Mathematical Model and Theoretical Investigation of the Performance of Journal Bearing using a discretized Reynolds Lubrication Equation with Finite Width

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Abstract: - Hydrodynamic journal bearings are widely used for a large number of applications such as rotating turbo machinery, axial flow compressors, axial flow turbines, and internal combustion engines. These types of journals are used to support high loads high speed rotating turbo machinery elements mainly shafts and attached components and also for enhancing the quality of these machines. In this work, the importance of journal bearing and an overview of the Reynolds hydrodynamic equation in three dimensions have been studied using the Boundary Value finite-difference method. The second-order nonlinear partial differential equation has been solved using numerical iterations approach with MATLAB software. The numerical solution of the transient Reynolds equation has been studied to investigate the Pressure distribution, pressure gradient as well film lubricant thickness on journal bearings. A numerical solution using ANSYS FLUENT has been applied to investigate the main parameters that have the main influence on the performance of the journal bearing and bearing performance.

Key-Words: - journal bearings, hydrodynamic, Reynolds equation, rotating shafts.

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1 Introduction

The main purpose of journal bearings is to support rotating shafts in a large number of applications mainly turbo machinery, crankshafts, and camshafts applications such as in internal combustion engines, axial flow compressors, and axial flow turbines, [1].

Both laminar and turbulent flow regions between rotating and stationary elements are governed by complex three-dimensional Nervier stokes equations in both Cartesian cylindrical coordinates and sometimes there is a special need for body-fitted coordinates which need special transformation equations, [2].

The main design parameters of journal bearings are lubricant type, temperature variation between the journal and the shaft, coefficient of friction, lubricant volumetric flow rate, type of load, and minimum film thickness, [3].

Three dimensional Reynolds equation for plane slider bearing has been used. The bearing curvature effect has been neglected which means a plane journal bearing where there is a pressure gradient in both radial and tangential directions only exists. Minimum film thickness, maximum operating temperature, and minimum design factor of safety at running load are the most important design parameters that should be considered, [4].

A steel base metal journal bearing is the most popular type and it has a bore equal to the nominal diameter of the shaft diameter plus the desired tolerance.

During the start, there is metal-to-metal contact and excessive vibration and heat generation due to this contact limiting the life of the bearing. The main advantage of a journal bearing is to dissipate the generated heat from contact surfaces, especially at high speeds. Journal bearings are also used to minimize the damping effect for rotating shafts, and motors and also have a great ability to take up shock and vibration and also to minimize the noise of moving machine parts, [5]. Metal-to-metal contact, lubricant working temperature, start-up friction factor, and eccentricity effect at high speed, all these factors have a great effect on the journal performance. If the eccentricity is large, there is a metal-to-metal contact especially at high dynamic loads causing premature fatigue, [6].

Going back to 1959, [7] , investigated the effect of changing . Load-carrying capacity, lubricant film thickness, Coefficient of static and dynamic friction factors, and Pressure distribution using a continuity equation. The separation of variables method has been used to solve a simplified model of the Reynolds equation.

Analytical exact solution for a standard form of one dimension Reynolds equation has been studied and solved for special cases under certain boundary conditions. Two and three dimensions of the Reynolds equation have been solved using numerical methods such as finite difference and finite element depending on the specified special cases of boundary conditions and geometry.

A boundary value problem of the Reynolds equation in one and two dimensions has been studied for specified cases of the independent variable. Boundary conditions that correspond to oil film thickness for both maximum and minimum data were specified for different independent variables, such as viscosity, journal speed, bearing pressure, and bearing radial clearance.

The study [8], presented a detailed solution for a modified Reynolds equation for the lubrication of finite-length journal bearing for non-Newtonian fluids based on momentum, continuity equations, and stress constitutive equations. The velocity components for the two-dimensional flow of a non-Newtonian fluid were obtained. For different solutions obtained, differentiation and integration techniques have been applied for different cases of boundary conditions they found that the performances of journal bearings lubrication with a non-Newtonian fluid can be compared with the case of Newtonian lubricant through the variation of the non-Newtonian parameter, i.e., the nonlinear factorψ. Inertia forces and centrifugal forces have been neglected in calculations, also fluid inertia and surface roughness are neglected in calculations.

The authors of [1] , [2], [9] , demonstrated an analytical technique to perform an analytical solution of finite length journal bearing. perturbation method has been proposed to find pressure distribution through the entire length of the bearing. Pressure coefficient, shear stress, as well as friction factor, and fluid film pressure distribution were calculated and analyzed as a function of azimuthal position factor, dimensionless bearing Somerfield number, minimum film thickness, and eccentricity ratio .

The Study [10], demonstrated the relation between turbulence and inertia effects on pressure distribution . They found that Convective inertia effects also boost the load capacity and shift the journal position to a lower eccentricity, depending on the magnitude of Reynolds number values. The turbulence effect has a great influence on static characteristics parameters of journal bearing that is by increasing the load capacity .

The study [11], demonstrated the main performance characteristics of a journal bearing lubricated with a Bingham fluid. he derived a three-dimensional computational fluid dynamic (3-D CFD) technique. The FLUENT software package is used to calculate the hydrodynamic balance of the journal bearing using the so-called "dynamic mesh" technique. The obtained results agree with experimental and analytical data from investigations on Bingham fluids. The main advantage of CFD code is that it uses the full Naiver–Stokes equations and provides a solution to the flow problem at the end of their research they conclude that at a high value of relative eccentricity, a core is formed and adheres to a small region of the journal. As the value of eccentricity increases, the solid on the bearing separates into two or three parts, and a hollow core between these parts is observed.

The authors in [12], research a 3D CFD model to compute power friction losses due to journal bearings. Computations were carried out for various oil entrance temperatures and rotational speeds. Results are presented and discussed, making comparisons with some sets of experiments carried out in the CNAM laboratory using a special turbocharger test rig equipped with a torque meter.

The authors in the study [13], solved the Reynolds equation using the Gauss-Seidel iterative algorithm in the case of turbocharger bearings.

The authors in the article [14], studied the solution of the Reynolds equation in two dimensions, results show that friction power loss is over estimated compared to experiment data which is mainly due to the isothermal assumption which neglects viscous heating and modification of the viscosity. In this work we developed a 3D CFD model, to solve three dimensional Reynolds equation.

2 Problem Formulation

Reynolds has derived a generalized form of a three-dimensional governing equation with variable fluid film thickness, which depends mainly on density, film thickness, lubricant pressure, and transverse velocities. The equation derived initially by

Reynolds was restricted to incompressible fluids as shown in Figure 1. Thus, it had been developed generally enough to incorporate the results of compressibility and dynamic loading and was aforementioned to be Generalized Reynolds Equation . So, the main form of the Generalized Reynolds Equation was as given in three dimensions is given by:

$$\left(\frac{\rho h^3}{\mu}\frac{dp}{dx}\right)+\left(\frac{\rho h^3}{\mu}\frac{dp}{dz}\right)=6U\frac{d(\rho h)}{dx}+12\rho V_o d \quad (1)$$

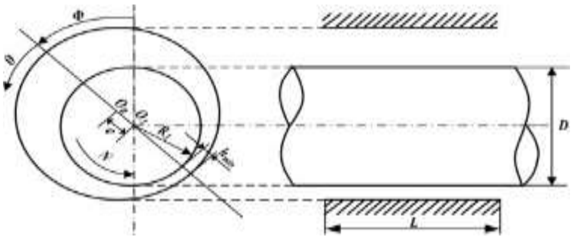

Fig. 1: Journal Bearing Layout

Where ρ is the density, μ is the dynamic viscosity, h is the lubricant film thickness, U1, and U2 are the surface velocities, e is the journal eccentricity , N is the journal rotational speed, L is the journal length, O is the journal center and V is that the general velocity. Within equation (1), the velocity difference term introduced was because of the bearing velocities on the lubricator film and depended on whether or not the bearing surfaces have angular or translational velocities. In most cases, the bearing (bushing) is stationary and the journal (shaft) is the runner also, the shaft within the journal bearings is moving, therefore U1=U and U2=0. Currently, the most general form of Reynolds equation for incompressible viscous flow in Cartesian coordinates is as given:

$$\frac{d}{dx}\left(\frac{\rho h^3}{\mu}\frac{dp}{dx}\right)+\frac{d}{dz}\left(\frac{\rho h^3}{\mu}\frac{dp}{dz}\right)=6U\frac{d(\rho h)}{dx}+12\rho V_o d \quad (2)$$

Where U is the net sliding velocity, and V is the motion of the journal center, [4]. This equation is a two-dimensional partial differential equation time independent boundary condition, which is familiar in different applications in physics and engineering. These applications involve heats equation, wave equation and rigid body motion . These problems focus on the determination of normal modes using eigenfunctions and differential operators.

Based on the principles of transformation equations between Cartesian and polar coordinates as shown in Figure 2, the Reynold equation can be easily converted into the polar coordinate system with the following form final form [3], [4]:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{6\mu}\frac{\partial p(x,\theta)}{\partial x}\right)+\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(\frac{h^3}{6\mu}\frac{\partial p(x,\theta)}{\partial\theta}\right)=\omega\frac{\partial h}{\partial\theta} \quad (3)$$

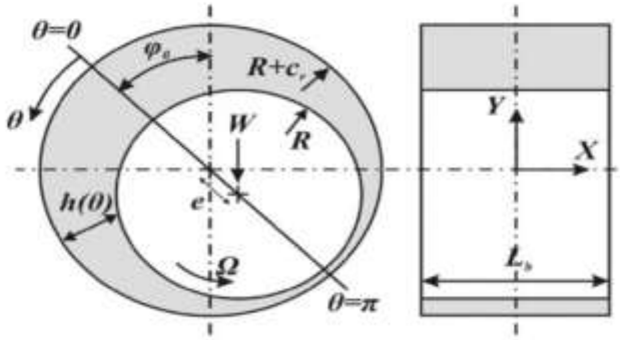

Fig. 2: Polar diagram showing the notations used for journal bearing

From the geometry of the journal bearing, fluid film thickness can be expressed as the following expression:

$$h=c_r+e.\cos(\theta) \quad (4)$$

where (c_r and e) constant value.

The boundary value problem has a specified boundary conditions at the boundaries of a well known geometry of the independent variables. Reynolds equation which is the main governing equation that describes the relation between coordinates x , y , z and the pressure through the thin film of the lubricant. This equation needs to specify the boundary conditions of the lubricant film thickness and the coordinate dimensions of the journal.

Any second-order linear or nonlinear ordinary or partial differential equation can be solved using a

3 Problem Solution and Results

3.1 Problem Solution

The main major assumptions that are used to derive the Reynolds equation resulted from realizing that the lubricant fluid film thickness is thin compared to the bearing radius so that we can assume the bearing is a Plane journal bearing i.e. bearing is two dimensions bearing that is pressure and film thickness only a simple function in the x-y plane . The main assumptions can be summarised as follows:

- The fluid is assumed to be Newtonian, where stress is directly proportional to the strain rate also the lubricant is assumed to be incompressible fluid.

- Inertia effects and body forces are assumed to be negligible compared to the viscous terms.
- pressure variation across the film lubricant thickness is assumed to be zero.
- Flow is laminar which means no turbulent effect in the boundary layer film of the lubricant.
- The curvature effect is taken across, implying that the thickness of the lubricant film is much smaller than the length or width of the bearing and allows the use of a polar coordinate system.

Different forms of one-dimensional Reynold equations for both Cartesian and cylindrical coordinates are available for different boundary conditions. Martin , Boyd , and others obtained an exact solution for plane minimum film thickness journal bearing according to simplified geometries . These solutions are not accurate for the cases when there is pressure gradient through the film thickness.

In 1949, Grubin obtained an approximate exact solution for one dimensional and plane journal bearing lubrication problem assuming a negligible pressure gradient through the bearing film thickness. The model is, therefore, accurate at high loads when the hydrodynamic pressure tends to be close to the Hertz contact pressure. The code below shows the methodology for solving the equation to investigate the pressure distribution and pressure gradient profiles (11).

Table 1. The main used parameters for the journal bearing

The radius of curvature [m]	Ambient Pressure [Pa]	Average sliding speed[m/s]
0.1680	10^5	0.33
Width or length[m]	Viscosity [Pa.s]	Cavitation pressure [Pa]
0.0112	0.007	-50P$_0$

Thus, when doing the main transformation of parameters of the Reynolds equation to polar coordinates we can get the following final form of the Reynolds equation as:

$$\left(\frac{(c_r+e.\cos(\theta))^3}{6\mu}\right)\left(\frac{\partial^2 P(\theta,x)}{\partial x^2}\right)-\left(\frac{3(c_r+e.\cos(\theta))^2 e.\sin(\theta)}{6\mu R^2}\right)\left(\frac{\partial P(\theta,x)}{\partial \theta}\right)+\left(\frac{(c_r+e.\cos(\theta))^3}{6\mu R^2}\right)\left(\frac{\partial^2 P(\theta,x)}{\partial \theta^2}\right)$$
$$=-e\omega\sin(\theta) \quad (5)$$

Any change in pressure relative to the depth is zero, because the pressure values were taken at the beginning and end of bearing in these two areas, therefore the value of the pressure gradient is zero:

$$\frac{\partial P}{\partial x}=0 \quad (6)$$

As for the change in pressure relative to the angle must find a relationship that combines these two variables can be combined in the following equation [5], [7] :

$$P(\theta)=\left(\frac{6\mu R^2}{c}\right)\left(\frac{n}{n+2}\right)\Omega\sin(\theta)\left[\left(\frac{1}{1+n}\right)\cos^2(\theta)+\left(\frac{1}{1+n}\right)\cos(\theta)\right] \quad (7)$$

Also , the pressure gradient can be found by differentiating equation 7 concerning angle Θ, the final form of pressure gradient can be written as :

$$\frac{\partial P}{\partial \theta}=\frac{a_1-a_2}{c} \quad (8)$$

Where:

$$a_1=18\mu\omega R^2\cos(\theta)(n\cos^2(\theta)+n\cos(\theta)+2)$$

$$a_2=18\mu\omega R^2\sin(\theta)(n\sin(\theta)+2n\cos(\theta)\sin(\theta))$$

Also, the second derivative of the pressure gradient can be written as:

$$\frac{\partial^2 P}{\partial \theta^2}=\frac{b_1+b_2+b_3}{c} \quad (9)$$

Where:

$$b_1=-36\mu\omega R^2\cos(\theta)\,(n\sin(\theta)+2n\cos(\theta)\sin(\theta))$$

$$b_2=-18\mu\omega R^2\sin(\theta)(n(\cos(\theta))^2+n\cos(\theta)+2)$$

$$b_3=-18\mu\omega\sin(\theta)(n\cos(\theta)-2n(\sin(\theta))^2+2n(\cos(\theta))^2)$$

Then the general form of the Reynolds equation can be written as :

$$\frac{d_1(a_1-a_2)}{c}+\frac{d_2(b_1+b_2+b_3)}{c}=-e\Omega\sin(\theta) \quad (10)$$

Where:

$$d_1=\frac{3(c+e^2.\cos^2(\theta))}{6\mu R^2}$$

$$d_2=\frac{(c+e.\cos(\theta))^3}{6\mu R^2}$$

3.2 Results

Using parameters from Table 1, the pressure function from equation 7 and the following parameters variables: μ=0.02756 Pa .s, R=19 mm, c=0.038, h_o=0.016 mm, e=0.022 mm, Ø=53° and $Ɔ_b$=zero, (all these values are constant).

The effects of rotational speed (ω) and pressure angle (θ) on pressure can be obtained as illustrated in Figure 3 and Figure 4 respectively. Figure 6 shows the relation between pressure and cleannce at constant rotational speed. Figure 8 shows the relation between between pressure and attitude angle at constant rotational speed. Figure 9 shows a three dimensional pressure distribution with 20 *20nodes. Figure 10 shows 3-D-Pressure distribution with 30*30 nodes . All other numerical results are shown in Figure 11, Figure 12, Figure 13, Figure 14, Figure 15, Figure 16 and Figure 17.

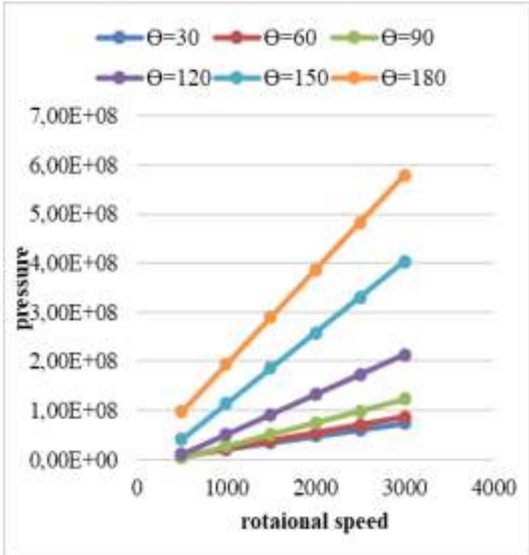

Fig. 3: pressure and rotational speed at a constant angle

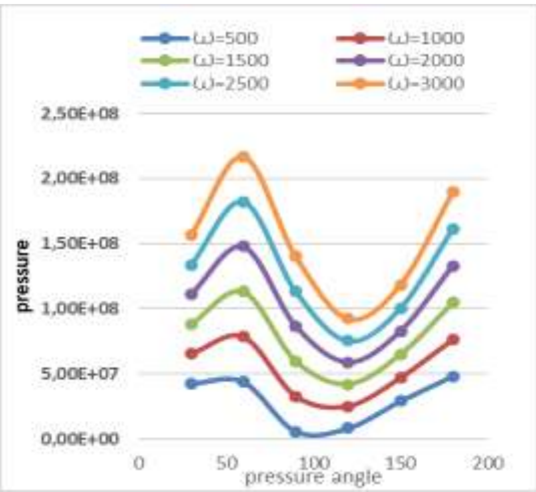

Fig. 4: pressures with the angle at constant rotational speed

The relation between the clearance and the pressure for different pressure angles is illustrated in Figure 5.

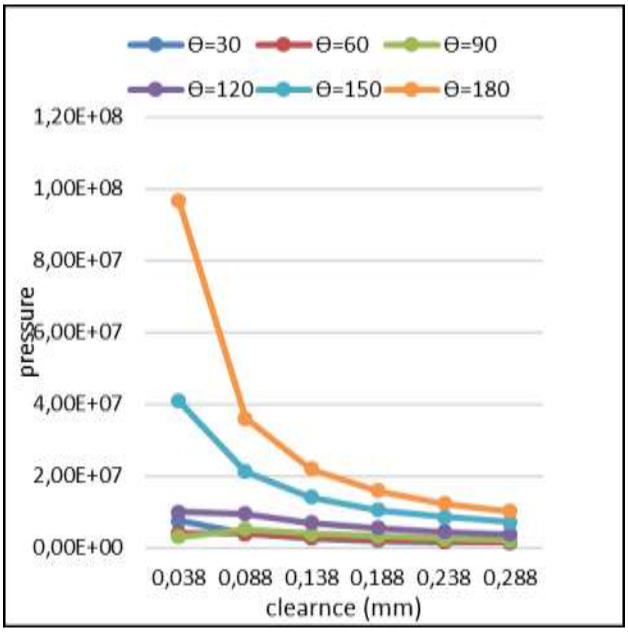

Fig. 5: pressure variation with journal clearance at different pressure angles

Also, the pressure variation with journal clearance constant rotational speeds can be shown as:

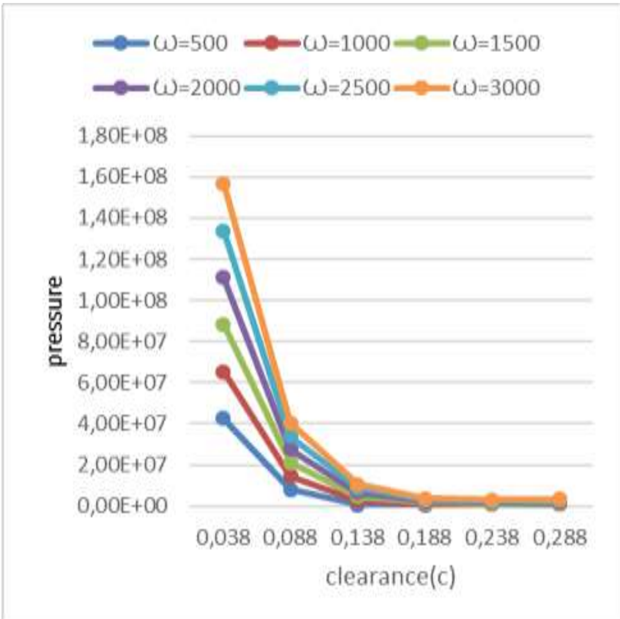

Fig. 6: The chart between pressure with clearance at a constant rotational speed

When changing the Attitude angle value with a change in rotational speed at a constant pressure angle we obtain a verity value from pressure:

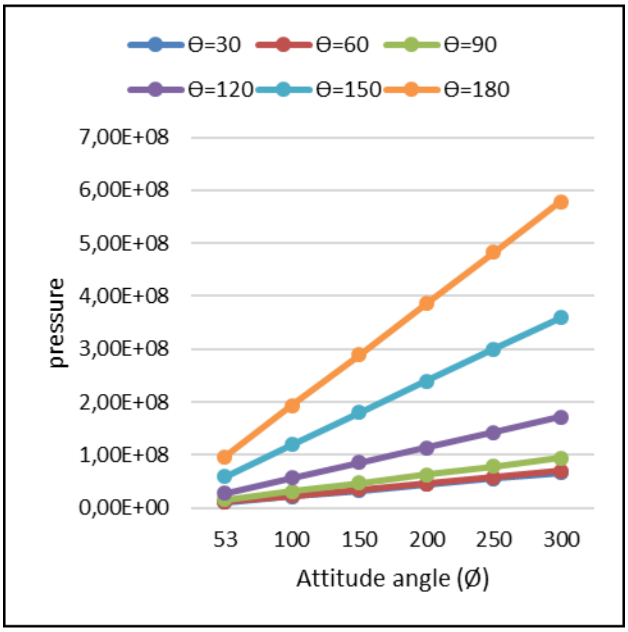

Fig. 7: The chart between pressure with Attitude angle at constant pressure angle

When changing the Attitude angle value with a change pressure angle at constant rotational speed we obtain verity value from pressure:

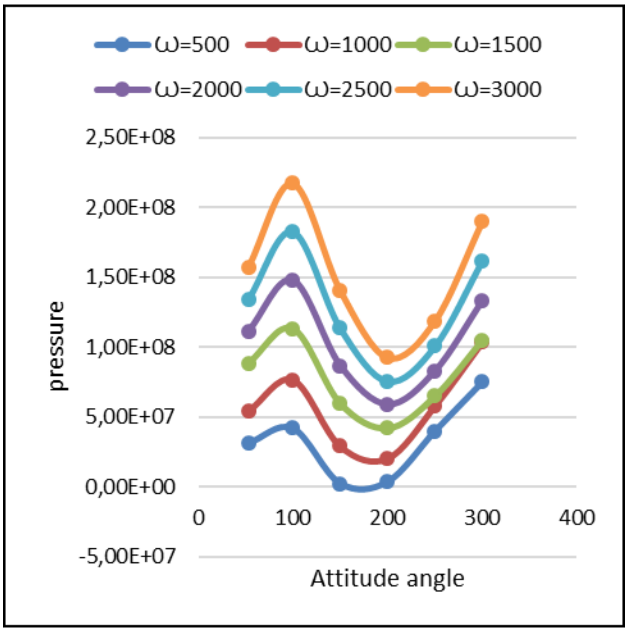

Fig. 8: Chart between pressure with attitude angle at constant rotational speed

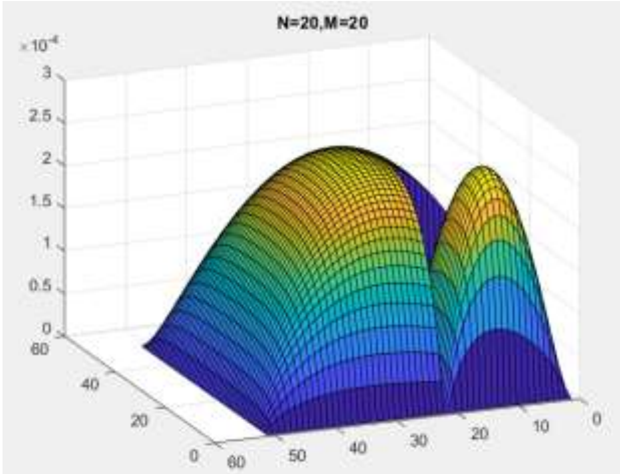

Fig. 9: 3-D -Pressure distribution with 20*20nodes

Several forms of generalized Reynolds equations were derived from weakening the assumptions used to derive the classical form. For example, compressible, non-Newtonian lubricant behavior can be considered. Reynolds equation is used to predict the thickness of the lubricant film, but also to predict friction developed by the lubricant on the surfaces. Since many tribological contacts operate in the highly loaded regime and thin films, the shear rates can be very high (in the order of 10^7-10^9). Many of the typical lubricants start to behave non-Newtonian in the contact conditions, and therefore, the Reynolds equation was generalized to the case of non-Newtonian lubricants.

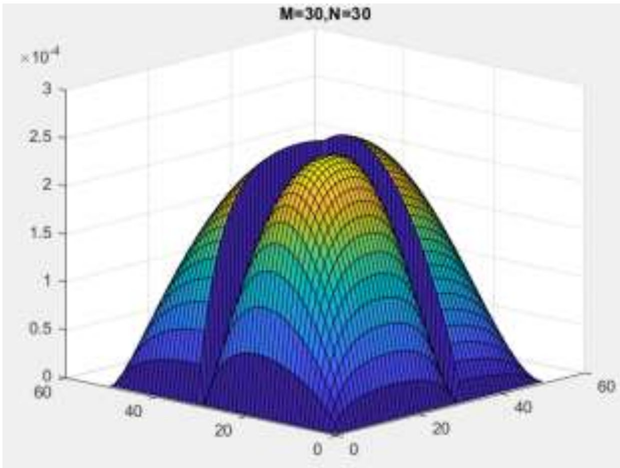

Fig. 10: 3-D-Pressure distribution with 30*30 nodes

Another generalization includes slip boundary conditions. This form of the Reynolds equation is used to calculate film thicknesses and friction in textured surfaces or surfaces with high slip.

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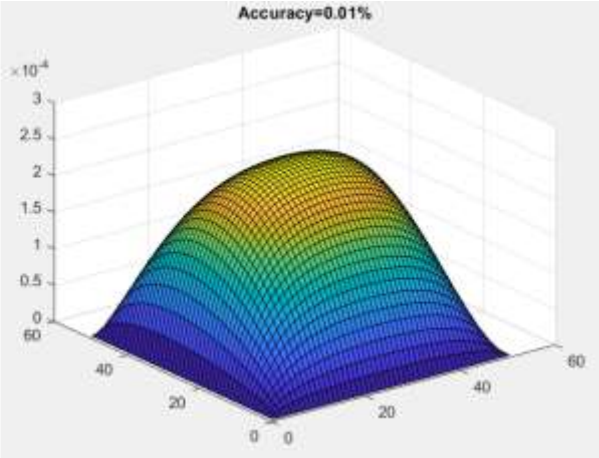

Fig. 11: 3-D-Pressure distribution with a tolerance of 0.01%

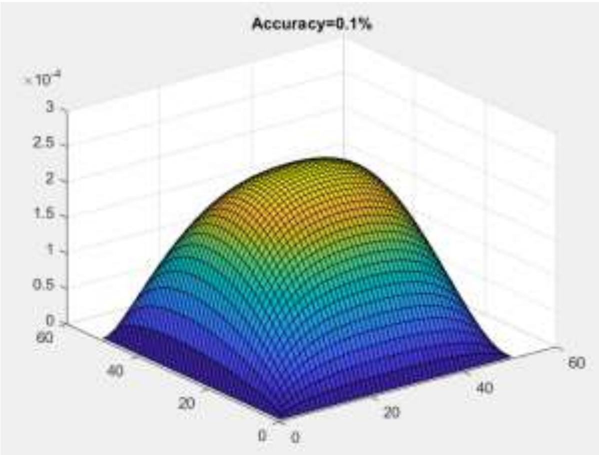

Fig. 12: 3-D -Pressure distribution, Accuracy of 0.1%

Here is simple a drawing journal bearing with a diameter of 15 mm, and dynamic viscosity of magnitude of [0.007 pa.s], and a radial clearance of 0.03mm as shown in Figure 13.

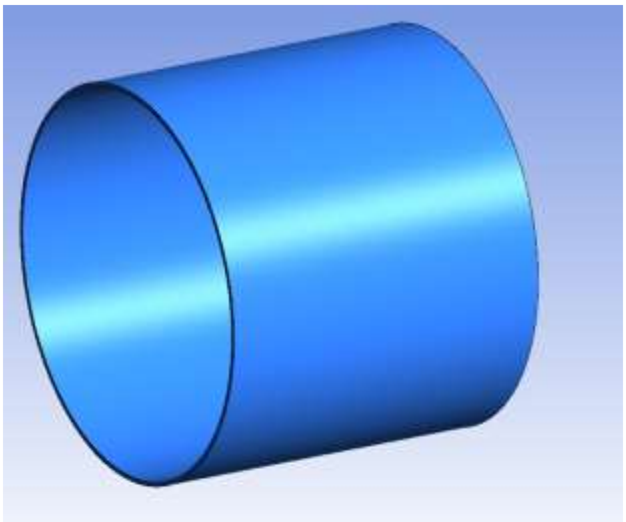

Fig. 13: Journal Bearing

By assuming the angular speed of the journal 1500rpm with constant oil lubricant temperature, the following parameters have been investigated as follows:

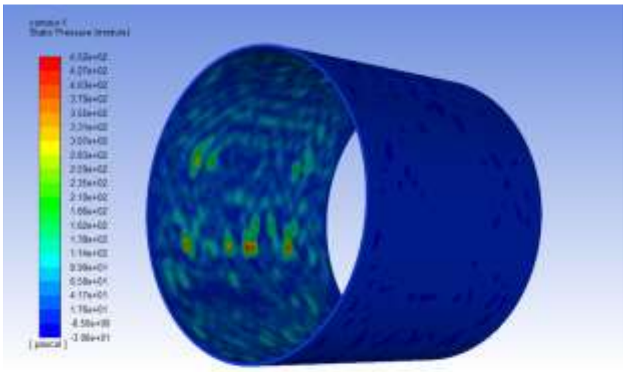

Fig. 14: static pressure distribution with maximum static pressure 452 Pascal

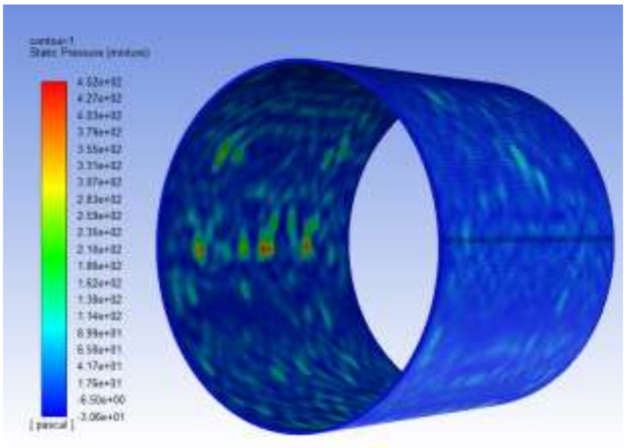

Fig. 15: The dynamic pressure distribution with 625 Pascal at the interior region of the bearing

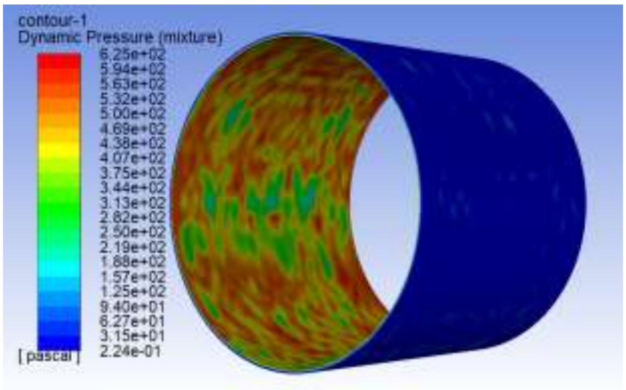

Fig. 16: Dynamic pressure

The total pressure is 745 as a maximum value and 1.97 pascals as the minimum value

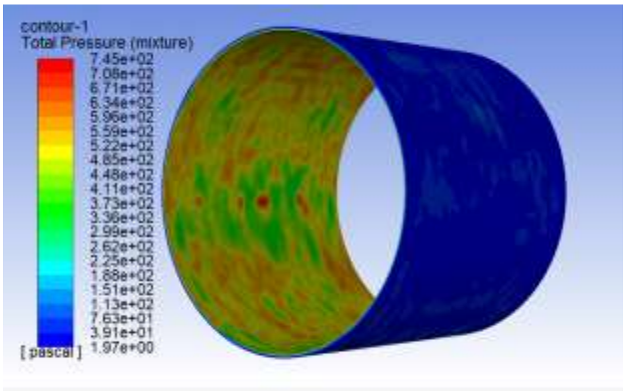

Fig. 17: Total pressure

4 Conclusion

Journal bearings are one of the most important applications of hydrodynamic lubricant devices, The Reynold equation has a primary role in investigating the pressure gradient and pressure distribution in the journal bearing. A simple boundary condition solution has been done for firstly 30 elements. The pressure distribution and gradient are studied and investigated. Also, the comparison between the transient and steady-state has been motioned.

After that general finite-difference solution was applied, and the pressure gradient profile plotted and has the most appropriate shape with 50*50 element mesh and 0.001%tolerance.

ANSYS Fluent has been used to investigate numerical solutions and to analysis to the journal-bearing performance characteristic parameters including , static pressure, dynamic pressure, velocity distribution, and turbulence effect, all these factors have been investigated.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- Muhammad Gogazeh: carried out the investigation simulation, literarure review, editing and the optimization.
- Hassan al dabass: resources and writing
- Nabil mousa: executed formulation and mathematical modelling.

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