Calculation of Rayleigh Damping Coefficients for a Transient Structural Analysis

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Abstract: - Direct numerical integration of differential equations of motion is widely used by engineers to describe the behavior of structures under dynamic loading. The method entails directly integrating the motion equations over time. In the direct method, the damping matrix is formed as a linear combination of mass and stiffness matrices multiplied by the Rayleigh damping coefficients α and β, respectively. The Rayleigh damping coefficients have a significant effect on the response of building structures under dynamic loading. Therefore, the design values of the damping coefficients α and β have crucial importance to ensure accurate and reliable results in a dynamic analysis. The paper presents a time domain analysis for a building subjected to seismic excitations using the modal superposition and direct integration methods. The direct method considers the damping properties of building structures by Rayleigh damping coefficients obtained using various approaches. The building's response to seismic load is compared by response spectra. The authors proposed the least conservative approach for calculating the Rayleigh damping coefficients for analyzing a building in the time domain.

Key-Words: - Rayleigh damping coefficients, transient analysis, time-history analysis, choosing, direct method, response spectra.

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1 Introduction

Time history analysis is used for actual time-varying loads (such as earthquakes) to the structure and predicting its response over time. The direct integration and modal superposition methods are the main methods that used in structural analysis considering inertial forces and damping. The direct integration method solves the equations of motion for the entire structure directly in the time domain. The modal superposition method builds the solution on the scaled mode shapes and sums them to capture the dynamic response.

When the dynamic analysis of a structure using the direct integration method is performed, the proportional damping (Rayleigh damping) can be applied to account for the damping properties of the structure, [1]. This includes creating a damping matrix [C] using a linear combination of mass [M] and stiffness [K] matrices, which are multiplied by proportional coefficients α and β as follows [2], [3], [4], [5], [6]:

$$[\mathrm{C}] = \alpha[\mathrm{M}] + \beta[\mathrm{K}] \tag{1}$$

where α and β are Rayleigh coefficients that decide how much the system damps. Engineers can change α and β to better match the true way the building moves. The values of α and β can be determined based on the modal damping ratios ξ_i, which represent the actual damping compared to critical damping for a specific mode shape i. If ω_i is the natural circular frequency of mode i, α and β can be related as follows, [1]:

$$\xi_i = \alpha/(2\omega_i) + \beta\omega_i/2 \tag{2}$$

Rayleigh damping can accurately match modal damping values at one or two natural frequency points, making it suitable for structures with dominant frequencies. However, for structures with numerous modes across a wide range of natural frequencies, Rayleigh damping may result in significant deviations in response compared to modal damping.

The paper aims to determine the most suitable method for calculating Rayleigh damping coefficients for a structure experiencing dynamic loads. This will be achieved through a comparison of response spectra generated from both direct and modal superposition analyses, ensuring the accurate consideration of the structure's damping characteristics.

2 Problem Formulation

The objective is to determine Rayleigh damping coefficients for a dynamic structural analysis that most accurately corresponds to the data. The accuracy of the structure's fit to a data point is evaluated through its residual, which is the variance between the prescribed damping ratio (ψ) and the estimated value from the chosen method within the designated frequency range:

$$r_i = \psi - \xi_i \tag{3}$$

The least squares method determines the best parameter values by reducing the objective function, S, which is the sum of squared residuals for each mode shape i:

$$S = \sum_{i=1}^{N} r_i^2 \tag{4}$$

Setting the gradient to zero helps to find the minimum of the sum of squares. In the Rayleigh damping model, which has two coefficients, there are two gradient equations that need to be considered:

$$\begin{cases} \frac{\partial S}{\partial \alpha} = 2\sum_{i=1}^{N} r_i \frac{\partial r_i}{\partial \alpha} = 0; \\ \frac{\partial S}{\partial \beta} = 2\sum_{i=1}^{N} r_i \frac{\partial r_i}{\partial \beta} = 0. \end{cases} \tag{5}$$

In the following approaches to various types of objective functions, S is taken into consideration, [7], [8], [9], [10].

2.1 Least Squares Method Approaches

The method of least squares is commonly used to estimate the solution of over-determined systems, such as sets of equations with more equations than unknowns. Given that, real structures typically have more natural modes than unknowns, the least squares method is applied to calculate Rayleigh damping coefficients (α and β).

2.1.1 Conventional approach (CA)

In this method, by inputting ξ, ω_{min}, and ω_{max}, two simultaneous equations (2) can be solved to determine α and β:

$$\alpha = 2\xi \frac{\omega_{max}\omega_{min}}{\omega_{max}+\omega_{min}};\ \beta = \frac{2\xi}{\omega_{max}+\omega_{min}}, \tag{6}$$

where ξ represents the damping ratio (specified in the National Regulatory Guides); ω_{min} is the lowest undamped circular frequency of the structure; and ω_{max} is the highest undamped circular frequency that affects the structure's response.

2.1.2 Pure approach (Pure)

When expression (2) is replaced by (4), the objective function, S, simplifies as follows:

$$S = \sum_{i=1}^{N} \left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right)^2 \tag{7}$$

2.1.3 Inverse Frequency Weighted Approach (IFWA)

S is considered as the objective function:

$$S = \sum_{i=1}^{N} \frac{1}{\omega_i} \left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right)^2 \tag{8}$$

2.1.4 Mass Participation Weighted Approach (MPWA)

S is considered as the objective function:

$$S = \sum_{i=1}^{N} \frac{m_i}{M} \left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right)^2 \tag{9}$$

2.2 Finite Element Model of a Building

The design of a Shielded Control Room building (SCR) is intended for the management of a nuclear plant during both regular operations and emergencies. Constructed primarily with reinforced concrete, the SCR building includes concrete block partitions and features a concrete strength of 17 MPa and an elasticity modulus of 32.5 GPa. The reinforcing bars have a modulus of 200 GPa. Anchored partly in soil, the influence of soil-structure interaction effects on Rayleigh damping coefficients is not considered in this study.

A three-dimensional finite-element model of the SCR building, depicted in Figure 1, is meshed with BEAM188 and SHELL181 finite elements based on the building's design. An additional mass of 300 kg/m^2 is distributed evenly on the floor elements, while 250 kg/m^2 is placed on the roof elements. The foundation elements of the model are constrained with a fixed support boundary condition.

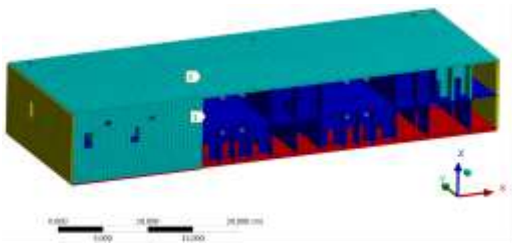


Fig. 1: Finite Element Model of the SCR building

2.3 Earthquake Ground Motion

Figure 2 and Figure 3 display the vertical and horizontal spectra components of the seismic input ground motions for the Safe Shutdown Earthquake (SSE). The predicted intensity of the seismic input ground motions is 0.201g for the horizontal component and 0.134g for the vertical component.

In Figure 4, a three-component accelerogram is presented for the SSE seismic input ground motions. The components of the accelerogram are statistically independent, with a coefficient of mutual correlation not exceeding 0.1. The accelerogram time discretization is 0.005 s, allowing for the consideration of frequencies up to 100 Hz.

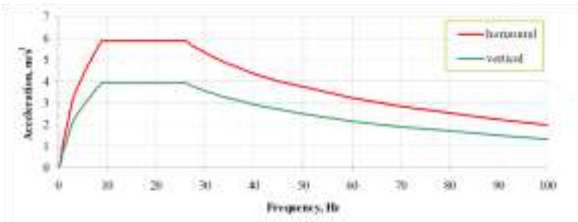


Fig. 2: Horizontal and vertical spectra components of the seismic input ground motion

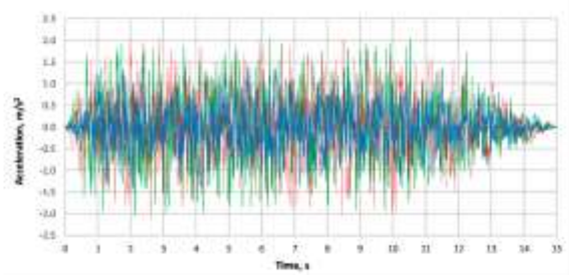


Fig. 3: Ground motion three-component accelerogram

3 Problem Solutions

The process of selecting the optimal method for establishing Rayleigh damping coefficients involved:

a) conducting modal analysis on the SCR building;

b) computing Rayleigh damping coefficients for different methods;

c) performing transient analysis on the SCR building using the direct and modal superposition methods with specified damping coefficients;

d) creating response spectra for both direct and modal superposition analyses;

e) comparing the resulting response spectra.

3.1 Modal Analysis

Based on the results of the modal calculation, Table 1 shows the vibration characteristics of the SCR building.

Table 1. Vibration characteristics of the SCR building

Mode	f_i, Hz	m_{xi}, kg	m_{yi}, kg	m_{zi}, kg
1	15.6	0	41	2350
2	15.7	2	30	118000
3	17.4	6	700	23400
5	21.1	696	2830000	318
…				
11	27.2	2240000	263	123
…				
108	50.8	1250	2480	79200
…				
500	106.8	34	127	11
Total mass of the building M = 0.505E+07 kg				
Note: f_i and ω_i represent the phase and circular natural frequencies respectively for mode shape i; m_{xi}, m_{yi}, m_{zi} indicate the effective horizontal and vertical masses corresponding to mode shape i; $m_i=(m_{xi}^2 + m_{yi}^2 + m_{zi}^2)^{0.5}$ is the total mass associated with mode shape i.				

3.2 Rayleigh Damping Coefficients for the Approaches Considered

The values of Rayleigh damping coefficients are provided in Table 2 based on the approaches described in Section 2.

Table 2. Rayleigh damping coefficients for the considered approaches

Approach	Rayleigh Damping Coefficients		Formula
	α	β	
CA	11.8909	0.000193	(6)
Least squares method:			
- Pure	21.397	0.000187	(11)
- IFWA	17.9289	0.000209	(13)
- MPWA	15.3401	0.000242	(15)

3.2.1 Pure Approach (Pure)

The system of equations (5) representing the objective function (7) is written as:

$$\begin{cases} \frac{\partial S}{\partial \alpha} = \sum_{i=0}^{N} \frac{1}{\omega_i}\left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right) = 0; \\ \frac{\partial S}{\partial \beta} = \sum_{i=0}^{N} \omega_i\left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right) = 0. \end{cases} \quad (10)$$

The solution to the system of equations (10) for α and β will be written in the form:

$$\begin{cases} \alpha = \frac{2\left(\sum_{i=1}^{N}\frac{\psi}{\omega_i}\sum_{i=1}^{N}\omega_i^2 - N\sum_{i=1}^{N}\psi\omega_i\right)}{\sum_{i=1}^{N}\frac{1}{\omega_i^2}\sum_{i=1}^{N}\omega_i^2 - N^2}; \\ \beta = \frac{2\left(\sum_{i=1}^{N}\frac{1}{\omega_i^2}\sum_{i=1}^{N}\psi\omega_i - N\sum_{i=1}^{N}\frac{\psi}{\omega_i}\right)}{\sum_{i=1}^{N}\frac{1}{\omega_i^2}\sum_{i=1}^{N}\omega_i^2 - N^2}. \end{cases} \quad (11)$$

3.2.2 Inverse Frequency Weighted Approach (IFWA)

The system of equations (5) representing the objective function (8) is written as:

$$\begin{cases} \frac{\partial S}{\partial \alpha} = \sum_{i=0}^{N} \frac{1}{\omega_i}\left(\frac{\alpha}{2\omega_i^2} + \frac{\beta}{2} - \frac{\psi}{\omega_i}\right) = 0; \\ \frac{\partial S}{\partial \beta} = \sum_{i=0}^{N} \left(\frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} - \psi\right) = 0. \end{cases} \quad (12)$$

The solution of the system of equations (12) for α and β is:

$$\begin{cases} \alpha = \frac{2\left(\sum_{i=1}^{N}\omega_i\sum_{i=1}^{N}\frac{\psi}{\omega_i^2} - \sum_{i=1}^{N}\xi\psi\sum_{i=1}^{N}\frac{1}{\omega_i}\right)}{\sum_{i=1}^{N}\frac{1}{\omega_i^3}\sum_{i=1}^{N}\omega_i - \sum_{i=1}^{N}\frac{1}{\omega_i}\sum_{i=1}^{N}\frac{1}{\omega_i}}; \\ \beta = \frac{2\left(\sum_{i=1}^{N}\frac{1}{\omega_i^3}\sum_{i=1}^{N}\psi - \sum_{i=1}^{N}\frac{1}{\omega_i}\sum_{i=1}^{N}\frac{\psi}{\omega_i^2}\right)}{\sum_{i=1}^{N}\frac{1}{\omega_i^3}\sum_{i=1}^{N}\omega_i - \sum_{i=1}^{N}\frac{1}{\omega_i}\sum_{i=1}^{N}\frac{1}{\omega_i}}. \end{cases} \quad (13)$$

3.2.3 Mass Participation Weighted Approach (MPWA)

The system of equations (5) representing the objective function (9) is written as:

$$\begin{cases} \frac{\partial S}{\partial \alpha} = \frac{1}{M}\sum_{i=0}^{N}\left(\frac{\alpha m_i}{2\omega_i^2} + \frac{\beta m_i}{2} - \frac{\psi m_i}{\omega_i}\right) = 0; \\ \frac{\partial S}{\partial \beta} = \frac{1}{M}\sum_{i=0}^{N}\left(\frac{\alpha m_i}{2} + \frac{\beta\omega_i^2 m_i}{2} - \psi\omega_i m_i\right) = 0. \end{cases} \quad (14)$$

The solution of the system of equations (14) for α and β is:

$$\begin{cases} \alpha = \frac{\sum_{i=1}^{N}\frac{\omega_i^2 m_i}{2}\sum_{i=1}^{N}\frac{\psi m_i}{\omega_i} - \sum_{i=1}^{N}\psi m_i\omega_i\sum_{i=1}^{N}\frac{m_i}{2}}{\sum_{i=1}^{N}\frac{m_i}{2\omega_i^2}\sum_{i=1}^{N}\frac{m_i}{2\omega_i^2} - \sum_{i=1}^{N}\frac{m_i}{2}\sum_{i=1}^{N}\frac{m_i}{2}}; \\ \beta = \frac{\sum_{i=1}^{N}\xi m_i\omega_i\sum_{i=1}^{N}\frac{m_i}{2\omega_i^2} - \sum_{i=1}^{N}\frac{m_i}{2}\sum_{i=1}^{N}\frac{\xi m_i}{\omega_i}}{\sum_{i=1}^{N}\frac{m_i}{2\omega_i^2}\sum_{i=1}^{N}\frac{m_i}{2\omega_i^2} - \sum_{i=1}^{N}\frac{m_i}{2}\sum_{i=1}^{N}\frac{m_i}{2}}. \end{cases} \quad (15)$$

3.3 Rayleigh Damping Curves for the Approaches Considered

Figure 4 displays the Rayleigh damping curves for the suggested methods.

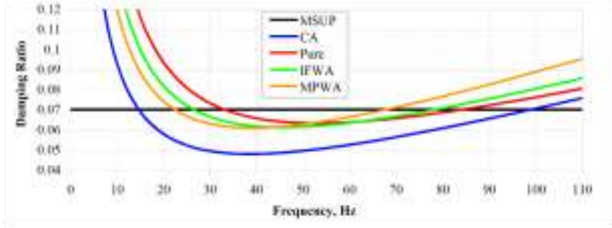
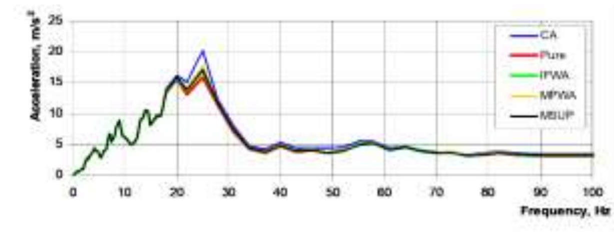


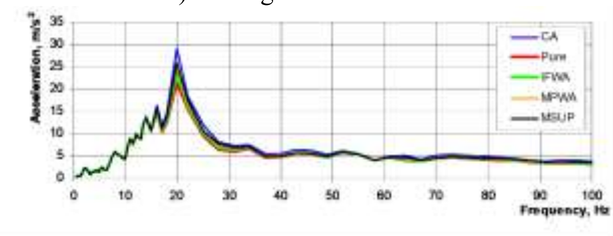
Fig. 4: Rayleigh damping curves for the suggested methods

3.4 Response Spectra

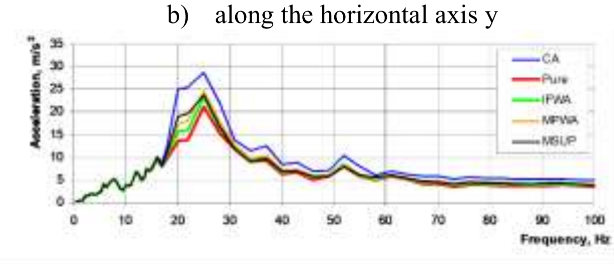
Response spectra of 2% are generated for the points illustrated in Figure 1. The reference response spectrum is represented by the MSUP curve as it accurately incorporates damping through the modal superposition method.



a) along the horizontal axis x

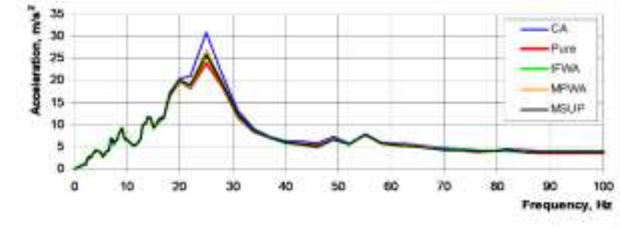


b) along the horizontal axis y

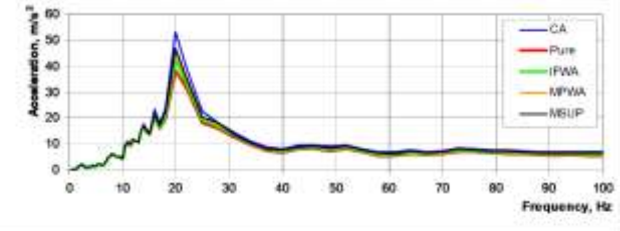


c) along the vertical axis z

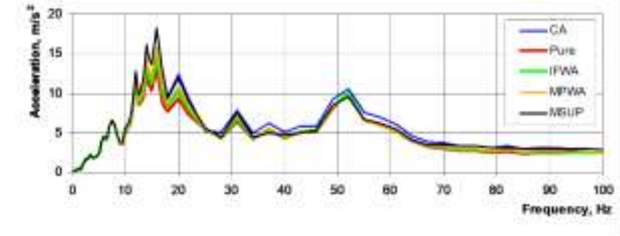
Fig. 5: Response spectra at point 1



a) along the horizontal axis x



b) along the horizontal axis y



c) along the vertical axis z

Fig. 6: Response spectra at point 2

The CA, Pure, IFWA, and MPWA curves display response spectra generated by the direct method with the Rayleigh damping coefficients listed in Table 2. The response spectra for points 1 and 2 are depicted in Figure 5 and Figure 6.

4 Conclusion

Selecting Rayleigh damping coefficients is crucial in a direct dynamic analysis. Four methods were suggested to determine these coefficients. The SCR building underwent transient analyses using both the direct and modal superposition methods. Among the methods studied, MPWA is closest to the ideal solution (MSUP). The spectral acceleration values are similar across all methods, possibly due to the building's frequency high-frequency first mode of vibration.

More research into soil-structure interaction and flexible building designs will lead to better recommendations for selecting Rayleigh damping coefficients.

Future studies will focus on more flexible buildings and consider soil-structure interaction to identify the best approach for Rayleigh parameters.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.