Plane Strain Expansion of a Cylindrical Cavity in an Infinite Porous Rigid/Plastic Medium Obeying a General Yield Criterion

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Abstract: - A semi-analytical plane-strain solution for an expanding cylindrical cavity surrounded by an infinite porous rigid/plastic medium is presented. The constitutive equations are a general yield criterion and its associated flow rule. The yield criterion depends on the relative density and the linear and quadratic stress invariants. No restriction is imposed on this dependence, except for the standard requirements imposed on the yield criteria. The boundary value problem reduces to a Cauchy problem for three ordinary differential equations. This system of equations must be solved numerically. Numerical results are presented for Green's yield criterion. This yield criterion involves two functions of the relative density. The influence of the choice of these functions on the distributions of the relative density, the radial velocity, and the stress components is revealed.

Key-Words: - cylindrical cavity, rigid/plastic medium, porous material, general yield criterion, plane strain, Green's yield criterion.

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1 Introduction

Under certain conditions, the behavior of isotropic porous and powder ductile materials is successfully described by plasticity theory, assuming that the yield criterion depends on the relative density and stress invariants. The associated flow rule is usually used as the plastic flow rule. A comprehensive description of this theory is provided in [1]. In many cases, elastic strains can be neglected, leading to rigid/plastic models, [2], [3]. The present paper is restricted to such models.

The linear stress invariant is responsible for the plastic compressibility of materials. Therefore, this invariant must be involved in the yield criterion. The effect of the cubic stress invariant on the plastic behavior of porous and powder materials is often ignored. The corresponding yield criteria have been proposed in [4], [5], [6], among others. The present paper assumes an arbitrary dependence of the yield criterion on the linear and quadratic stress invariants satisfying the general standard requirements imposed on the yield criteria. The von Mises yield criterion is a particular case of this general criterion.

Analytical and semi-analytical solutions to non-stationary problems are rare in plasticity, even in the case of rigid perfectly plastic incompressible materials, [7]. For the class of models specified above, two solutions for instantaneous flow have been derived in [8], [9]. The evolution of the relative density has been considered in these solutions. Meanwhile, analytical and semi-analytical solutions that account for the relative density evolution have theoretical interest. Moreover, such solutions are important for verifying numerical codes, [10], [11].

Self-similar processes are an important class of processes for which analytical and semi-analytical solutions can be found. The most known solution of this class is for a spherical cavity expanding in an infinite medium from a zero radius. An elastic perfectly plastic solution has been provided in [12]. Several papers have been devoted to dynamic spherical cavity expansion in various elastic plastic media. A review of these solutions can be found in [13]. The process of cylindrical cavity expansion has attracted less attention. An elastic perfectly plastic solution has been provided in [12]. The effect of inertia has been taken into account in [14], assuming that strains are infinitesimal. A solution for the quasi-static expansion of a cylindrical cavity in hypoelastic compressible Mises and Tresca solids at large strains has been derived in [15]. In contrast to the solutions above, the present paper considers a rigid plastic model. The material model provided in [1], is employed. Neglecting elastic strains changes the boundary value problem significantly. In particular, some equations contain the expression 0/0, which requires some analytical treatment of these equations before using a numerical method. The numerical solution is based on Green's yield criterion, [4]. This yield criterion involves two functions of the relative density. A review of these functions is provided in [16]. The functions proposed in [1] and [4], are adopted in the numerical solution. The effect of the choice of these functions on the distributions of the relative density, the radial velocity, and the stress components is revealed.

2 Statement of the Problem

A cylindrical cavity of a zero initial radius expands in an infinite porous rigid/plastic medium under plane strain conditions. The relative density is uniformly distributed at the initial instant and equals ρ_0. The flow theory of plasticity is used. A comprehensive description of this theory has been provided in [4].

The constitutive equations constitute a yield criterion and its associated flow rule. The present paper is restricted to the yield criteria independent of the third invariant of the stress tensor. Therefore, the yield criterion can be represented as:

$$\Phi(\sigma, \tau, \rho) = 0. \tag{1}$$

Here ρ is the relative density, σ is the first stress invariant, τ is the second stress invariant, and Φ is an arbitrary function of its arguments satisfying the standard requirements imposed on the yield criteria. The stress invariants are expressed in terms of the principal stresses σ_1, σ_2 and σ_3 as:

$$\sigma = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad \text{and}$$

$$\tau = \frac{1}{2}\left[(\sigma_1 - \sigma)^2 + (\sigma_2 - \sigma)^2 + (\sigma_3 - \sigma)^2\right]. \tag{2}$$

The plastic flow rule associated with (1) is:

$$\xi_1 = 3\lambda \frac{\partial \Phi}{\partial \sigma_1}, \quad \xi_2 = 3\lambda \frac{\partial \Phi}{\partial \sigma_2}, \quad \text{and } \xi_3 = 3\lambda \frac{\partial \Phi}{\partial \sigma_3}, \tag{3}$$

where ξ_1, ξ_2 and ξ_3 are the principal strain rates and $\lambda \geq 0$. The stress and strain rate tensors are coaxial. This condition is automatically satisfied in the problem under consideration. Substituting (1) into (3) and employing (2) yields:

$$\begin{aligned} \xi_1 &= \lambda\left[\Phi,_\sigma + (2\sigma_1 - \sigma_2 - \sigma_3)\Phi,_\tau\right], \\ \xi_2 &= \lambda\left[\Phi,_\sigma + (2\sigma_2 - \sigma_3 - \sigma_1)\Phi,_\tau\right], \\ \xi_3 &= \lambda\left[\Phi,_\sigma + (2\sigma_3 - \sigma_1 - \sigma_2)\Phi,_\tau\right], \end{aligned} \tag{4}$$

where $\Phi,_\sigma = \partial\Phi/\partial\sigma$ and $\Phi,_\tau = \partial\Phi/\partial\tau$.

Using a cylindrical coordinate system (r, θ, z) whose z-axis coincides with the symmetry axis is natural. The normal stresses in this coordinate system are denoted as σ_r, σ_θ, and σ_z. These stresses are the principal stresses. Similarly, the normal strain rates are denoted as ξ_r, ξ_θ, and ξ_z. These strain rates are the principal strain rates. One may choose $\sigma_r = \sigma_1$, $\sigma_\theta = \sigma_2$, and $\sigma_z = \sigma_3$. Consequently, $\xi_r = \xi_1$, $\xi_\theta = \xi_2$, and $\xi_z = \xi_3$. The radial velocity is denoted as u. The other velocity components vanish. Plane strain conditions demand:

$$\xi_z = 0. \tag{5}$$

The other principal strain rates are expressed through the radial velocity as:

$$\xi_r = \frac{\partial u}{\partial r} \quad \text{and} \quad \xi_\theta = \frac{u}{r}. \tag{6}$$

The third equation in (4) and (5) combine to give:

$$\Phi,_\sigma + (2\sigma_z - \sigma_r - \sigma_\theta)\Phi,_\tau = 0. \tag{7}$$

Eliminating λ between the first and second equations in (4) gives:

$$\xi_r = \xi_\theta \frac{\left[\Phi,_\sigma + (2\sigma_r - \sigma_\theta - \sigma_z)\Phi,_\tau\right]}{\left[\Phi,_\sigma + (2\sigma_\theta - \sigma_z - \sigma_r)\Phi,_\tau\right]}. \tag{8}$$

Equations (6) and (8) combine to give:

$$\frac{\partial u}{\partial r} = \frac{u}{r} \frac{\left[\Phi,_\sigma + (2\sigma_r - \sigma_\theta - \sigma_z)\Phi,_\tau\right]}{\left[\Phi,_\sigma + (2\sigma_\theta - \sigma_z - \sigma_r)\Phi,_\tau\right]}. \tag{9}$$

The only equilibrium equation that is not identically satisfied is:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{10}$$

In the case under consideration, the equation of mass conservation is:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) = 0, \tag{11}$$

where t is the time. One can eliminate the derivative $\partial u/\partial r$ in (11) using (9) to arrive at:

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\frac{3\rho u \Phi,_{\sigma}}{r\left[\Phi,_{\sigma}+\left(2\sigma_{\theta}-\sigma_{z}-\sigma_{r}\right)\Phi,_{\tau}\right]}=0. \tag{12}$$

Equations (1), (9), (10), and (12) constitute a system for determining σ_r, σ_θ, u, and ρ.

3 General Solution

A plastic region propagates from the cavity. The current radius of the rigid/plastic boundary is denoted as a. The material is rigid in the region $r \geq a$. remainder is rigid. In this region, it is only necessary to show that a stress field satisfying the equilibrium equations and not violating the yield criterion exists. The radial stress, the radial velocity, and the relative density must be continuous across the rigid/plastic boundary. The current radius of the rigid/plastic boundary can be regarded as a time-like parameter. Moreover, since the material model is rate-independent, one may put:

$$da/dt=1 \tag{13}$$

without loss of generality. Then, u becomes dimensionless. Like all similar problems (for example [4]), the solution depends on the ratio $\mu = r/a$ rather than r and a separately. Taking into account (13), one can express the derivatives with respect to r and t in terms of the derivative with respect to μ as:

$$\frac{\partial}{\partial r}=\frac{1}{a}\frac{d}{d\mu} \quad \text{and} \quad \frac{\partial}{\partial t}=\frac{\partial}{\partial a}=-\frac{r}{a^2}\frac{d}{d\mu}=-\frac{\mu}{a}\frac{d}{d\mu}. \tag{14}$$

Then, equations (9), (10), and (12) become:

$$\mu\frac{du}{d\mu}=u\frac{\left[\Phi,_{\sigma}+\left(2\sigma_{r}-\sigma_{\theta}-\sigma_{z}\right)\Phi,_{\tau}\right]}{\left[\Phi,_{\sigma}+\left(2\sigma_{\theta}-\sigma_{z}-\sigma_{r}\right)\Phi,_{\tau}\right]}, \tag{15}$$

$$\mu\frac{d\sigma_r}{d\mu}+\sigma_r-\sigma_\theta=0, \tag{16}$$

and

$$\mu\left(u-\mu\right)\frac{d\rho}{d\mu}+\frac{3\rho u \Phi,_{\sigma}}{\Phi,_{\sigma}+\left(2\sigma_{\theta}-\sigma_{z}-\sigma_{r}\right)\Phi,_{\tau}}=0, \tag{17}$$

respectively.

Since the radial velocity and relative density must be continuous across the rigid/plastic boundary,

$$u=0 \quad \text{and} \quad \rho=\rho_0 \tag{18}$$

for $\mu=1$. It is then seen from equations (15) and (17) that physically reasonable solutions are possible only if:

$$\Phi,_{\sigma}+\left(2\sigma_{\theta}-\sigma_{z}-\sigma_{r}\right)\Phi,_{\tau}=0 \tag{19}$$

for $\mu=1$. Equations (1), (7) and (19) allows all the principal stresses to be found at the rigid/plastic boundary. This boundary condition and (18) lead to a Cauchy problem for equations (15) to (17). This problem must be solved numerically.

Let R be the current radius of the cavity. At the initial instant, the mass of the material contained in the unit length of the cylinder of radius a is determined as:

$$M=\pi a^2 \rho_0. \tag{20}$$

After any amount of deformation, this mass can be calculated as:

$$M=2\pi\int_R^a \rho r dr=2\pi a^2\int_{R/a}^{1}\rho\mu d\mu. \tag{21}$$

Equations (20) and (21) combine to give:

$$2\int_{R/a}^{1}\rho\mu d\mu=\rho_0. \tag{22}$$

A numerical solution of equations (15) to (17) provides the integrand as a function of μ. Therefore, equation (22) determines R/a.

4 Green's Yield Criterion

The yield criterion proposed in [4], can be represented as

$$\frac{\sigma^2}{p_s^2}+\frac{\tau}{\tau_s^2}-1=0, \tag{23}$$

where p_s and τ_s are functions of the relative density. An ellipse represents this yield criterion in a two-dimensional space where the linear and quadratic stress invariants are taken as Cartesian coordinates. The length of its major and minor axes depends on the relative density. The length of the major axis approaches infinity as the relative density approaches unity. In this case, Green's yield criterion approaches Mises' yield criterion. It follows from (2) that:

$$\Phi,_{\sigma}=\frac{2\sigma}{p_s^2} \quad \text{and} \quad \Phi,_{\tau}=\frac{1}{\tau_s^2}. \tag{24}$$

Substituting (24) into (7), one gets:

$$\sigma_z = \gamma\left(\sigma_r + \sigma_\theta\right), \quad (25)$$

where

$$\gamma = \frac{2-\beta^2}{4+\beta^2} \quad \text{and} \quad \beta = \frac{2\tau_s}{\sqrt{3}p_s}. \quad (26)$$

Substituting (25) into (2) yields:

$$\sigma = \frac{(1+\gamma)(\sigma_r+\sigma_\theta)}{3} \quad \text{and}$$

$$\tau = \frac{\left[1+\gamma(\gamma-1)\right]\left(\sigma_r^2+\sigma_\theta^2\right)+\left[2\gamma(\gamma-1)-1\right]\sigma_r\sigma_\theta}{3}. \quad (27)$$

Equations (23) and (27) combine to give:

$$A\left(\sigma_r^2+\sigma_\theta^2\right)+2B\sigma_r\sigma_\theta = 1, \quad (28)$$

where

$$A = \frac{4\left[1+\gamma(\gamma-1)\right]+\beta^2(1+\gamma)^2}{12\tau_s^2} \quad \text{and}$$

$$B = \frac{2\left[2\gamma(\gamma-1)-1\right]+\beta^2(1+\gamma)^2}{12\tau_s^2}. \quad (29)$$

Equation (28) is satisfied by the following substitution:

$$\sigma_r = a\sin\varphi \quad \text{and} \quad \sigma_\theta = b\sin\varphi + c\cos\varphi, \quad (30)$$

where

$$a = \frac{1}{\sqrt{A}\sqrt{1-\alpha^2}}, \quad b = -\frac{\alpha}{\sqrt{A}\sqrt{1-\alpha^2}}, \quad c = \frac{1}{\sqrt{A}},$$

and $\alpha = \dfrac{\beta^2-2}{2\left(\beta^2+1\right)}$. (31)

Substituting (24), (25), and (30) into (19) results in

$$\frac{\sqrt{1+\beta^2}}{\tau_s^2\sqrt{4+\beta^2}}\cos\varphi = 0 \quad (32)$$

at $\mu = 1$. Since $\sigma_r < 0$, it follows from the first equation in (30) and (32) that:

$$\varphi = -\frac{\pi}{2} \quad (33)$$

for $\mu = 1$.

Employing (24), (26), (27), and (30), one transforms equations (15) to (17) to:

$$\mu\frac{du}{d\mu} - \frac{u\left[\left(\beta^2-2\right)\cos\varphi+\sqrt{3}\beta\sqrt{4+\beta^2}\sin\varphi\right]}{2\left(1+\beta^2\right)\cos\varphi} = 0, \quad (34)$$

$$\mu\left(\frac{d\rho}{d\mu}\frac{da}{d\rho}\sin\varphi + a\cos\varphi\frac{d\varphi}{d\mu}\right) + \frac{\sqrt{3}\beta\sqrt{4+\beta^2}\sin\varphi-\left(4+\beta^2\right)\cos\varphi}{\sqrt{4+\beta^2}\sqrt{1+\beta^2}} = 0, \quad (35)$$

and

$$\mu(u-\mu)\frac{d\rho}{d\mu} + \frac{\sqrt{3}\rho u\beta\left[\sqrt{4+\beta^2}\sin\varphi+\sqrt{3}\beta\cos\varphi\right]}{2\left(1+\beta^2\right)\cos\varphi} = 0. \quad (36)$$

It is advantageous to introduce the following variable:

$$\eta = \ln\mu. \quad (37)$$

Then, equations (34) and (36) become:

$$\frac{du}{d\eta} - \frac{u\left[\left(\beta^2-2\right)+\sqrt{3}\beta\sqrt{4+\beta^2}\tan\varphi\right]}{2\left(1+\beta^2\right)} = 0 \quad (38)$$

and

$$\frac{d\rho}{d\eta} = \frac{\sqrt{3}\rho u\beta\left[\sqrt{4+\beta^2}\tan\varphi+\sqrt{3}\beta\right]}{2\left(1+\beta^2\right)\left(e^\eta-u\right)}. \quad (39)$$

The derivative $d\rho/d\mu$ in (35) can be eliminated by employing (36). Then, upon using (37), equation (35) becomes:

$$a\frac{d\varphi}{d\eta} + \frac{\sqrt{3}\rho u\beta\left[\sqrt{4+\beta^2}\tan\varphi+\sqrt{3}\beta\right]}{2\left(1+\beta^2\right)\left(e^\eta-u\right)}\frac{da}{d\rho}\tan\varphi + \frac{\sqrt{3}\beta\tan\varphi-\sqrt{4+\beta^2}}{\sqrt{1+\beta^2}} = 0. \quad (40)$$

The boundary conditions to equations (38) to (40) follow from (18), (33), and (37) in the form:

$$u = 0, \quad \rho = \rho_0, \quad \text{and} \quad \varphi = -\frac{\pi}{2} \quad (41)$$

for $\eta = 0$.

Some terms in equations (38) to (40) reduce to the expression $0/0$ at $\eta = 0$. Therefore, the solution's behavior near this point must be investigated before using a numerical method. A Taylor series is assumed to represent each unknown function in the neighborhood of $\eta = 0$. Then, using (41),

$$u = U_1\eta + o(\eta), \quad \rho = \rho_0 + \rho_1\eta + o(\eta),$$

and

$$\varphi = -\frac{\pi}{2} + \varphi_1\eta + o(\eta), \quad (42)$$

as $\eta \to 0$. Substituting (42) into equations (38) to (40) leads to:

$$U_1 = -\frac{\sqrt{3}\beta}{\sqrt{1+\beta^2}\rho_0\, da/d\rho}, \quad \rho_1 = \rho_0 U_1,$$

and

$$\varphi_1 = -\frac{\sqrt{3}\beta\sqrt{4+\beta^2}}{2\left(1+\beta^2\right)}. \tag{43}$$

It is understood here that β and $da/d\rho$ are calculated at $\rho = \rho_0$.

5 Numerical Solution

Equations (38) to (40) have been solved numerically using (42) and (43). The solution has been calculated for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1] and [4]. In particular,

$$p_s(\rho) = \frac{2k}{\sqrt{3}}\frac{\rho^2}{\sqrt{1-\rho}} \quad \text{and} \quad \tau_s(\rho) = k\rho^{3/2} \tag{44}$$

and

$$p_s(\rho) = -2\sqrt{3}k\ln(1-\rho), \ \tau_s(\rho) = \frac{3k\left(1-\sqrt[3]{1-\rho}\right)}{3-2\sqrt[4]{1-\rho}}, \tag{45}$$

respectively. In these equations, k is the shear yield stress of the pore-free material. Equations (26), (29), (31), (44), and (45) allow the coefficients involved in equations (38) to (40) to be expressed in terms of the relative density. Similarly, the coefficients involved in (42) can be expressed in terms of the initial relative density.

Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5 illustrate the solution for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]. The variation of the relative density with μ is depicted in Figure 1. The rightmost points correspond to the rigid plastic boundary where $\rho = \rho_0$. The leftmost points correspond to the cavity surface. It can be verified by substituting the solution for ρ into (22).

In particular, the dependence of R/a on ρ_0 found from this equation is presented in Table 1. Interestingly, the relative density equals one at the cavity surface. Figure 2 shows the variation of the dimensionless radial velocity with μ. The rightmost points correspond to the rigid plastic boundary where $u = 0$. Figure 3, Figure 4 and Figure 5 illustrate the distributions of the principal stresses in the plastic region. The radial stress must be continuous across the rigid plastic boundary. Therefore, the values of this stress at the rightmost points in Figure 3 must be used for extending the stress field into the rigid region.

Figure 6, Figure 7, Figure 8, Figure 9 and Figure 10 illustrate the solution for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]. Qualitatively, the solution behavior is similar to that for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]. However, the solutions significantly differ quantitatively, emphasizing a need for accurate representations of the functions $p_s(\rho)$ and $\tau_s(\rho)$ for practical applications.

In particular, Table 2 presents the dependence of R/a on ρ_0.

Figure 11, Figure 12, Figure 13, Figure 14 and Figure 15 show the influence of the choice of the functions of the distributions of the relative density, the dimensionless radial velocity, and the principal stresses at $\rho_0 = 0.6$.

Table 1. Dependence of R/a on ρ_0 for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]

ρ_0	0.4	0.5	0.6	0.7	0.8
R/a	0.559	0.488	0.416	0.34	0.255

Table 2. Dependence of R/a on ρ_0 for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

ρ_0	0.4	0.5	0.6	0.7	0.8
R/a	0.34	0.291	0.246	0.201	0.159

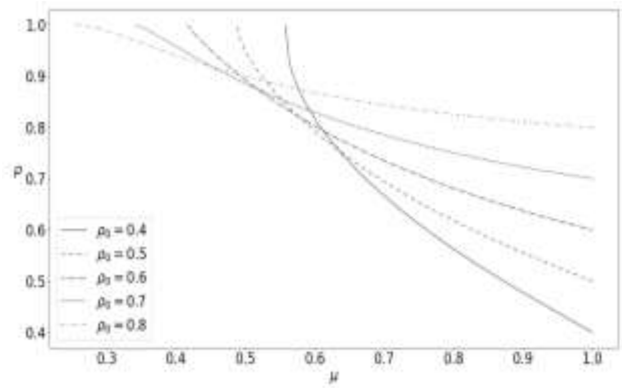

Fig. 1: Variation of the relative density with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]

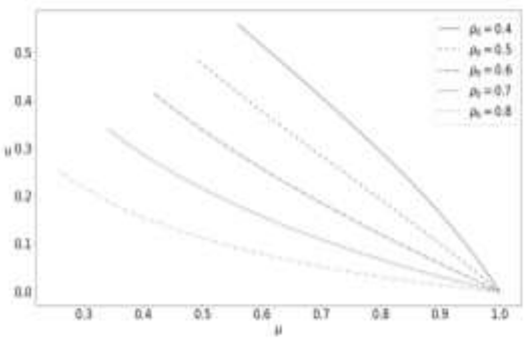

Fig. 2: Variation of the dimensionless radial velocity with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]

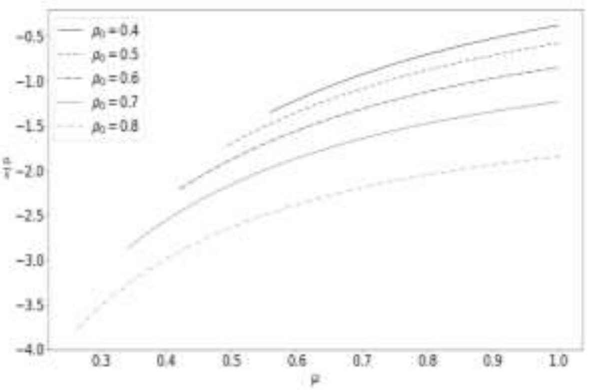

Fig. 3: Variation of the radial stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]

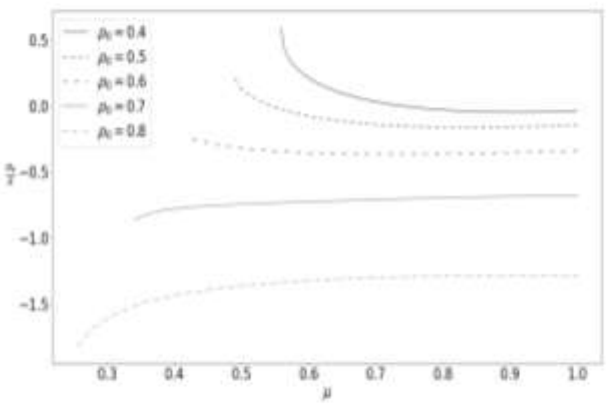

Fig. 4: Variation of the circumferential stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [1]

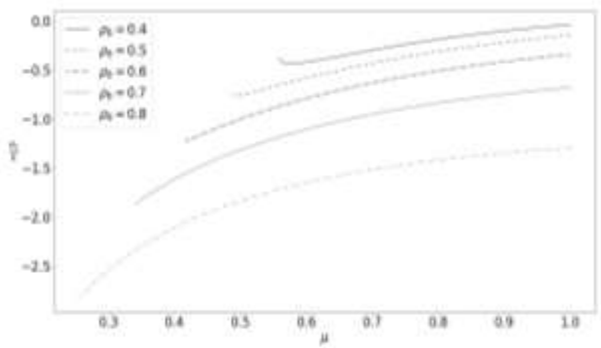

Fig. 5: Variation of the axial stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

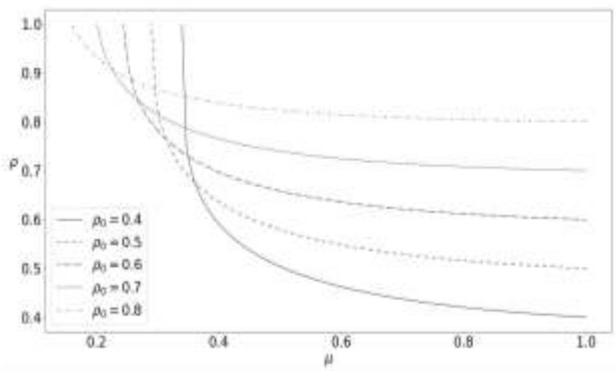

Fig. 6: Variation of the relative density with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

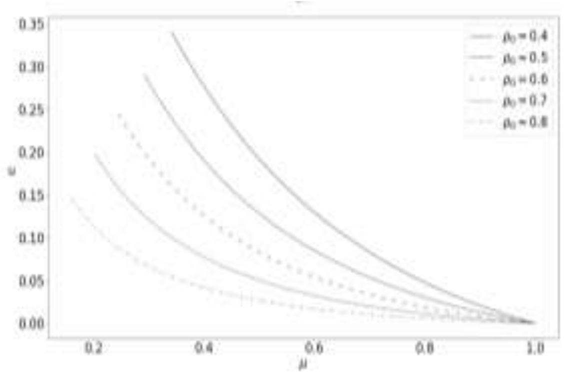

Fig. 7: Variation of the dimensionless radial velocity with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

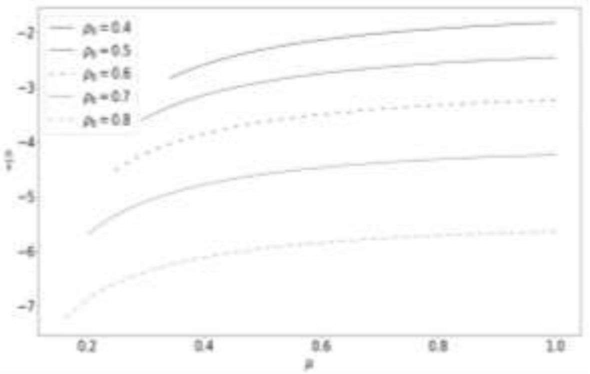

Fig. 8: Variation of the radial stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

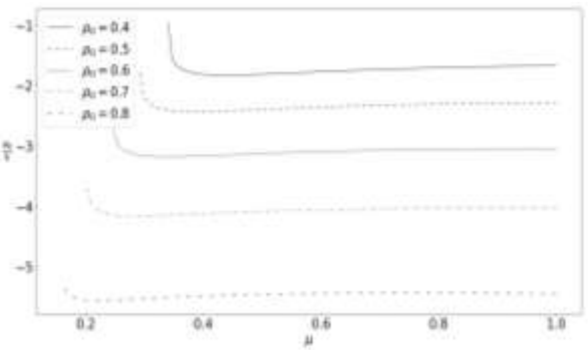

Fig. 9: Variation of the circumferential stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

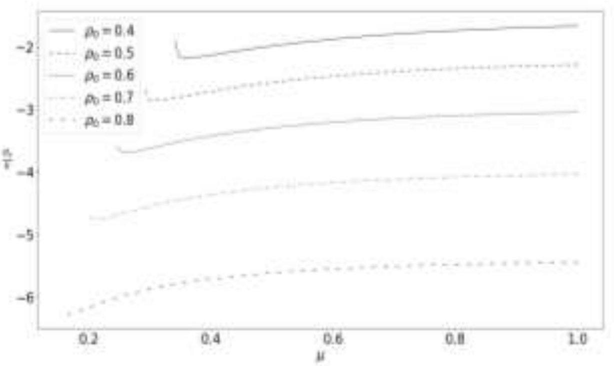

Fig. 10: Variation of the axial stress with m for the functions $p_s(\rho)$ and $\tau_s(\rho)$ proposed in [4]

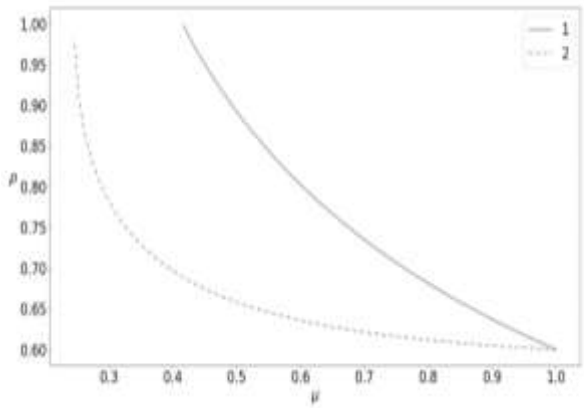

Fig. 11: Influence of the functions $p_s(\rho)$ and $\tau_s(\rho)$ on the distribution of the relative density (Curve 1 corresponds to the functions proposed in [1] and Curve 2 to the functions proposed in [4])

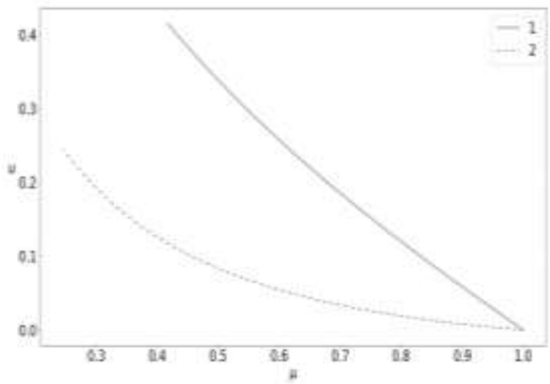

Fig. 12: Influence of the functions $p_s(\rho)$ and $\tau_s(\rho)$ on the distribution of the radial velocity (Curve 1 corresponds to the functions proposed in [1] and Curve 2 to the functions proposed in [4])

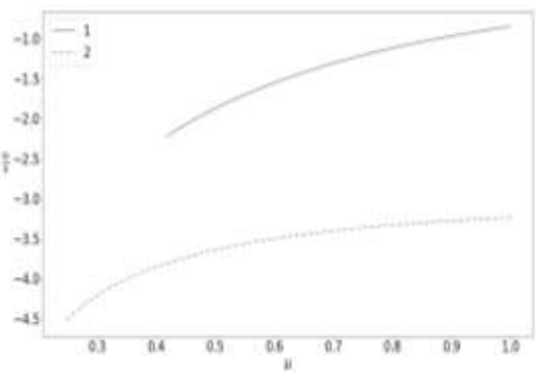

Fig. 13: Influence of the functions $p_s(\rho)$ and $\tau_s(\rho)$ on the distribution of the radial stress (Curve 1 corresponds to the functions proposed

in [1] and Curve 2 to the functions proposed in [4])

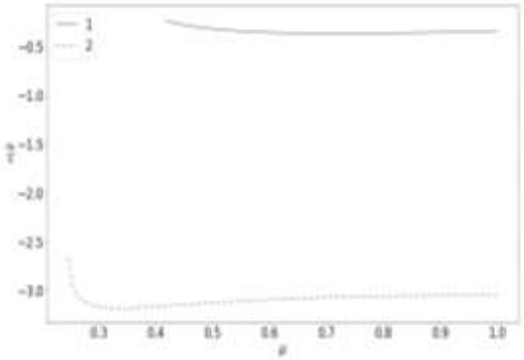

Fig. 14: Influence of the functions $p_s(\rho)$ and $\tau_s(\rho)$ on the distribution of the circumferential stress (Curve 1 corresponds to the functions proposed in [1] and Curve 2 to the functions proposed in [4])

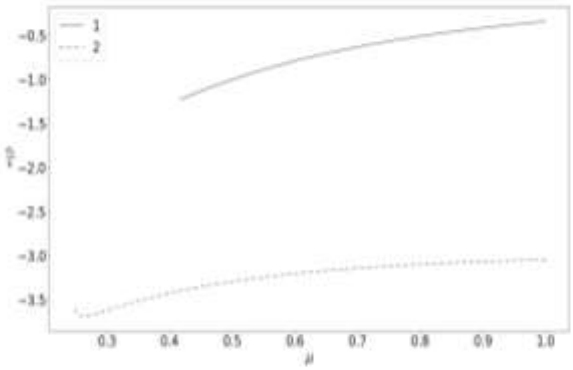

Fig. 15: Influence of the functions $p_s(\rho)$ and $\tau_s(\rho)$ on the distribution of the axial stress (Curve 1 corresponds to the functions proposed in [1] and Curve 2 to the functions proposed in [4])

6 Conclusion

The quasi-static expansion of a cylindrical cavity of a zero initial radius in an infinite porous rigid/plastic medium has been investigated under plane strain assumptions. The problem has been reduced three ordinary differential equations. Some of these equations contain expressions 0/0 at the rigid/plastic boundary. Therefore, an asymptotic analysis of the equations has been carried out before using a numerical method. The numerical solution has been provided for Green's yield criterion, giving the distributions of the relative density, the radial velocity, and the principal stresses in the plastic region. The relative density approaches unity at the cavity's surface. Green's yield criterion involves two functions of the relative density. The choice of these functions significantly affects the solution's behavior.

The solution can be adopted to describe the expansion of a non-zero initial radius cavity using the approach proposed in [12]. The resulting solution may be used to verify the accuracy of numerical solutions, which is a necessary step for using such solutions for practical applications, [10], [11].

The subject of a subsequent investigation is to consider strain hardening and to reveal the effect of neglecting elasticity on the solution.

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Conflict of Interest

The authors have no conflicts of interest to declare.