Optimization of a Two-stage Bevel Helical Gearbox using Multiple Objectives to Increase Efficiency and Reduce Gearbox Bottom Area

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Abstract: This study aims to look at multi-target optimization of two-stage bevel helical gearboxes to determine the best major design factors for reducing gearbox bottom area (GBA) and increasing gearbox efficiency (GE). Grey relation analysis (GRA) and the Taguchi technique were used to address the problem in two steps. Prioritizing the closure of the variable level gap, the single-objective optimization problem was addressed, followed by the multi-objective optimization problem, which identified the ideal primary design variables. Additionally, the first-stage gear ratio, allowable contact stresses (ACS), and first and second-stage coefficients of wheel face width (CWFW) were calculated. The outcomes of the study were used to determine the best values for five essential design features of a two-stage bevel helical gearbox (BHG).

Key-Words: - Bevel helical gearbox, Two-stage gearbox, Optimization, Multi-objective, Gear ratio, Gearbox efficiency, Gearbox bottom area.

Received: April 2, 2023. Revised: November 14, 2023. Accepted: December 11, 2023. Published: January 16, 2024.

1 Introduction

Extensive studies have been shown in the optimization of gearboxes. In [1], the author focused on optimizing gear ratios for a drive system with a three-stage BLH and chain drive, with the objective being minimizing the system cross section area. The authors in [2], analyzed input parameters such as total gear ratio, face width coefficients, contact stress, and output torque to minimize system length, deriving optimal gear ratios for a system with a two-stage helical gearbox with first stage double gear sets and a chain drive. Furthermore, in [3], authors focused on minimizing cross-sectional height, considering gear pitting resistance and movement equilibrium, and from that deriving optimal partial ratio, allowing accurate and efficient calculations. The authors in [4], developed a prototype of an active driven knee with BHG. Moreover, in [5], scientists used a simulation experiment to propose an equation for optimal gear ratios for three-step BHG to reduce the height of the gearbox. In [6], the authors presented a novel approach to determine optimal partial transmission ratios in a drive system using a chain drive and two-stage BHG, with the optimization aimed at minimizing the system's cross section dimension. Besides, a study to minimize the mass of a two-stage BHG was introduced, [7]. Another cost optimization study presented insights into input factor effects and proposed models for optimal gear ratios for a two-stage BHG, [8]. With an optimization problem that minimized geabox volume using eight main input parameters, the best gear ratios in a three step BHG were proposed in [9]. Furthermore, the authors in [10], focused on optimizing the gear ratios of a drive system with a three-step BHG and a V-belt for minimal system height.

From the above analysis, it is found that up to now there have been several studies on optimization and multi-objective optimization of different gearboxes. However, up to now, there has been no research on multi-objective optimization of BHG with single-objective functions minimal GBA and maximal GE.

The current study aims to investigate multi-target optimization learning for a two-step BHG. The work being done had two distinct goals: lowering GBA and optimizing GE. Furthermore, five main design elements were evaluated: the CWFW and the ACS of steps 1 and 2, and the gear ratio of step 1. Also, a multi-target optimization task for gearbox design was tackled in two stages by integrating the Taguchi technique with the GRA. The ideal values for five essential design factors were also suggested for creating a two-step BHG.

2 Optimization Problem

2.1 Determining Gearbox Length

The gearbox bottom area can be found in (Figure 1):

$$A_b = L \cdot B \quad (1)$$

Where, L and B are the gearbox length and gearbox width which are determined by (Figure 1):

$$L = 2 \cdot l_0/3 + d_{e21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot k \quad (2)$$

$$B = b \cdot cos\delta_2 + b_w + 4 \cdot S_G \quad (3)$$

In (2), $k = 8 \div 12$ [11]; $l_0 = 3 \cdot d_{s1}$ with d_{s1} is the initial shaft diameter obtained by [11]:

$$d_{s1} = [T_{11}/(0{,}2 \cdot [\tau])]^{1/3} \quad (4)$$

d_{e21} is the outside diameter of bevel gear (mm); d_{e21} can be calculated by [11]:

$$d_{e21} = 2 \cdot u_1 \cdot R_e/(1 + u_1^2)^{1/2} \quad (5)$$

Wherein, R_e is the cone distance (mm), [11]:

$$R_e = k_R \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{11} \cdot k_{h\beta 1}/[(1 - k_{be}) \cdot k_{be} \cdot u_1 \cdot [\sigma_H]^2]} \quad (6)$$

In which, $k_R = 50$ (MPa) is a coefficient, [11]; $k_{H\beta 1} = 1.04 \div 1.18$ is the contacting load coefficient of stage 1 [11]; $k_{be} = b/R_e = 0.25 \div 0.3$ is coefficient of face width; T_{11} is the torque on pinion (Nmm).

In (1), d_{w12} and d_{w22} are the pitch diameters of the pinion and gear of stage 1, [11]:

$$d_{w12} = 2 \cdot a_w/(u_2 + 1) \quad (7)$$

$$d_{w22} = 2 \cdot a_w \cdot u_2/(u_2 + 1) \quad (8)$$

In the above equation, a_w is the center distance of stage 2 which is found as, [11]:

$$a_w = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta 2}/(AS_2^2 \cdot u_2 \cdot X_{ba})} \quad (9)$$

In where, $k_{H\beta 2} = 1.05 \div 1.27$ is contacting load ratio of stage 2 [11]; AS_2 represents the permitted contact stress (MPa); $k_a = 43$ denotes a coefficient, [11]; X_{ba} is the coefficient of wheel face width and T_{12} is the torque on the pinion (Nmm) of stage 2:

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{bg} \cdot \eta_{hg} \cdot \eta_b^3\right) \quad (20)$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (31)$$

In which, T_{out} is the output torque (N.mm); $\eta_{bg} = 0.95 \div 0.97$ and $\eta_{hg} = 0.96 \div 0.98$ are the efficiency of stage 1 and 2, [11]; η_b is the rolling bearing efficiency (η_h=0.99÷0.995, [11]).

In (3), S_G is determined by, [12]:

$$S_G = 0.005 \cdot L + 4.5 \quad (42)$$

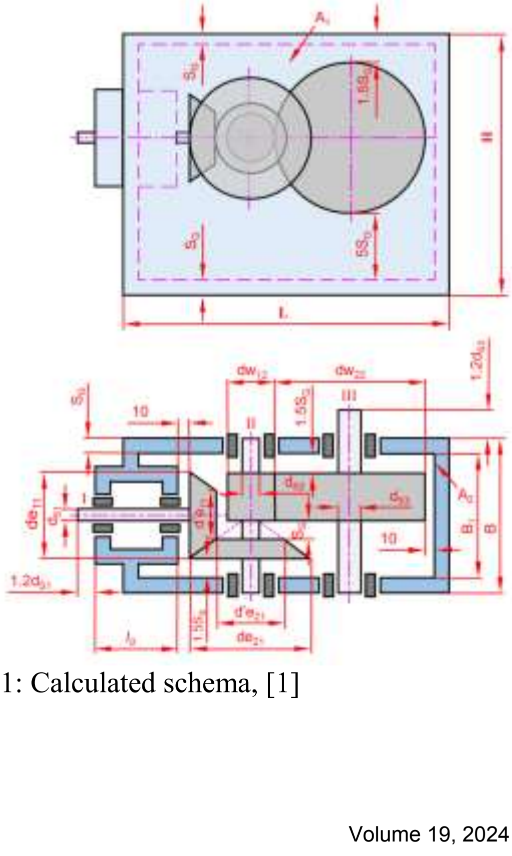

Fig. 1: Calculated schema, [1]

2.2 Calculating Gearbox Efficiency

The efficiency of the gearbox can be determined by in the following manner:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}} \tag{53}$$

Where, P_l is the overall power loss of the gearbox, [13]:

$$P_l = P_{lg} + P_{lb} + P_{ls} \tag{64}$$

In (14) P_{lg}, P_{lb}, and P_{ls} are the total power loss of gears, bearings, and seals which are calculated by:

+) The total power loss of gear:

$$\mathrm{P}_{lg} = \textstyle\sum_{i=1}^{2} \mathrm{P_{lgi}} \tag{75}$$

In which, P_{lgi} is the losses of gear power of step i:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right) \tag{86}$$

Where, η_{gi} the efficiency of the i step, [16]:

$$\eta_{gi} = 1 - \left(\frac{1+1/\mathrm{u_i}}{\beta_{\mathrm{ai}}+\beta_{\mathrm{ri}}}\right) \cdot \frac{\mathrm{f_i}}{2} \cdot \left(\beta_{\mathrm{ai}}^2 + \beta_{\mathrm{ri}}^2\right) \tag{97}$$

In which, u_i is gear ratio of the i step, f is coefficient of friction; β_{ai} and β_{ri} are the arcs of approach and recess on the i step that can be determined by, [14]:

+) For stage 1:

$$\beta_{\mathrm{ai}} = \frac{\left(\mathrm{R}_{eev2}^2 - \mathrm{R}_{0v2}^2\right)^{1/2} - \mathrm{R}_{v2} \cdot \sin\alpha}{\mathrm{R_{01i}}} \tag{108}$$

$$\beta_{\mathrm{ri}} = \frac{\left(\mathrm{R}_{aev1}^2 - \mathrm{R}_{0v1}^2\right)^{1/2} - \mathrm{R}_{v1} \cdot \sin\alpha}{\mathrm{R_{01i}}} \tag{119}$$

In which R_{aev1} and R_{aev2} are the equivalent pinion and gear outer radiuses; R_{v1} and R_{v2} are the equivalent pinion and gear pitch radiuses; R_{0v1} and R_{0v2} are the equivalent pinion and gear base radiuses; and α is the pressure angle.

$$\mathrm{R_{v1}} = \mathrm{R_1}/\cos\delta_1 \tag{20}$$

$$\mathrm{R_{v2}} = \mathrm{R_2}/\cos\delta_2 \tag{21}$$

In where R_1 and R_2 are the large pitch radius of the bevel pinion and gear; and δ_1 and δ_2 are the pitch angles of the bevel pinion and gear, correspondingly.

$$\mathrm{R_{aev1}} = \mathrm{R_{v1}} + \mathrm{a_p} \tag{22}$$

$$\mathrm{R_{aev2}} = \mathrm{R_{v2}} + \mathrm{a_g} \tag{23}$$

+) For stage 2:

$$\beta_{\mathrm{ai}} = \frac{\left(\mathrm{R_{e2i}^2} - \mathrm{R_{02i}^2}\right)^{1/2} - \mathrm{R_{2i}} \cdot \sin\alpha}{\mathrm{R_{01i}}} \tag{24}$$

$$\beta_{\mathrm{ri}} = \frac{\left(\mathrm{R_{e1i}^2} - \mathrm{R_{01i}^2}\right)^{1/2} - \mathrm{R_{1i}} \cdot \sin\alpha}{\mathrm{R_{01i}}} \tag{25}$$

In where, R_{e1i} and R_{e2i} are the outer radiuses; R_{1i} and R_{2i} are the pitch radiuses, and R_{01i} and R_{02i} are the base-circle radiuses of the pinion and gear; α is the pressure angle.

In (17), the friction coefficient is found by, [14]:

- If the sliding velocity v ≤ 0.424 (m/s):

$$f = -0.0877 \cdot v + 0.0525 \tag{26}$$

- If v > 0.424 (m/s):

$$f = 0.0028 \cdot v + 0.0104 \tag{27}$$

+) The bearing power loss can be determined by, [13]:

$$P_{lb} = \textstyle\sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \tag{28}$$

In which, $f_b = 0.0011$ is friction coefficient of a radical ball bearings, [13]; F is load on bearing (N), v is the peripheral speed of i^{th} bearing (i = 1÷6).

+) The power loss in seals is determined as, [13]:

$$P_{\mathrm{s}} = \textstyle\sum_{i=1}^{2} \mathrm{P_{si}} \tag{29}$$

In which, i =1÷2 is the ordinal seal number; P_{si} is the power loss in a single seal (w):

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \tag{30}$$

In which, VG_{40} is the ISO Viscosity Grade number.

2.3 Objective Function and Constrains

2.3.1 Objectives Functions

The multi-target work has two different objectives:

Minimizing gearbox bottom area:

$$\min f_2(X) = \mathrm{L} \tag{31}$$

Maximizing gearbox efficiency:

$$\min f_1(X) = \eta_{gb} \tag{32}$$

Where, X is the vector indicating variables. As five input parameters including u_1, Xba_1, Xba_2, AS_1, and AS_2 were selected as variables, we have:

$$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\} \tag{33}$$

2.3.2 Constraints

The multi-objective function must follow the following constraints:

$$1 \le u_1 \le 6 \ \text{ and } 1 \le u_2 \le 9 \tag{34}$$

$$0.25 \le k_{be} \le 0.3 \ \text{ and } 0.25 \le X_{ba} \le 0.4 \tag{35}$$

$$350 \le AS_1 \le 420 \text{ and } 350 \le AS_2 \le 420 \quad (36)$$

3 Methodology

In this study, five input parameters have been selected for investigation. Table 1 illustrates the minimum and maximum values for various parameters. The Taguchi approach and GRA have been used for solving the optimization work. The L25 (5^5) design was used to optimize the total amount of levels for each parameter. Nevertheless, u_1 has a wide range (ranging from 1 to 6 - Table 1) among the factors tested. Even with five levels, the difference in the values of these traits remained advantageous (in this case, the difference was 1.5 ((6-1)/4).

The 2-stage technique for solving the multi-target optimization problem was used to help decrease the difference between values of a variable spread across a wide range (Figure 2), [15]. This technique's first stage addresses a single-target optimization prob-lem, while the second stage deals a multi-target optimization work to determine the optimal primary design features.

4 Optimization problem

4.1 Single-objective Optimization

In this paper, the single-objective optimization issue is solved using the direct search strategy. Two single-objective challenges were also solved using a Matlab-based computer program: maximizing gearbox efficiency and minimizing gearbox bottom area. Figure 3 shows the connection between the total gear-box ratio u_t and the ideal gear ratio of the first stage u_1, based on the program's results. Furthermore, as Table 2 shows, new constraints have been developed for the variable u_1.

4.2 Multi-objective Optimization

The purpose of this work is to find the best primary design variables for a given total gear-box ratio while meeting two single-target functions: decreasing gearbox bottom area and optimizing gearbox efficiency. A computer experiment was constructed to address the given multi-objective optimization issue. Table 3 displays the key design components and their values for u_t = 15. The experimental design was created using the Taguchi technique using L25 (5^5) design, and the data was analyzed using Minitab R18 software. The experimental design and results for ut = 15 are shown in Table 4.

Table 1. Main design factors and their maximum and lowest limits

Factor	Notation	Lower limit	Upper limit
Gear ratio of step 1	u_1	1	6
CWFW of step 1	k_{be}	0.25	0.3
CWFW of step 2	X_{ba}	0.25	0.4
ACS of step 1 (MPa)	AS_1	350	420
ACS of step 2 (MPa)	AS_2	350	420

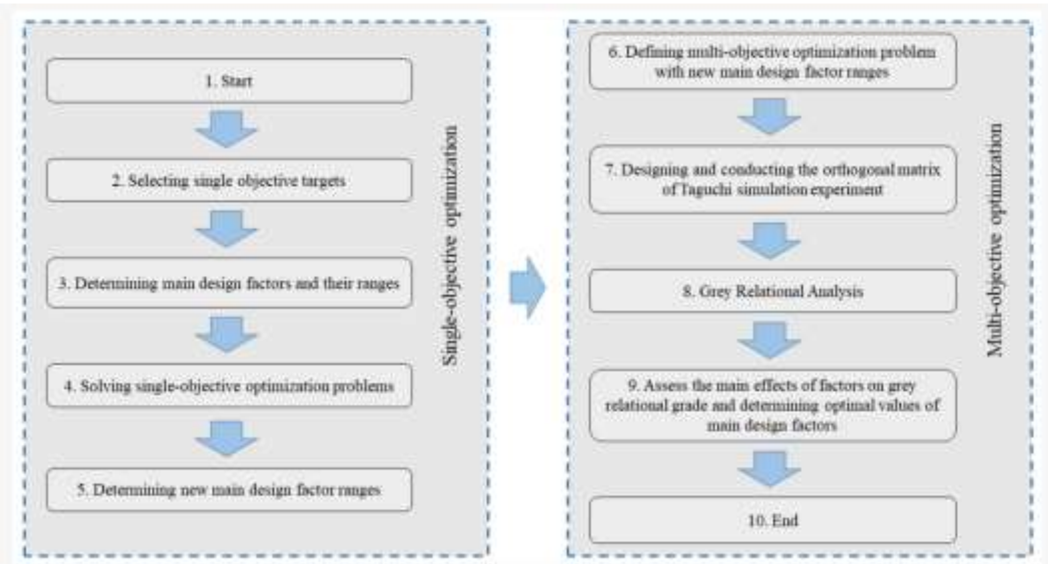

Fig. 2: Method for solving multi-objective problem, [16]

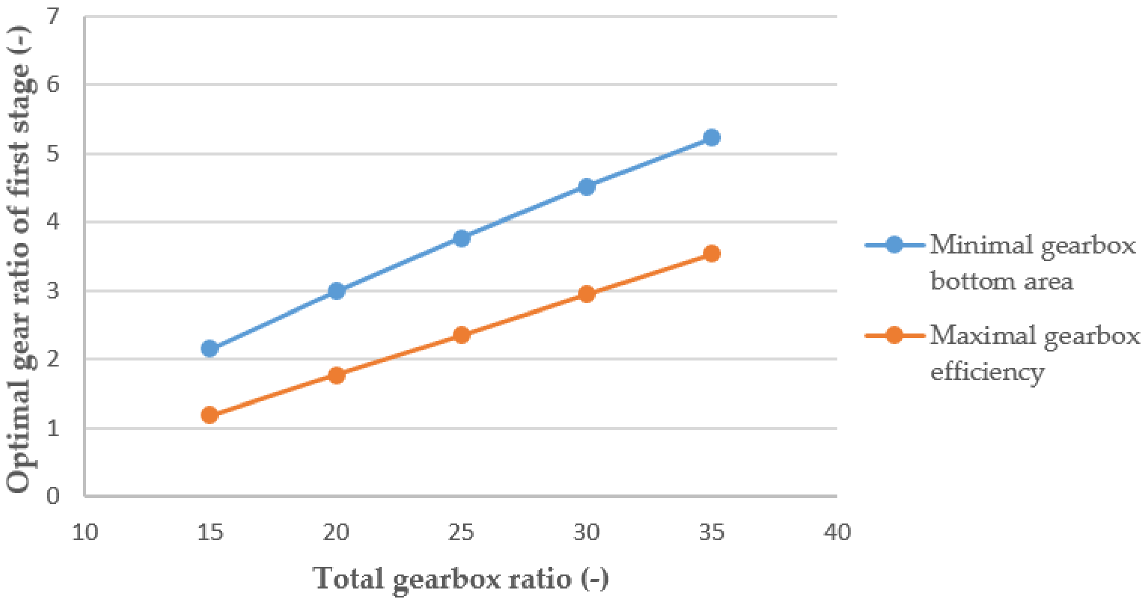

Fig. 3: Relation between optimal values of u_1 and u_t

Table 2. New constraints of u_1

u_t	u_1	
	Lower limit	Upper limit
10	1.17	2.16
15	1.76	3
20	2.34	3.78
25	2.93	4.52
30	3.52	5.24
35	4.1	5.93

Table 3. Input parameters and their levels for u_t = 15.

Factor	**Notation**	**Level**				
		1	2	3	4	5
Gear ratio of step 1	u_1	1.76	2.33	2.90	3.47	4.04
CWFW of step 1	k_{be}	0.25	0.2625	0.275	0.2875	0.3
CWFW of step 2	X_{ba}	0.25	0.2875	0.325	0.3625	0.4
ACS of step 1 (MPa)	AS_1	350	368	386	404	420
ACS of step 2 (MPa)	AS_2	350	368	386	404	420

Table 4. Experimental plan and results when u_t = 15

Exp. No.	Input parameters					Ab	η_{gb}
	u_1	K_{be}	X_{ba}	AS_1	AS_2	(dm^2)	(%)
1	1.76	0.2500	0.2500	350	350	5.520	95.188
2	1.76	0.2625	0.2875	368	368	5.413	95.073
3	1.76	0.2750	0.3250	386	386	5.308	95.043
4	1.76	0.2875	0.3625	404	404	5.204	94.917
5	1.76	0.3000	0.4000	420	420	5.132	94.882
6	2.07	0.2500	0.2875	386	404	4.732	95.047
7	2.07	0.2625	0.3250	404	420	4.685	95.013
8	2.07	0.2750	0.3625	420	350	5.761	94.990
9	2.07	0.2875	0.4000	350	368	5.824	94.943
10	2.07	0.3000	0.2500	368	386	4.897	95.091
11	2.38	0.2500	0.3250	420	368	5.165	94.958
12	2.38	0.2625	0.3625	350	386	5.270	95.000
13	2.38	0.2750	0.4000	368	404	5.161	94.930
14	2.38	0.2875	0.2500	386	420	4.394	94.991
15	2.38	0.3000	0.2875	404	350	5.379	95.026
16	2.69	0.2500	0.3625	368	420	4.756	94.923
17	2.69	0.2625	0.4000	386	350	5.778	94.913
18	2.69	0.2750	0.2500	404	368	4.861	95.012
19	2.69	0.2875	0.2875	420	386	4.793	94.892
20	2.69	0.3000	0.3250	350	404	4.947	94.992
21	3.00	0.2500	0.4000	404	386	5.165	94.871
22	3.00	0.2625	0.2500	420	404	4.387	94.932
23	3.00	0.2750	0.2875	350	420	4.587	94.962
24	3.00	0.2875	0.3250	368	350	5.548	94.964
25	3.00	0.3000	0.3625	386	368	5.425	94.930

Table 5. S/N values of each experiment when u_t=15

Exp. No.	Input Factors					Ab		η_{gb}	
	u_1	K_{be}	X_{ba}	AS_1	AS_2	(dm^2)	S/N	(%)	S/N
1	1.76	0.2500	0.2500	350	350	5.520	-14.8388	95.188	39.5716
2	1.76	0.2625	0.2875	368	368	5.413	-14.6688	95.073	39.5611
3	1.76	0.2750	0.3250	386	386	5.308	-14.4986	95.043	39.5584
4	1.76	0.2875	0.3625	404	404	5.204	-14.3267	94.917	39.5469
5	1.76	0.3000	0.4000	420	420	5.132	-14.2057	94.882	39.5437
6	2.07	0.2500	0.2875	386	404	4.732	-13.5009	95.047	39.5588
7	2.07	0.2625	0.3250	404	420	4.685	-13.4142	95.013	39.5557
8	2.07	0.2750	0.3625	420	350	5.761	-15.2100	94.990	39.5536
9	2.07	0.2875	0.4000	350	368	5.824	-15.3044	94.943	39.5493
10	2.07	0.3000	0.2500	368	386	4.897	-13.7986	95.091	39.5628
11	2.38	0.2500	0.3250	420	368	5.165	-14.2614	94.958	39.5506
12	2.38	0.2625	0.3625	350	386	5.270	-14.4362	95.000	39.5545
13	2.38	0.2750	0.4000	368	404	5.161	-14.2547	94.930	39.5481
14	2.38	0.2875	0.2500	386	420	4.394	-12.8572	94.991	39.5536
15	2.38	0.3000	0.2875	404	350	5.379	-14.6140	95.026	39.5568
16	2.69	0.2500	0.3625	368	420	4.756	-13.5448	94.923	39.5474
17	2.69	0.2625	0.4000	386	350	5.778	-15.2356	94.913	39.5465
18	2.69	0.2750	0.2500	404	368	4.861	-13.7345	95.012	39.5556
19	2.69	0.2875	0.2875	420	386	4.793	-13.6121	94.892	39.5446
20	2.69	0.3000	0.3250	350	404	4.947	-13.8868	94.992	39.5537
21	3.00	0.2500	0.4000	404	386	5.165	-14.2614	94.871	39.5427
22	3.00	0.2625	0.2500	420	404	4.387	-12.8434	94.932	39.5483
23	3.00	0.2750	0.2875	350	420	4.587	-13.2306	94.962	39.5510
24	3.00	0.2875	0.3250	368	350	5.548	-14.8827	94.964	39.5512
25	3.00	0.3000	0.3625	386	368	5.425	-14.6880	94.930	39.5481

Table 6. Values of $\Delta_i(k)$ and $\overline{y_i}$

No.	S/N		Zi		Δ_i (k)		Grey relation value y_i		$\overline{y_i}$
	Ab	η_{gb}	Ab	η_{gb}	Ab	η_{gb}	Ab	η_{gb}	
			Reference values						
			1.000	1.000					
1	-15.3387	39.4711	0.2853	1.0000	0.715	0.000	0.412	1.000	0.706
2	-15.1828	39.4590	0.3496	0.6200	0.650	0.380	0.435	0.568	0.501
3	-15.0240	39.4582	0.4150	0.5970	0.585	0.403	0.461	0.554	0.507
4	-14.8639	39.4451	0.4810	0.1876	0.519	0.812	0.491	0.381	0.436
5	-14.7534	39.4391	0.5266	0.0000	0.473	1.000	0.514	0.333	0.423
6	-14.1616	39.4549	0.7705	0.4933	0.229	0.507	0.685	0.497	0.591
7	-14.0847	39.4466	0.8022	0.2366	0.198	0.763	0.717	0.396	0.556
8	-15.8478	39.4510	0.0754	0.3722	0.925	0.628	0.351	0.443	0.397
9	-16.0088	39.4524	0.0091	0.4154	0.991	0.585	0.335	0.461	0.398
10	-14.4708	39.4564	0.6431	0.5394	0.357	0.461	0.583	0.520	0.552
11	-14.9808	39.4448	0.4328	0.1789	0.567	0.821	0.469	0.378	0.424
12	-15.2145	39.4538	0.3365	0.4587	0.663	0.541	0.430	0.480	0.455
13	-15.0394	39.4449	0.4087	0.1818	0.591	0.818	0.458	0.379	0.419
14	-13.6049	39.4492	1.0000	0.3174	0.000	0.683	1.000	0.423	0.711
15	-15.3401	39.4524	0.2847	0.4154	0.715	0.585	0.411	0.461	0.436
16	-14.3584	39.4493	0.6894	0.3203	0.311	0.680	0.617	0.424	0.520
17	-16.0308	39.4500	0.0000	0.3405	1.000	0.660	0.333	0.431	0.382
18	-14.4921	39.4521	0.6343	0.4068	0.366	0.593	0.578	0.457	0.517
19	-14.3800	39.4419	0.6805	0.0895	0.320	0.911	0.610	0.354	0.482
20	-14.7072	39.4441	0.5456	0.1587	0.454	0.841	0.524	0.373	0.448
21	-15.0732	39.4416	0.3947	0.0779	0.605	0.922	0.452	0.352	0.402
22	-13.6121	39.4427	0.9970	0.1126	0.003	0.887	0.994	0.360	0.677
23	-14.0417	39.4527	0.8199	0.4241	0.180	0.576	0.735	0.465	0.600
24	-15.6852	39.4522	0.1425	0.4097	0.858	0.590	0.368	0.459	0.413
25	-15.4976	39.4408	0.2198	0.0549	0.780	0.945	0.391	0.346	0.368

The Taguchi and GRA approaches are used for dealing with multi-objective optimization problems. The following are the major steps in this approach:

+) Using the following equations, calculate the signal-to-noise ratio (S/N):

The better the S/N, the shorter the gearbox bottom area:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \tag{37}$$

The greater the S/N ratio, the better for the gearbox efficiency goal:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2}) \tag{38}$$

Where y_i is the output result and m is the number of experiment repeats. Since the experiment is a simulation, m = 1 and no repeats are needed. Table 5 shows the estimated S/N indices of output objectives.

The data amounts for the two single-target functions were dissimilar. To guarantee similarity, the data must be normalized, or brought to a standard scale. The normalization value Zij, which changes from 0 to 1, is used to normalize the data. This value is calculated using the following formula:

$$Z_i = \frac{SN_i - min(SN_i, = 1,2,..n)}{max(SN_i, j = 1,2,..n) - min(SN_i, = 1,2,..n)} \tag{39}$$

In which, n=25 is the total test runs.

+) The grey relational (GR) factor can be found by:

$$y_i(k) = \frac{\Delta_{min} + ξ.\Delta_{max}(k)}{\Delta_i(k) + ξ.\Delta_{max}(k)} \tag{40}$$

$\Delta_j(k) = \|Z_0(k) - Z_j(k)\|$; i=1,2,...,n; k=2 is the number of objectives; Δ_{min} and Δ_{max} are the minimum and maximum values of $\Delta_i(k)$, respectively; and ζ =0.5 is the distinguishing factor.

+) Identifying the level of grey in a situation: It is calculated by averaging the GR coefficients associated with the output targets:

$$\overline{y_i} = \frac{1}{k}\sum_{j=0}^{k} y_{ij}\,(k) \qquad (41)$$

In which y_{ij} is the GR value of the i^{th} experiment's j^{th} output aim. For each trial, Table 6 presents the projected GR number y_i as well as the average GR value $\overline{y_i}$.

A greater average GR value is advised to promote harmony between the output factors. As a result, a multi-target task can be reduced to a single-target work, yielding the mean GR value.

Table 7. Analysis of variance for means

Analysis of Variance for Means

Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u1	4	0.012636	0.012636	0.003159	1.31	0.400	10.54
Kbe	4	0.032398	0.032398	0.008100	3.36	0.134	27.02
Xba	4	0.031287	0.031287	0.007822	3.24	0.141	26.10
AS1	4	0.015194	0.015194	0.003798	1.57	0.335	12.67
AS2	4	0.018730	0.018730	0.004683	1.94	0.268	15.62
Residual Error	4	0.009650	0.009650	0.002413			8.05
Total	24	0.119896					

Model Summary

S	R-Sq	R-Sq(adj)
0.0491	91.95%	51.71%

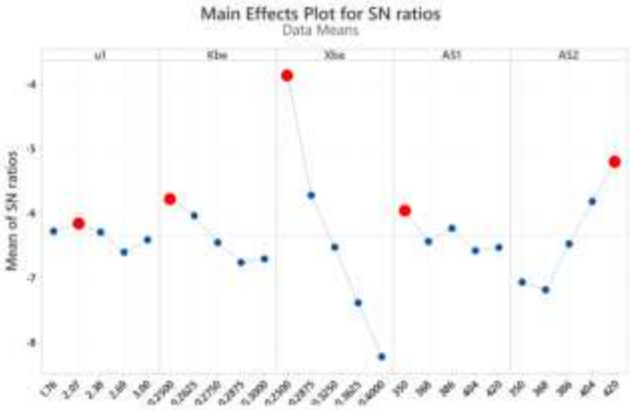

Fig. 4: Main effects plot for S/N ratios

Table 8. Optimum input parameters

No.	Input factors	Code	Optimum Level	Optimum Value
1	Gear ratio of first stage	u_1	2	2.07
2	CWFW of first step	k_{be}	1	0.25
3	CWFW of second step	X_{ba}	1	0.25
4	ACS of first step (MPa)	AS_1	1	350
5	ACS of second step (MPa)	AS_2	5	420

Table 9. Optimal values of main design factors

u_t	10	15	20	25	30	35
u_1	1.4175	2.07	2.5	3.1	3.52	4.1
K_{be}	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba}	0.25	0.25	0.25	0.25	0.25	0.25
AS_1	350	350	350	350	350	350
AS_2	420	420	420	420	420	420

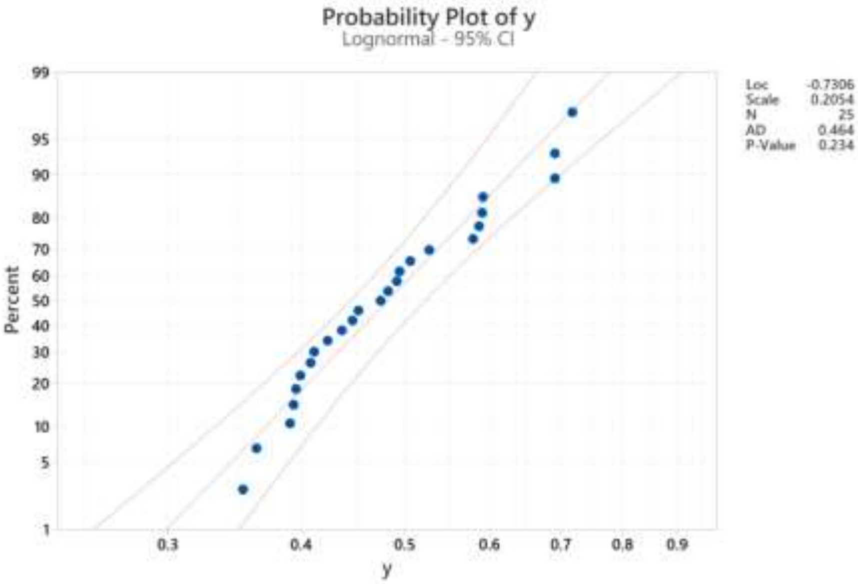

Fig. 5: Probability plot of $\bar{y}$

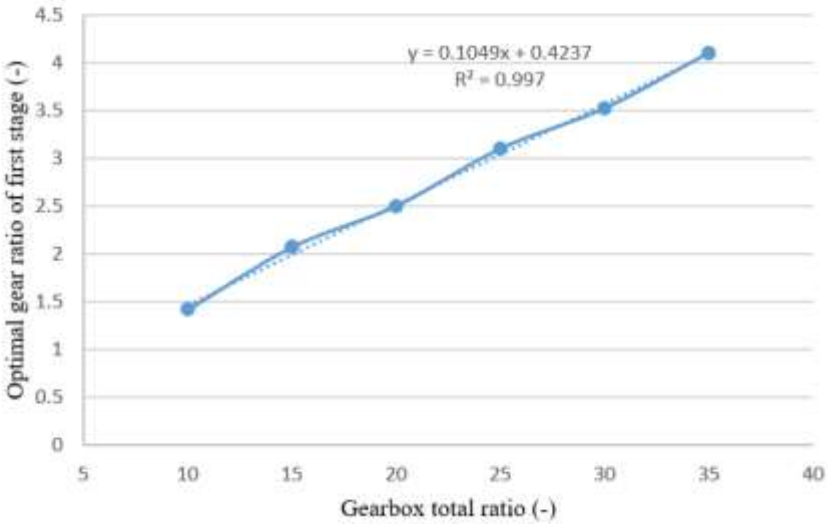

Fig. 6: Relation between u_1 and u_t

4.3 Results and Discussion

Table 7 displays the results of an ANOVA test run to assess the influence of the key input factors on the average GR value $\overline{y_i}$. According to this table, k_{be} has the highest impact on $\overline{y_i}$ (27.02 %), followed by X_{ba} (26.10 %), AS_2 (15.62 %), AS_1 (12.67 %), and u_1 (10.54 %).

+) Identifying the best primary design parameters: In theory, the best factor set would include fundamental design elements with the highest S/N values. Therefore, the influence of the key input aspects on the S/N ratios was calculated (Figure 4). From Figure 4, the optimal levels of the input factors for multi-target work (corresponding to the red points) have been easily determined. These optimal levels was described in Table 8.

+) Evaluating the experimental modeling: Figure 5 displays the Anderson-Darling approach findings, which are used to examine the adequacy of the suggested model. The data points that match the findings from the experiment (shown in the graph as blue points) are among the top 95% standard deviation zone specified by the top and bottom limits. Moreover, the significance level of $\alpha = 0.05$ is significantly lower than the p-value of 0.234. These results demonstrate the applicability of the experimental model for assessment in this work.

Continue in the same manner as with u_t=15, but with the lasting u_t values including 10, 20, 25, 30, and 35. Table 9 shows the optimal values for each of the five key design parameters at different u_t. Figure 6 shows the connection between the proper first-stage gear ratio and the total gearbox ratio. This table provided the following results:

- k_{be} and X_{ba} choose the least value feasible. This is because these variables were used to optimize the average grey relation value $\overline{y_i}$.

- Ideal AS_1 values are the lowest, while ideal AS_2 values are the highest. This is due to the fact that these modifications increased the average grey relation value $\overline{y_i}$.

- Figure 6 presents the association between the suitable first-stage gear ratio and the overall gearbox ratio. In addition, the following regression formula (with R^2=0.997) is provided to calculate the best values of u_1:

$$u_1 = 0.049 \cdot u_t + 0.4237 \tag{42}$$

After calculating u_1, the optimal value of u_2 is found using $u_2=u_t/u_1$.

5 Conclusion

This article introduces the results of a multi-objective optimization work on the optimization of a two-stage BHG to reduce gearbox bottom area and boost gearbox efficiency. This study optimized the gear ratio of step 1, the CWFW for steps 1 and 2, and the ACS for steps 1 and 2. To address this issue, a simulation experiment based on the Taguchi L25 type was developed and performed. The impact of significant design elements on the multi-objective goal was also investigated. It was noted that X_{ba} has the highest impact on $\overline{y_i}$ (72.15%), followed by AS_2 (17.56%), k_{be} (4.4%), AS_1 (2.43%), and u_1 (1.02%). In addition, the ideal settings for the important gearbox parameters have been recommended. A regression approach (Equation (42)) for calculating the appropriate first stage u_1 gear ratio was also described. However, it is necessary to conduct further research on multi-objective optimization for the three-stage bevel helical gearbox.

Acknowledgement:
This research has been supported by Thai Nguyen University of Technology.

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Tran Huu Danh, Dinh Van Thanh,
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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)
The paper's concept was set out by Luu Anh Tung. Luu Anh Tung and Tran Huu Danh carried out the optimization and simulation. Luu Anh Tung wrote the manuscript with assistance from Nguyen Manh Cuong and Tran Huu Danh. After reading the manuscript, each author gave their approval.

Sources of Funding for Research Presented in a Scientific Article or Scientific Article Itself
No funding was received for conducting this study.

Conflict of Interest
The authors have no conflicts of interest to declare.