seismic activity regime.



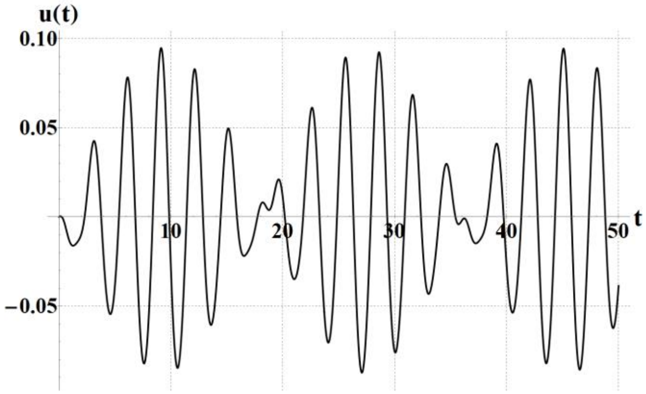
a)



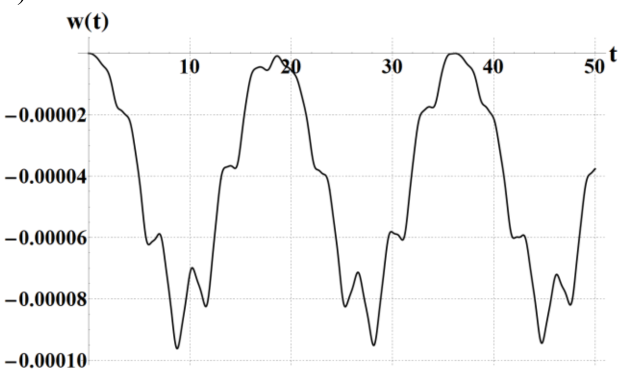
b)

Fig.2: Drill String vibration at Magnitude 5.
a) – transverse displacement,
b) – longitudinal displacement.
Source: created by the authors.

In the absence of seismic waves (Figure 1) and under the action of magnitude 5 waves (Fig.ure2), oscillating transverse vibrations of the drill string are observed. The longitudinal vibrations are harmonic in nature.
The results of studies on the effects of magnitude 6, 7, and 8 seismic waves (Figure 3, Figure 4, and Figure 5) indicate a complex nonlinear oscillatory process with a significant increase in vibration amplitude.



a)

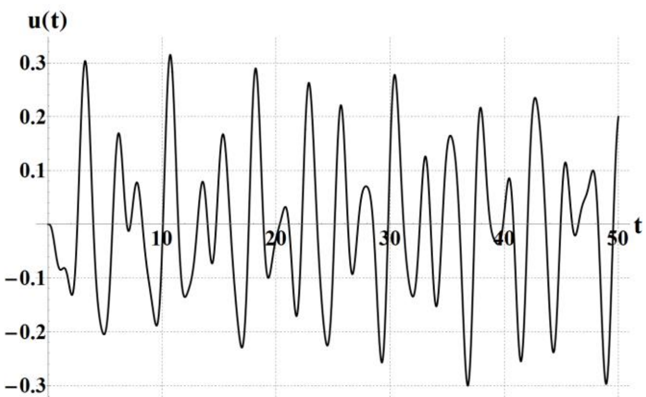


b)

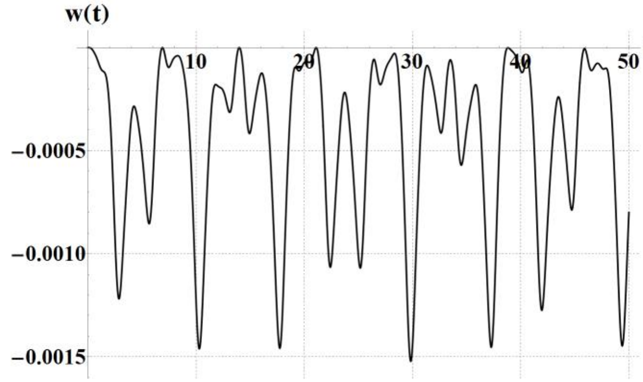
Fig.3: Drill String vibration at Magnitude 6.

a) – transverse displacement, b) – longitudinal displacement.

Source: created by the authors.



a)



b)

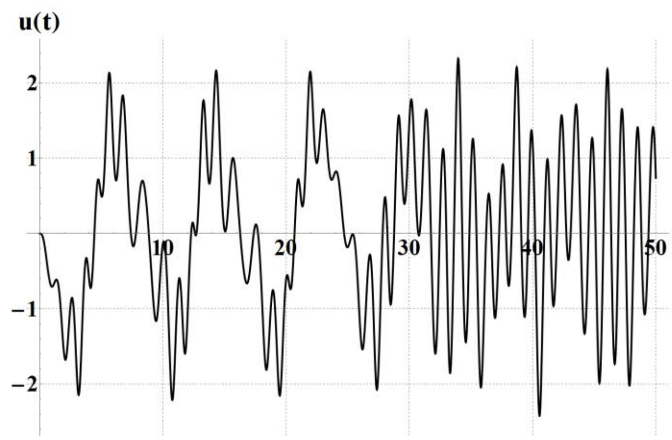
Fig.4: Drill String vibration at Magnitude 7.

a) – transverse displacement, b) – longitudinal displacement.

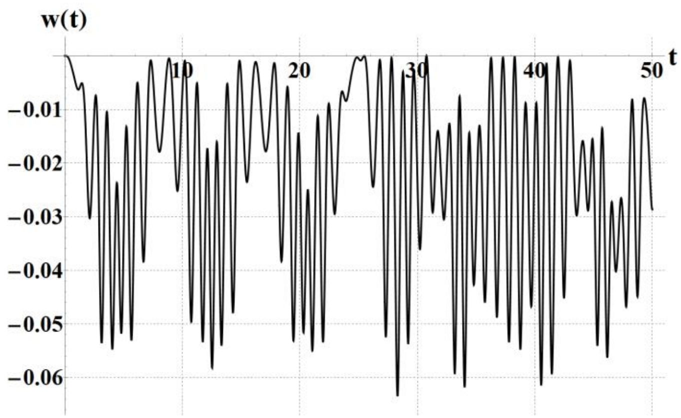
Source: created by the authors.

Seismic activity of magnitude 6 (Figure 3) leads to a noticeable increase in vibrations and a change in the dynamics of the drill string. This is particularly true for its longitudinal vibrations.

In cases of environmental seismic activity with a magnitude of 7 and above (Figure 4, and Figure 5), qualitative and quantitative changes in the amplitude and frequency characteristics of the drill string vibrations are observed. Figure 4, illustrating the case with the magnitude of 7, shows a significant increase in vibration amplitude, which causes significant dynamic loads on the drill string.



a)



b)

Fig.5: Drill String vibration at Magnitude 8.
a) – transverse displacement, b) – longitudinal displacement.
Source: created by the authors.

At a magnitude of 8 (Figure 5), substantial changes in the graphs are recorded: the presence of the beating effect, as well as an extreme growth in amplitude.

These data indicate that the drill string is in a state of critical instability due to reaching the resonant frequency or highly nonlinear, large-amplitude vibrations. This poses a direct risk of an accident, significantly increasing the fatigue of the drilling tool, damaging the drill bit, and destroying the wellbore walls.

4 Conclusion

This work focuses on the analysis of the nonlinear dynamics of the drill string under seismic loading. During the research, the authors constructed an improved nonlinear mathematical model to describe the spatial vibration of a rotating string under finite deformations. The seismic nature was integrated into the model through the dynamic displacement of the wellbore walls, specified via the kinetic energy of the entire system.

The main effects identified in the work include a substantial increase in the amplitude of string vibration, which directly indicates an increase in dynamic loads, pushing the system toward critical limits.

On the other hand, a change in the very nature of the vibration was discovered, specifically the emergence of modulation vibrations (beating effect) resulting from the superposition of the string's natural frequency and the frequency of the external seismic perturbation. For the string, the danger of beating lies in its periodicity with a chaotic growth in amplitude, which is more destructive than conventional steady-state vibration. Thus, the beating observed here, caused by sub-harmonic vibrations, directly indicates zones of dangerous operating regimes for the drill string. This occurs at a specific ratio between the natural and seismic frequencies of the nonlinear system.

Our future research will focus on the dynamics of drill strings subjected to seismic loads while considering environmental factors, specifically gas and liquid flow interactions.

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Contribution of individual authors to the creation of a scientific article (ghostwriting policy)

L. A. Khajiyeva was responsible for the conceptualization and methodology of the problem studied.

A.T. Gaisin developed the nonlinear mathematical model of drill string dynamics in a seismically active environment and conducted a numerical analysis under different loading scenarios.

All the authors participated in the preparation of the article.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.