Optimization of a Two-stage Helical Gearbox with Second Stage Double Gear Sets to Reduce Gearbox Mass and Increase Gearbox Efficiency

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Abstract: - This work aims to identify the key design components for lowering gearbox mass and raising gearbox efficiency by multi-target optimizing a two-stage helical gearbox (THG) with second-stage double gear sets (SSDGS). In the study, the Taguchi technique and grey relation analysis (GRA) were applied to handle optimization work in two phases. The single-target problem was addressed first to narrow the separation between variable levels, and then multi-target work was addressed to establish the best primary design variables. The first and second-stage CWFW coefficients, as well as the allowed contact stresses (ACS) and the gear ratio of the first stage, were computed. The outcomes of the study can be applied to determine the best values for the main essential design factors of a THG with SSDGS.

Key-Words: - Helical gearbox, Double gear sets, Optimization, Multi-objective, Gear ratio, Gearbox mass, Gearbox efficiency.

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1 Introduction

A gearbox, a chain driver, and a motor can all be found in a mechanical drive system (Figure 1). The primary component of the system is a gearbox. Its purpose is to lessen the torque and speed transfer from the motor shaft to the working shaft. As a result, several studies are concentrating on the best gearbox design.

To date, the optimal gearbox design has been done on a range of issues. To lower the gearbox cost, [1], studies the impact of eleven input parameters on second and third-stage ratios. The study, [2], outlines a study that optimized the gear ratios in a system including a gearbox and a chain drive. The authors in, [3], conducted a simulation study to design a two-stage gearbox for getting minimal gearbox volume. The optimal gear ratios for designing a helical gearbox were proposed in, [4], to get the minimum area of the cross-section of the gearbox. For this type of gearboxes, a design with the used of hybrid composite gears was suggested in, [5]. The study, [6], proposed two methods for determining optimal gear ratios to minimize helical reducer cross-sectional area in the same region of concern. Besides, a new design of gearbox housing was introduced in, [7]. Apart from that, [8], conducted a simulation experiment to study the link between partial gear ratios and input factors, from which models to find the optimum gear ratio were established. For a three-stage helical gearbox with SSDGS, [9], proposed models to determine ideal partial ratios to minimize the length, the mass, and the cross-section area of the gearbox. Recognizing the significance of gearbox cost reduction in both design and construction, [10], computed cost for helical reducers with SSDGS utilizing component mass. Furthermore, optimal gear ratios of a two-step helical reducer with SSDGS to reduce the system length were introduced in, [11]. Also, for the same type of gearboxes, [12], presented the gear ratio model to minimize the gearbox cross-section area. A variety of multi-target optimization studies for helical gearboxes, [13], and bevel helical gearboxes, [14], have recently been published.

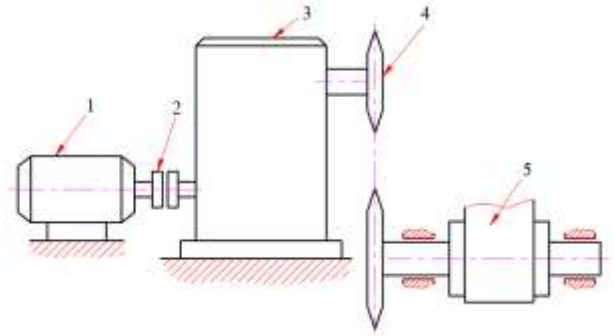

Fig. 1: A belt conveyor drive system, [15].
Note: 1) Motor 2) Coupling 3) Gearbox 4) Chain drive 5) Belt conveyor

Scientists are interested in the ideal design of a two-step reducer with SSDGS. In, [16], the findings of optimal partial gear ratios in a two-stage reducer with SSDGS for obtaining the smallest cross-section area are presented. The study took into account the impact of the input variables such as the overall ratio, the CWFW, the ACF, and the output torque. Also for this gearbox, the problem of finding optimal ratios was conducted in, [15], for minimum gearbox cost. The impact of the key design characteristics and cost components on the ideal ratios was explored in this work. A model for calculating the ideal gear ratio has also been provided. In, [17], optimal gear ratios were determined for a drive system with a helical reducer using SSDGS and a chain drive to find the minimum system length.

According to the previous study, a lot of studies have been conducted on optimizing gearbox design in general, and THG with SSDGS in particular, up to this point. The research on a THG with SSDGS, on the other hand, only has a single-target optimization problem.

The goal of this research is to explore multi-target optimization learning for a THG utilizing SSDGS. In this effort, two single goals were pursued: reducing gearbox mass and increasing gearbox efficiency. In addition, the CWFW and the ACS of stages 1 and 2, and the gear ratio for stage 1 were also examined. Furthermore, the multi-target work in this study was solved in two stages by using the Taguchi approach with GRA. The optimum key design parameters were also provided to design reducers with SSDGS.

2 Optimization Problem

2.1 Gearbox Mass Determination

The gearbox mas m_{gb} is determined by (Figure 1):

$$m_{gb} = m_g + m_{gh} + m_s \tag{1}$$

In (1), m_g, m_{gh}, and m_s are the mass of gears, gearbox housing, and shafts. The parts that follow specify these mass elements.

2.1.1 Gear Mass Determination

The mass of gears is determined by:

$$m_g = m_{g1} + 2 \cdot m_{g2} \tag{2}$$

In which, m_{g1} and m_{g2} denote the mass of gears of stages 1 and 2:

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 . d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 . d_{w21}^2 \cdot b_{w1}}{4}\right) \tag{3}$$

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 . d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 . d_{w22}^2 \cdot b_{w2}}{4}\right) \tag{4}$$

In (3) and (4), ρ_g is gear density (kg/m^3); e_1 and e_2 are volume coefficients of pinion and gear; e_1=1 and e_2 =0.6, [1], b_{w1} and b_{w2} are the gear widths of stage 1 and stage 2; d_{w11}, d_{w12}, d_{w21}, and d_{w22} are the pitch diameters of the pinion and the gear sets which are calculated by, [4]:

$$b_{w1} = X_{ba1} \cdot a_{w1} \tag{5}$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \tag{6}$$

$$d_{w11} = 2 \cdot a_{w1}/(u_1 + 1) \quad (7)$$

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \quad (8)$$

$$d_{w12} = 2 \cdot a_{w2}/(u_2 + 1) \quad (9)$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2/(u_2 + 1) \quad (10)$$

In the above equations, X_{ba1} and X_{ba2} are CWFW of stage 1 and 2; a_{w1} and a_{w2} are the center distances of stages 1and 2, [4]:

$$a_{w1} = k_a \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot k_{H\beta}/([\sigma_{Hi}]^2 \cdot u_1 \cdot X_{ba1})} \quad (11)$$

$$a_{w2} = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta}/(\mathrm{AS}_2^2 \cdot u_2 \cdot X_{ba2})} \quad (12)$$

Where u_1 is the gear ratio of stage 1; $k_{H\beta} = 1.01 \div 1.21$ is the contacting load ratio; $[\sigma_{H1}]$ is the ACS of stage 1; k_a=43 (Mpa$^{1/3}$) is the material factor, [4]; T_{11} and T_{12} are the torque on the pinions of stage 1 and 2 (Nmm) which can be found by:

$$T_{11} = T_{out}/(u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3) \quad (13)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (14)$$

Where T_{out} is the output torque (Nmm); η_{hg} is the helical gear efficiency ($\eta_{hg} = 0.96 \div 0.98$, [4], η_b is the bearing efficiency (η_b=0.99÷0.995, [4]).

2.1.2 Gear Mass Determination

The mass of the gearbox housing can be determined by:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \quad (15)$$

In which V_{gh} is the mass of gearbox housing (m^3) which is calculated by:

$$V_{gh} = L \cdot B \cdot H \quad (16)$$

Where, L, B, and H are found by (Figure 1):

$$L = \frac{d_{w11}}{2} + a_{w1} + a_{w2} + \frac{d_{w22}}{2} + 2 \cdot S_G + 2 \cdot k \quad (17)$$

$$B = b_{w1} + b_{w2} + 7 \cdot S_G \quad (18)$$

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot S_G \quad (19)$$

In (17), $k = 8 \div 12$, [4], d_{w11}, d_{w21}, d_{w22} are gear pitch diameters of stages 1 and 2 which are found by, [4]:

$$d_{w11} = 2\, a_{w1}/(u_1 + 1) \quad (20)$$

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \quad (21)$$

$$d_{w22} = 2\, a_{w2} u_2\,/(u_2 + 1) \quad (22)$$

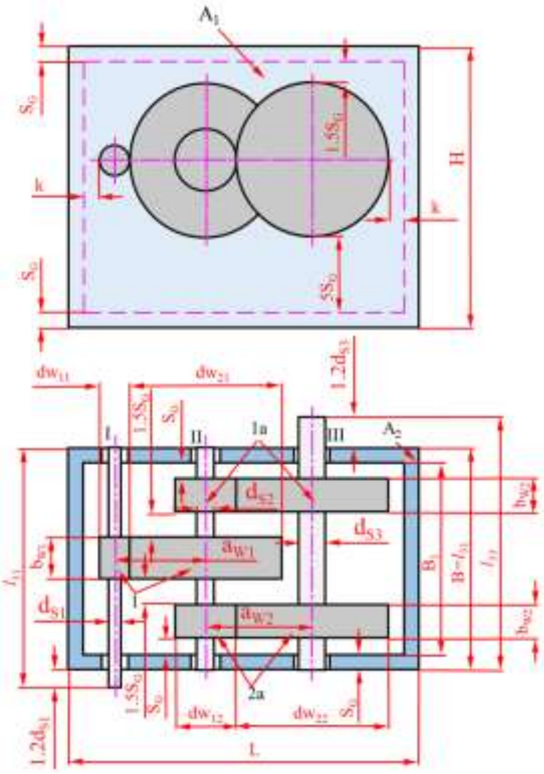

Fig. 2: Calculated schema

Where, a_{w1} and a_{w2} are the center distances and b_{w1} and b_{w2} are the gear width of stages 1 and 2. These components can be calculated by, [4]:

$$a_{w1} = k_a(u_1 + 1)\sqrt[3]{T_{11} k_{H\beta1}/([\sigma_{H1}]^2 u_1 X_{ba1})} \quad (23)$$

$$a_{w2} = k_a(u_2 + 1)\sqrt[3]{T_{12} k_{H\beta2}/([\sigma_{H2}]^2 u_2 X_{ba2})} \quad (24)$$

$$b_{w1} = X_{ba1} \cdot a_{w1} \quad (25)$$

$$b_{w2} = X_{ba2} \cdot a_{w1} \quad (26)$$

In the above equation, $k_a = 43$ is the material coefficient, [4], $k_{H\beta1}$ and $k_{H\beta2}$are the contacting load ratio of stages 1 and 2; $k_{H\beta1} = 1.0 \div 1.06$ and $k_{H\beta2} = 1.02 \div 1.28$, [4]. $[\sigma_{H1}]$and $[\sigma_{H2}]$ are ACS (MPa) and u_1 and u_2 denote the gear ratios of stages 1 and 2. X_{ba1} and X_{ba2} are CWFW and T_{11} and T_{12} are the pinion torque (Nmm) of stages 1 and 2:

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \quad (27)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (28)$$

Where T_{out} is the output torque (N.mm); η_{hg} is the gear efficiency ($\eta_{hg} = 0.96 \div 0.98$); η_b is the rolling bearing efficiency (η_h=0.99÷0.995), [4].

2.1.3 Shaft Mass Determination

The shaft mass (kg) is determined by:

$$m_s = m_{s1} + m_{s2} + m_{s3} \quad (29)$$

Where

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1}/4 \quad (30)$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2}/4 \quad (31)$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3}/4 \quad (32)$$

In which, m_{s1}, m_{s2}, and m_{s3} indicate the mass of shafts 1, 2, and 3 (kg); ρ_s is the shaft density (kg/m^3); l_{s1}, l_{s2}, and l_{s3} denote the length of the shafts 1, 2, and 3 which are determined by (Figure 2):

$$l_{s1} = B + 1.2 \cdot d_{s1} \quad (33)$$

$$l_{s2} = B \quad (34)$$

$$l_{s2} = B + 1.2 \cdot d_{s3} \quad (35)$$

In Equations (24) to (26), d_{s1}, d_{s2}, and d_{s3} are the shaft diameters (mm) which are found by, [4]:

$$d_{s1} = [T_{11}/(0{,}2 \cdot [\tau])]^{1/3} \quad (36)$$

$$d_{s2} = [T_{12}/(0{,}2 \cdot [\tau])]^{1/3} \quad (37)$$

$$d_{s3} = [T_{13}/(0{,}2 \cdot [\tau])]^{1/3} \quad (38)$$

In (36) to (38), $[\tau]$=17 (MPa) is the permissible shear stress.

2.2 Gearbox Efficiency Determination

The gearbox efficiency can be found by:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}} \quad (39)$$

With P_l is the overall power loss of the gearbox, [18]:

$$P_l = P_{lg} + P_{lb} + P_{ls} \quad (40)$$

Where, P_{lg} is overall gear power loss; P_{lb} is bearing power loss; P_{ls} is seal power loss which is determined by:
+) The power loss in all gears:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \quad (41)$$

With P_{lgi} is the gear power losses of i stage:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right) \quad (42)$$

In which, η_{gi} is the anticipated gear efficiency of the i stage, [19]:

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot \left(\beta_{ai}^2 + \beta_{ri}^2\right) \quad (43)$$

In (43), u_i is the gear ratio of i stage; f is the friction coefficient; β_{ai} and β_{ri} are the arcs of approach and retreat on the i stage which can be determined by, [19]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} \quad (44)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} \quad (45)$$

In which, R_{e1i} and R_{e2i} are the outer radiuses; R_{1i} and R_{2i} are the pitch radiuses and R_{01i} and R_{01i} are the base-circle radiuses of the pinion and gear; α is the pressure angle.
The friction coefficient f in Equation (43) depends on sliding velocity v, [14]:
- If v ≤ 0.424 (m/s):

$$f = -0.0877 \cdot v + 0.0525 \quad (46)$$

- If v > 0.424 (m/s) :

$$f = 0.0028 \cdot v + 0.0104 \quad (47)$$

+) The power loss in bearings can be calculated by, [18]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i$$

Wherein, F denotes the bearing load (N); $f_b = 0.0011$ represents the bearing friction coefficient, [18], v indicates the peripheral speed; i mean the bearing ordinal number (i = 1÷6).
+) The total power loss in seals can be found by [18]:

$$P_s = \sum_{i=1}^{2} P_{si} \quad (49)$$

In which P_{si} is the single seal power loss (w):

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot \log\log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (50)$$

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With VG_{40} indicates the ISO Viscosity Grade number.

2.3 Target Function and Constraints

2.3.1 Target Functions

The following are the objectives of the multi-target optimization problem:

Reducing the mass of the gearbox:

$$\min f_2(X) = m_g \quad (51)$$

Enhancing the gearbox efficiency:

$$\min f_1(X) = \eta_{gb} \quad (52)$$

Where X denotes the design variable vector. As variables in this study, five primary design parameters were chosen: u_1, Xba_1, Xba_2, AS_1, and AS_2. As a result, we have:
$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\}$

2.3.2 Constraints

The constraints on the multi-objective function are as follows:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \quad (54)$$

$$0.25 \le X_{ba1} \le 0.3 \text{ and } 0.25 \le X_{ba2} \le 0.4 \quad (55)$$

$$350 \le AS_1 \le 420 \text{ and } 350 \le AS_2 \le 420 \quad (56)$$

3 Problem Solution

In this study, five main design variables were chosen for consideration. These variables and their minimum and maximum values are shown in Table 1. Compared to single-objective optimization issues, multi-objective optimization (MOO) problems are more complex because they require simultaneously maximizing several competing objectives. In reality, MOO problems can be solved using a variety of strategies and tactics; the approach you choose will rely on the specifics of the situation as well as the trade-offs you wish to make. Multi-objective optimization (also known as issues optimization) can be solved in several ways. For instance, swarm intelligence, evolutionary algorithms, and Pareto-based techniques. In this paper, the MOO issue was solved using GRA and the Taguchi approach. To optimize the number of levels for each variable, the L25 (5^5) design was employed. Among the variables investigated, however, u_1 has a nearly large range (u_1=1÷9, Table 1). Even with five levels, the difference in the values of these qualities remained significant (in this case, the difference is ((9-1)/4 = 2).

The 2-stage multi-objective optimization problem solution technique was proposed in, [13], (Figure 3). In this method, the first stage of this method addresses a single-target optimization problem to reduce the gap between variable values scattered throughout a wide range while the second addresses a multi-target optimization work to find the best core design features.

Table 1. Key design parameters and their maximum and lowest restrictions

Factor	Notation	Lower limit	Upper limit
Gear ratio of stage 1	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.3
CWFW of stage 2	X_{ba2}	0.25	0.4
ACS of stage 1 (MPa)	AS_1	350	420
ACS of stage 2 (MPa)	AS_2	350	420

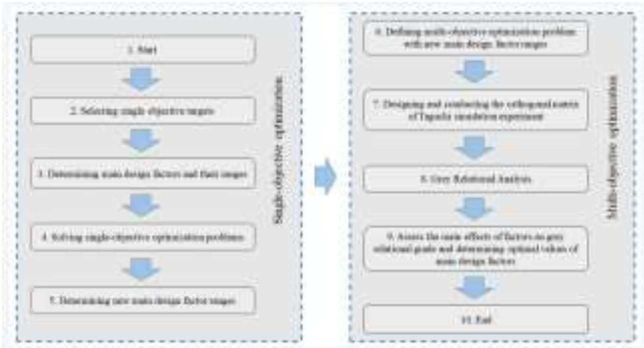

Fig. 3: Method to solve multi-objective problem, [13]

4 Single-target Optimization

As noted above, the single-target optimization problem was solved to reduce the gap between variable values scattered throughout a wide range. Also, in this work, the direct search technique was applied to deal with the single-target optimization problem. In addition, a Matlab-based computer program was developed to address two single-target problems: lowering gearbox mass and increasing gearbox efficiency. Based on the program's findings, Figure 4 displays a connection between the optimum gear ratio of stage 1 u_1 and the overall ratio u_t for both two single objectives. From the results of the figure, new limitations for the variable u_1 have been added (Table 2).

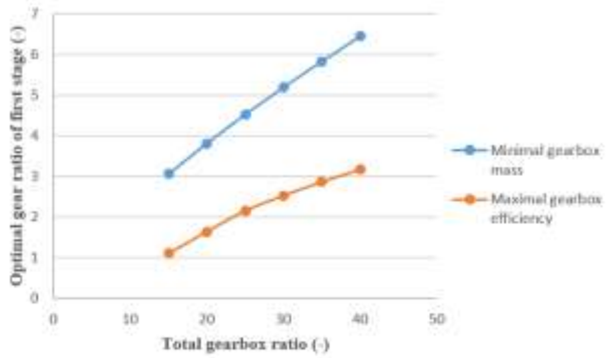

Fig. 4: Relation between u_1 and u_t

Table 2. New constraints of u_1

u_t	u_1	
	Lower limit	Upper limit
15	1.09	3.07
20	1.63	3.82
25	2.14	4.53
30	2.52	5.2
35	2.86	5.83
40	3.16	6.45

5 Multi-objective Optimization

The aim of this study's multi-target optimization work for a THG with SSDGS is to find the best primary design factors with a given total gear-box ratio that fulfills two single-target functions: reducing gearbox mass and optimizing gearbox efficiency. To accomplish this, a computer experiment was carried out. Table 3 shows the key design elements and their values for u_t=20. The experimental design was created using the Taguchi technique and the L25 (5^5) design, and the data was analyzed using Minitab R18 software. The design and results of the experiment for u_t=20 are shown in Table 4 (Appendix).

Multi-objective optimization issues are handled by the Taguchi and GRA approaches. The main stages of this approach are as follows:
+) Using the following equations, calculate the signal-to-noise ratio (S/N):

Table 3. Key parameters and levels for u_t = 20

Factor	Notation	Level				
		1	2	3	4	5
Gear ratio of stage 1	u_1	1.63	2.1775	3.725	3.2725	3.82
CWFW of stage 1	X_{ba1}	0.25	0.2625	0.275	0.2875	0.3
CWFW of stage 2	X_{ba2}	0.25	0.2875	0.325	0.3625	0.4
ACS of stage 1 (MPa)	AS_1	350	368	386	404	420
ACS of stage 2 (MPa)	AS_2	350	368	386	404	420

The higher the S/N, the smaller the gearbox mass:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \tag{57}$$

The greater the S/N, the more efficient the gearbox:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2}) \tag{58}$$

Where y_i is the output result and m=1 is the experimental repetition number because this is a simulation. Table 5 (Appendix) provides the computed S/N values for the objectives.

The data quantities for the two single-objective functions differ. To ensure comparability, the data must be normalized, or brought to a suitable scale. The normalization value Z_{ij}, which is 0 to 1, is used to normalize the data. This value is calculated using the following formula:

$$Z_i = \frac{SN_i - min(SN_i, =1,2,..n)}{max(SN_i, j=1,2,..n) - min(SN_i, =1,2,..n)} \tag{59}$$

With n=25 is the experimental run.
+) The grey relational factor is determined by:

$$y_i(k) = \frac{\Delta_{min} + \xi.\Delta_{max}(k)}{\Delta_i(k) + \xi.\Delta_{max}(k)} \tag{60}$$

Where, i=1,2,...,n; k=2 is the objective number; $\Delta_j(k) = \|Z_0(k) - Z_j(k)\|$ with Z0(k) and Zj(k) are the reference and particular comparison sequence; Δ_{min} and Δ_{max} are the minimum and maximum values of i(k); ζ=0.5 is the characteristic coefficient.
+) Determining the coefficient of grey relations by:

$$\overline{y_i} = \frac{1}{k}\sum_{j=0}^{k} y_{ij}(k) \tag{61}$$

Where y_{ij} is the grey relation value (GRV) of the ith experiment's j^{th} output target. Table 6 (Appendix) displays the estimated grey relation number y_i as well as the average GRV($\overline{y_i}$) for each experiment.

It is advised to use a higher average GRV to bring the output parts into harmony. This enables the objective function of the multi-objective issue to be converted into a single-objective optimization problem, the mean grey relation value being the result.

6 Results and Discussions

Table 7 (Appendix) displays the results of an ANOVA test run to investigate the effect of the major design factors on the average GRV($\overline{y_i}$). According to Table 7 (Appendix), u_1 has the most influence on $\overline{y_i}$ (36.36%), followed by AS_2 (28.73%), X_{ba2} (26.39%), X_{ba1} (3.50), and AS_1 (2.79%). Table 8 displays the order of the effect of the key parameters on ($\overline{y_i}$) using ANOVA analysis.

Table 4. Response table for means

Level	u_1	Xba_1	Xba_2	AS_1	AS_2
1	0.644	0.5722	0.626	0.5402	0.5013
2	0.5705	0.5745	0.5748	0.5452	0.5172
3	0.5383	0.5504	0.5458	0.5623	0.5549
4	0.5186	0.5489	0.5273	0.5546	0.5904
5	0.5092	0.5346	0.5067	0.5784	0.6168
Delta	0.1349	0.0398	0.1193	0.0382	0.1155
Rank	1	4	2	5	3
Average of grey analysis value: 0.556					

+) *Determining the best key design parameters:* In theory, the best factor set would incorporate essential design elements with the highest S/N values. As a result, the S/N ratio influence of the primary design features (Figure 5) was evaluated. Furthermore, the ideal set of multi-objective parameters (corresponding to the red points) may be easily calculated using the Figure 4 chart. Table 9 provides the appropriate levels and values for the important design variables of the multi-target function.

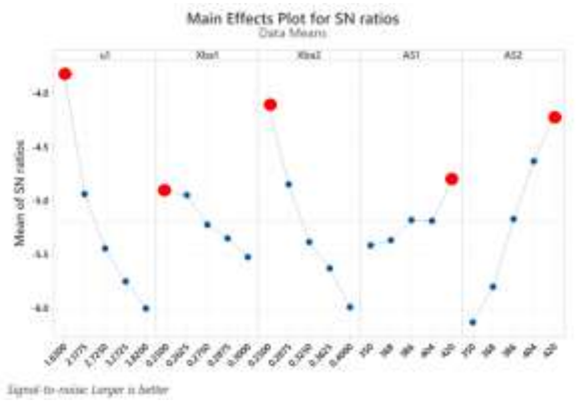

Fig. 5: Main effects plot for S/N ratios

Table 5. Optimal key factors

No.	Key factors	Code	Optimal Level	Optimal Value
1	Gear ratio of stage 1	u_1	1	1.63
2	CWFW of stage 1	Xba1	1	0.25
3	CWFW of stage 2	Xba2	1	0.25
4	ACS of stage 1 (MPa)	AS1	5	420
5	ACS of stage 2 (MPa)	AS2	5	420

+) *Evaluating proposed modeling:* Figure 6 displays the Anderson-Darling approach results, which are applied to estimate the proposed model's adequacy. The experimental observations' data points (shown as blue dots in the graph) fall within the 95% standard deviation zone specified by the top and bottom boundaries. Furthermore, the p-value of 0.105 exceeds the level of significance of α = 0.05. These results show that the proposed model is appropriate for evaluation.

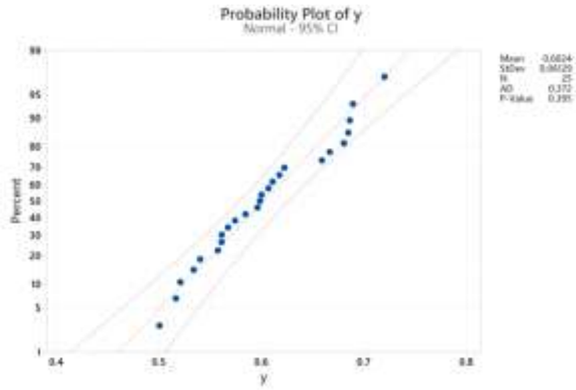

Fig. 6: Probability plot of $\overline{y}$

The Anderson-Darling approach is used to confirm the suitability of the suggested model (Figure 6). The data corresponding to the experimental points (blue dots) are all contained within the area bounded by two upper and lower limit lines with a 95% standard deviation limit, according to the results from this figure. Additionally, the applied empirical model is adequate because the P value of 0.105 is higher than the value = 0.05.

Proceed as with u_t=20, but with ut values 15, 25, 30, 35, and 40. Table 10 shows the optimum values of the five basic design parameters at various u_t. Figure 7 depicts the relationship between the optimal first-stage gear ratio and the overall gearbox ratio. To achieve the ideal values of u_1, the following regression formula (with R^2=0.9986) was presented:

$$u_1 = 2.1241 \cdot \ln(x) - 4.694 \tag{62}$$

After having u_1, $u_2=u_t/u_1$ determines the optimum gear ratio of the second stage.

Table 6. Optimal values of main design parameters

No.	u_t					
	15	20	25	30	35	40
u_1	1.09	1.63	2.14	2.52	2.86	3.16
X_{ba1}	0.25	0.25	0.2625	0.2625	0.2625	0.2625
X_{ba2}	0.25	0.25	0.25	0.25	0.25	0.25
AS_1	420	420	420	420	420	420
AS_2	420	420	420	420	420	420

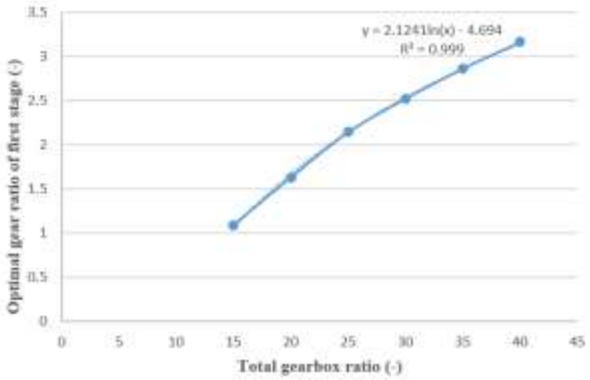

Fig. 7: Relation between optimal gear ratio of stage 1 and total ratio

7 Conclusion

This paper discusses the results of a multi-objective optimization study on optimizing a two-step helical gearbox with SSDGS to reduce gearbox across section area and enhance gearbox efficiency. The first stage of this research improved the gear ratio, wheel face width efficiency in stages 1 and 2, and permissible contact stress in steps 1 and 2. To address this issue, a simulation experiment based on the Taguchi L25 type was designed and carried out. The impact of major design elements on the multi-objective goal was also studied. The gear ratio u_1 was discovered to have the greatest influence on $\overline{y_i}$ (36.36%), followed by AS_2 (28.73%), X_{ba2} (26.39%), X_{ba1} (3.50), and AS_1 (2.79%). Additionally, the ideal settings for the important gearbox features have been recommended. To determine the ideal first stage u_1 gear ratio, a regression technique (Equation (62) was also implemented.

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APPENDIX

Table 7. Experimental plan and output results for u_t=20

No.	Input Factors					m_{gb}	η_{gb}
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(kg)	(%)
1	1.6300	0.2500	0.2500	350	350	231.475	95.572
2	1.6300	0.2625	0.2875	368	368	226.429	95.540
3	1.6300	0.2750	0.3250	386	386	222.282	95.534
4	1.6300	0.2875	0.3625	404	404	218.818	95.500
5	1.6300	0.3000	0.4000	420	420	216.390	95.498
6	2.1775	0.2500	0.2875	386	404	207.879	95.381
7	2.1775	0.2625	0.3250	404	420	205.853	95.353
8	2.1775	0.2750	0.3625	420	350	222.889	95.385
9	2.1775	0.2875	0.4000	350	368	223.271	95.326
10	2.1775	0.3000	0.2500	368	386	212.696	95.407
11	2.7250	0.2500	0.3250	420	368	211.044	95.211
12	2.7250	0.2625	0.3625	350	386	212.869	95.211
13	2.7250	0.2750	0.4000	368	404	210.344	95.188
14	2.7250	0.2875	0.2500	386	420	201.561	95.210
15	2.7250	0.3000	0.2875	404	350	216.044	95.276
16	3.2725	0.2500	0.3625	368	420	204.317	95.070
17	3.2725	0.2625	0.4000	386	350	217.962	95.107
18	3.2725	0.2750	0.2500	404	368	207.604	95.116
19	3.2725	0.2875	0.2875	420	386	205.258	95.092
20	3.2725	0.3000	0.3250	350	404	208.465	95.098
21	3.8200	0.2500	0.4000	404	386	208.831	94.972
22	3.8200	0.2625	0.2500	420	404	199.810	94.985
23	3.8200	0.2750	0.2875	350	420	204.252	94.981
24	3.8200	0.2875	0.3250	368	350	216.388	94.995
25	3.8200	0.3000	0.3625	386	368	213.575	94.995

Table 8. S/N values for each run when u_t=20

No.	Input Factors					mgb		η_{gb}	
	u_1	X_{ba1}	X_{ba2}	AS_1	AS2	(kg)	S/N	(%)	S/N
1	1.6300	0.2500	0.2500	350	350	231.475	-47.2901	95.572	39.6066
2	1.6300	0.2625	0.2875	368	368	226.429	-47.0986	95.540	39.6037
3	1.6300	0.2750	0.3250	386	386	222.282	-46.9381	95.534	39.6032
4	1.6300	0.2875	0.3625	404	404	218.818	-46.8017	95.500	39.6001
5	1.6300	0.3000	0.4000	420	420	216.390	-46.7047	95.498	39.5999
6	2.1775	0.2500	0.2875	386	404	207.879	-46.3562	95.381	39.5892
7	2.1775	0.2625	0.3250	404	420	205.853	-46.2711	95.353	39.5867
8	2.1775	0.2750	0.3625	420	350	222.889	-46.9618	95.385	39.5896
9	2.1775	0.2875	0.4000	350	368	223.271	-46.9766	95.326	39.5842
10	2.1775	0.3000	0.2500	368	386	212.696	-46.5552	95.407	39.5916
11	2.7250	0.2500	0.3250	420	368	211.044	-46.4875	95.211	39.5737
12	2.7250	0.2625	0.3625	350	386	212.869	-46.5622	95.211	39.5737
13	2.7250	0.2750	0.4000	368	404	210.344	-46.4586	95.188	39.5716
14	2.7250	0.2875	0.2500	386	420	201.561	-46.0881	95.210	39.5737
15	2.7250	0.3000	0.2875	404	350	216.044	-46.6908	95.276	39.5797
16	3.2725	0.2500	0.3625	368	420	204.317	-46.2061	95.070	39.5609
17	3.2725	0.2625	0.4000	386	350	217.962	-46.7676	95.107	39.5642
18	3.2725	0.2750	0.2500	404	368	207.604	-46.3447	95.116	39.5651
19	3.2725	0.2875	0.2875	420	386	205.258	-46.2460	95.092	39.5629
20	3.2725	0.3000	0.3250	350	404	208.465	-46.3807	95.098	39.5634
21	3.8200	0.2500	0.4000	404	386	208.831	-46.3959	94.972	39.5519
22	3.8200	0.2625	0.2500	420	404	199.810	-46.0123	94.985	39.5531
23	3.8200	0.2750	0.2875	350	420	204.252	-46.2033	94.981	39.5527
24	3.8200	0.2875	0.3250	368	350	216.388	-46.7047	94.995	39.5540
25	3.8200	0.3000	0.3625	386	368	213.575	-46.5910	94.995	39.5540

Table 9. Values of $\Delta_i(k)$ and $\bar{y}_i$

No	S/N		Zi		Δi (k)		GRVyi		$\bar{y}_i$
	mgb	ηgb	Mgb	ηgb					
			Reference values		Mgb	ηgb	mgb	ηgb	
			1.000	1.000					
1	-47.2901	39.6066	0.0000	1.0000	1.000	0.000	0.333	1.000	0.667
2	-47.0986	39.6037	0.1498	0.9468	0.850	0.053	0.370	0.904	0.637
3	-46.9381	39.6032	0.2755	0.9369	0.725	0.063	0.408	0.888	0.648
4	-46.8017	39.6001	0.3823	0.8803	0.618	0.120	0.447	0.807	0.627
5	-46.7047	39.5999	0.4581	0.8770	0.542	0.123	0.480	0.803	0.641
6	-46.3562	39.5892	0.7309	0.6823	0.269	0.318	0.650	0.612	0.631
7	-46.2711	39.5867	0.7975	0.6357	0.203	0.364	0.712	0.579	0.645
8	-46.9618	39.5896	0.2569	0.6890	0.743	0.311	0.402	0.617	0.509
9	-46.9766	39.5842	0.2453	0.5908	0.755	0.409	0.399	0.550	0.474
10	-46.5552	39.5916	0.5752	0.7256	0.425	0.274	0.541	0.646	0.593
11	-46.4875	39.5737	0.6282	0.3991	0.372	0.601	0.573	0.454	0.514
12	-46.5622	39.5737	0.5696	0.3991	0.430	0.601	0.537	0.454	0.496
13	-46.4586	39.5716	0.6507	0.3607	0.349	0.639	0.589	0.439	0.514
14	-46.0881	39.5737	0.9407	0.3974	0.059	0.603	0.894	0.453	0.674
15	-46.6908	39.5797	0.4690	0.5075	0.531	0.493	0.485	0.504	0.494
16	-46.2061	39.5609	0.8484	0.1638	0.152	0.836	0.767	0.374	0.571
17	-46.7676	39.5642	0.4089	0.2255	0.591	0.774	0.458	0.392	0.425
18	-46.3447	39.5651	0.7399	0.2406	0.260	0.759	0.658	0.397	0.527
19	-46.2460	39.5629	0.8171	0.2005	0.183	0.799	0.732	0.385	0.558
20	-46.3807	39.5634	0.7117	0.2105	0.288	0.789	0.634	0.388	0.511
21	-46.3959	39.5519	0.6998	0.0000	0.300	1.000	0.625	0.333	0.479
22	-46.0123	39.5531	1.0000	0.0217	0.000	0.978	1.000	0.338	0.669
23	-46.2033	39.5527	0.8505	0.0150	0.149	0.985	0.770	0.337	0.553
24	-46.7047	39.5540	0.4582	0.0384	0.542	0.962	0.480	0.342	0.411
25	-46.5910	39.5540	0.5471	0.0384	0.453	0.962	0.525	0.342	0.433

Table 10. Analysis of variance for means

Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u_1	4	0.059333	0.059333	0.014833	16.34	0.010	36.36
Xba_1	4	0.005714	0.005714	0.001429	1.57	0.336	3.50
Xba_2	4	0.043055	0.043055	0.010764	11.86	0.017	26.39
AS_1	4	0.004552	0.004552	0.001138	1.25	0.416	2.79
AS_2	4	0.046877	0.046877	0.011719	12.91	0.015	28.73
Residual Error	4	0.003630	0.003630	0.000908			2.22
Total	24	0.163161					

Model Summary

S	R-Sq	R-Sq(adj)
0.0301	97.77%	86.65%

Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The idea of the paper was proposed by Nguyen Van Trang. The simulation and the optimization were conducted by Nguyen Van Trang and Trieu Quy Huy. The manuscript was written by Nguyen Van Trang with support from Trieu Quy Huy and Tran Huu Danh. All authors have read and agreed to the manuscript.

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