

# A Mathematical method for Calculating the Retentive Force of Ring Clasps when Designing the Removable Partial Denture Metal Frame

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*Abstract:* - Practically, calculating the retentive force of the clasps used as a part of removable partial dentures testing of general retention. This test is done using a universal testing machine or a pull test kit. However, these techniques aren't feasible until after designing and fabricating the framework. Therefore, a calculating method or formula based on the variables affecting the retentive force of any clasp seems necessary while developing the framework and before the final casting of the partial denture. The objective of this experiment is to determine the formula that mathematically estimates the pulling force of four-ring clasps in vitro. The experiment included the fabrication and recording of pullout forces for four types of ring clasps; then, the variables were identified and related to the retentive force mathematically. An equation was generated to count the retentive force for the four clasps and assess the precision of the results. In addition, the retentive arm sizes were correlated to the pulling forces using the Pearson coefficient. In conclusion, a new mathematical method was presented to the profession using simple math. The method is a practical and simple way to enhance the selection of clasps concerning their retentive force in addition to the other design factors. Therefore, additional tools for creating the removable partial denture with a bio-protective concept were displayed, and further research should be done to analyze the force generated by other clasps to make a global system to be added to the software for designing the partial dental framework.

*Key-Words:* - Mathematical equation, ring clasp, removable, partial denture, partial denture design, retentive arm.

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## 1 Introduction

A partial denture is an artificial device used to replace the missing teeth and related structures to optimize reduced function and maintain healthy conditions for the remaining structures of the oral cavity for as long as possible, [1]. It is made of either acrylic resin alone or supported by a metal frame made of specially engineered alloys for this purpose.

The partial denture can be fixed or removable, depending on the final objectives and diagnosis of the remaining dental and oral structures. In this article, the removable types will be displayed, and their main retentive components will be described and analyzed. The removable partial denture (RPD) is composed of many components connected to form the final configuration of the frame. RPD gains its retention and support mainly by incorporating the bulbous form of the remaining teeth around the edentulous areas or any additional teeth that are

located away from the missing teeth, in addition to the remaining supporting tissues.

The components of removable partial dentures may be classified into:

1. The retentive components are responsible for retaining the denture in its designed place in the oral cavity. They include direct retainers, indirect retainers, and guiding planes.
2. The replaced masticatory units and their supporting structures; include artificial teeth and the supporting bases of the denture.
3. The connecting elements include the major and minor connectors, [1], [2].

In this research, the main intention is to study and mathematically analyze some types of retentive components of RPD. They are classified into primary and secondary parts.

Direct or primary retainers (DR), or clasps (C) are composed mainly of three parts joined together at the clasp shoulder or the origin (Figure 1).



Fig. 1: The components of the clasp

Two components of the clasp are responsible for retention (retentive arm RA) and stability (reciprocal or bracing arm). Without a reciprocal arm, no retention can be provided by the retentive arm. Spring action is the retentive principle on which the clasp functions in general; however, friction is another way to promote the function of some clasps.

The secondary retentive components are elements that participate in retaining the RPD by frictional resistance, adhesion, and cohesion or by following certain orientations that resist movement in different planes of RPD movement, like the guiding plane, minor connector, denture base, etc.

Many designs of clasps were conferred on the profession for a long time. However, in this article, only four types of circumferential clasps were studied. These are four designs of ring clasps (Figure 2). In previous studies, the retentive force of specific clasps was measured for variable clasp designs and materials, [1], [3], [4], [5], [6].

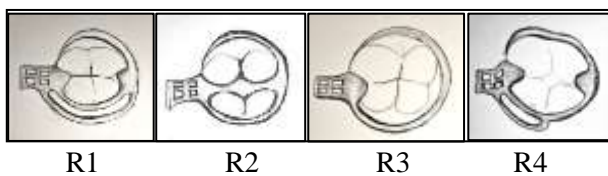


Fig. 2: Ring clasp designs

R1; conventional ring clasp, R2; ring clasp with continuous rest, R3; ring clasp with one rest, R4; ring clasp with short strut.

## 2 Problem Formulation

In human beings, being partially edentulous produces several combinations of edentulous configurations of the upper and lower dental arches that reach over 65,000 cases, [1], [2], [3]. Until today, dentists have used an arbitrary design method (either manual or using design software) to assign the type, form, and location of RPD components.

This approach might result in over- or under-estimation of the retentive force produced mainly by direct retainers due to changing variable values of arch size, form, and abutment anatomy like tooth circumference or height of contour, angle of cervical convergence, and undercut depth, in addition to the clasp design. The result is an over-destructive irritant force or a reduction of the necessary one to retain the RPD securely in its place during function.

Due to the presence of many variables in the oral cavity, it is necessary to estimate the knowledge of retentive forces for the most effective locking devices using a precise method. Therefore, we recommend utilizing a basic formula that takes into account the majority of the variables that impact the strength of retention.

### 2.1 Objectives

This work formulated a mathematical formula to find the retentive force of ring clasps as the first part of a series of investigations to cover the most usable retentive means in designing RPD.

The sizes of the retentive arm of ring clasps were calculated and correlated to the pullout force to find the possible relation between the two variables using Pearson correlation at  $p < .05$ .

The means of the calculated retentive force for the clasps were compared to the actual force estimated using UTM using a t-test at  $p < .05$ .

As mentioned before, RPD design is the most crucial step in providing treatment for partially edentulous patients, and it requires a simple, fast calculation to find the retention of different retentive components of the appliance. The new application can be added to already in-use software for RPD design, [3], [4].

### 2.2 Experiment Details

The analysis of the factors that influence the retentive force of a clasp is done on a laboratory basis. In this article, an attempt is made to display a simple mathematical way to calculate the force of retention of different clasps made of Co-Cr alloy exclusively during the designing phase and before the final casting approval of the RPD. This is an educational way to make dental professionals pay

attention to enhancing the RPD design and teaching program in general to enhance the service of still-growing treatment options. The work plan is summarized as follows (Figure 3):

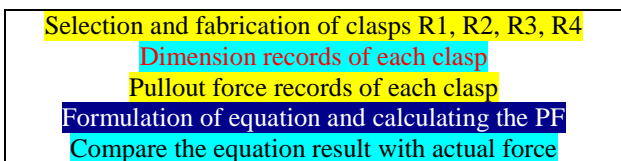


Fig. 3: The outline of the experiment

### 2.3 Factors that Influence Clasp Retentive Force

Theoretically, the parameters involved in clasp retention are shown in Figure 4.

The length of the retentive arm, which is measured from the clasp shoulder to the tip end, has an inverse or negative effect on the retentive force (RF). As the length decreases, the RF increases.

The width of the retentive arm (W) is the distance between the upper and lower borders of the RA, measured at the origin (W1) and the last third (W2) of the arm. Both W1 and W2 have a linear or positive correlation with RF.

The thickness of the retentive arm (TH) also has a linear or positive correlation with RF, meaning that an increase in thickness will increase the RF of the clasp. It was measured at the start (Th1) and end (Th2) of the clasp.

The angle of cervical convergence (A) also has a linear or positive correlation with RF, as a higher CCA produces increased RF (Figure 5).

The environment of the oral cavity, specifically salivary flow, can be unpredictable and may increase or decrease the frictional force between the DR and tooth surface, affecting the RF.

The material used for clasp fabrication also affects the RF, with a gold alloy having higher elasticity than a Co-Cr alloy and therefore lower RF for similar measurements. While Co-Cr alloy was used for the fabrication of the clasp samples in this research, the material was considered neutral and not investigated further.

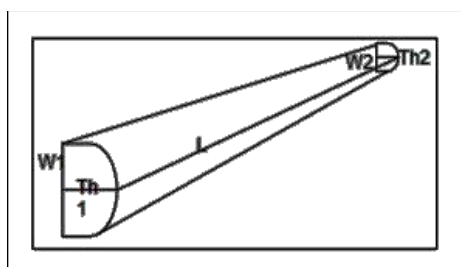


Fig. 4: The record areas of the retentive arm section

L; length, W1, W2; width at start and end, Th1, Th2; thickness at start and end

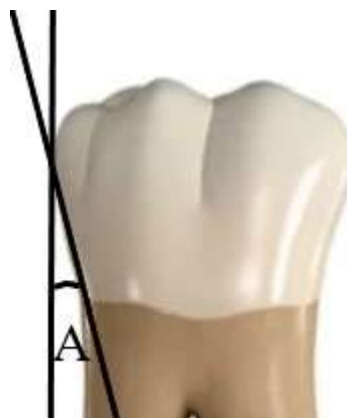


Fig. 5: Location of the angle of cervical convergence (A)

The retention of a clasp is a function of width, thickness, length, and cervical convergence angle (A).

The equation sets the:

$$RF = W.Th.A/L \tag{1}$$

where, RF increases when width (W), thickness (Th), and angle (A) increase in values, whereas, RF decreases when L increases (Figure 4, Figure 5).

This function is specific to each clasp design and dimension. Therefore, the validity of this theoretic calculation should be approved using the experimental findings of the retentive force of the designated clasp.

A constant (K) is added to the equation to formalize the equal mathematical expression of the two sides of the formula.

The final equation:

$$RF = K.W.Th.A/L \tag{2}$$

The RF is in grams, the dimensions (W, Th, L) are in mm., and A is in angular degrees.

To solve for and find the constant value K for the defined clasps, the retentive force (pullout force) was measured for four types of ring clasps.

### 2.4 Selection of the Clasp-Model Designs

Four ring clasps were selected as models for measuring the retentive pulling force. The designs were:

Classical ring clasp with two rests and reinforcing strut (Figure 6).

Modified ring clasp with one continuous occlusal rest (Figure 7).

Classical ring clasp with one rest only (Figure 8).

Modified ring clasp with a shortened strut located at the middle of the bracing arm (Figure 9).

The pullout locations for the four designs were on the proximal rest near the saddle.



Fig. 6: Classical ring



Fig. 7: Modified ring



Fig. 8: One rest ring



Fig. 9: Short connector ring

## 2.5 Fabrication of the Clasps

A maxillary plastic model (Frasaco AG-3 WOK 40, Germany) and four natural comparable second molars were used. The plastic model was duplicated with silicone (Wirosil®Bego, Germany) to produce four casting molds. Natural teeth were placed inside the silicone molds and then poured using dental stones to make four casts. Before setting the dental stone, two captive screws were fixed 3 mm away from the border of the silicone mold to mount and

lock the master model to a custom-made jig later. The four master casts surveyed at the zero-tilt position and 0.50 mm undercut depth were measured and marked off. Standardized occlusal rest seats were prepared with dimensions of 2.5 x 2.5 mm and a depth of 1.5 mm for the four master casts. Using a milling machine (AF 30, milling machine, Switzerland), the guiding plane was prepared roughly 2 mm on the proximal side in an occlusogingival direction, keeping the marginal ridge, [1], [2]. The master cast was duplicated using silicone to make working stone casts, then, re-surveyed using the original tilt. The clasp forms are traced on the molar using a mechanical pencil  $\varnothing$  0.5 mm. Undesirable undercuts were blocked out, and the entire lengths of the clasp arms were ledged. In addition, a small amount of wax was placed on the mesiobuccal line angles of the tooth as reference points to standardize the location of the tips and lengths of the clasp arms.

The corrected working casts were duplicated with reversible hydrocolloids. Twenty-four refractory casts were duplicated for placing the wax patterns of different ring clasps. The total refractory casts were divided into four groups. Each group consisted of six casts of each clasp used in this research. The waxed pullout rings were placed parallel to the path of insertion with the aid of a surveyor.

The waxed clasp is sprued, invested, preheated, and cast into Co-Cr alloy (Wironit, Bego, Germany). They were sandblasted, finished, and electropolished. The fitting surfaces of the retentive and reciprocal arms had never been touched during the removal of the burs, nodules, etc. The clasp assemblies were examined radiographically for internal porosity using a dental X-ray machine (Siemens, 1448 237 D3195, Germany) under standardized conditions with a source of 70 kV/7mA and exposure time of 1.2 seconds, located at a 50 cm distance.

## 2.6 Measuring the Dimensions of the Finished Clasps

Each ring clasp was measured in mm using a digital vernier at the following locations (Figure 4):

The width of the clasp at the origin and near the tip of the retentive arm.

The thickness of the clasp at the origin and near the tip.

The length of the clasp from the origin to the tip.

### 2.7 Measuring the Pullout Force of the Clasps

A movable, custom-made jig is fabricated to hold the master cast in a small container and to fix it perpendicular to the pulling-out chain. One end of the chain was attached to the upper jig of the universal testing machine (UTM) (Shimadzu testing machine AG-X, 10N-10KN, Japan), while the other end was attached to an S-shaped metal hook connected to the pullout ring of the clasp. Chain tension is set at zero so that no premature pull is introduced before starting the experiment. The clasp was seated manually, and a tensile load was applied until the dislodgement of the clasp and the UTM stopped automatically and the pulling force was displayed. The pulling cycle was repeated ten times for each clasp with a cross-head speed of 10 mm/min in dry conditions.

## 3 Problem Solution and Results

### 3.1 The Dimensions of the Fabricated Ring Clasps

The average dimensions of the retentive arm form in different ring clasps are displayed in Table 1.

The retentive arm size (in cubic measurement) is equal to  $= 1/2 \pi (W \cdot Th \cdot L)$  (3)

To be calculated for the used clasp and correlated to the RF.

Table 1. The means of the four ring clasps dimensions in mm

C	N	L	Th 1	Th 2	MT	W 1	W 2	MW
R1	6	13.83	1.1	1	1.05	1.8	1.23	1.51
R2	6	13.83	1.2	1	1.1	1.8	1.22	1.49
R3	6	13.91	1.1	1	1.05	1.8	1.23	1.5
R4	6	13.75	1.13	1.02	1.07	1.8	1.22	1.5

C; clasp type, N; the number of samples, L; length of the retentive arm, Th1; thickness of retentive arm at origin, Th2; thickness of retentive arm at the end, MTh; mean thickness of retentive arm, W1; width of the retentive arm at origin, W2; width at end of retentive arm, MW; mean width, R1; classical ring with 2 rests, R2; modifies ring with one continuous rest, R3; classical ring with one rest, R4; Modified ring clasp with strut located at the middle of the bracing arm

### 3.2 The Average Pulling Force of the Four-Ring Clasps Samples

Six samples of each ring clasp model were used in pullout procedures with the UTM machine. Each clasp was pulled out ten times in dry conditions. The average pullout force in (Newton) is shown in Table 2.

Table 2. The average pulling force of different clasps

CT	N	R1	R2	R3	R4
S1	10	20.03	14.69	13.85	11.38
S2	10	19.23	20.05	12.03	7.86
S3	10	12.25	19.11	13.31	12.09
S4	10	19.38	15.58	11.60	14.37
S5	10	16.52	21.44	11.66	10.14
S6	10	17.15	14.27	11.63	11.05

CT; clasp type, N; pullout number, R1; classical ring, R2; one continuous rest ring, R3; one rest ring, R4; classical ring with a strut ends at the middle of the tooth, S1; samples of R1,2,3,4 types, S2; samples of R1,2,3,4 types, S3; samples of R1,2,3,4 types, S4; samples of R1,2,3,4 types, S5; samples of R1,2,3,4 types, S6; samples of R1,2,3,4 types, M; mean

### 3.3 Formula Application

The measuring units of the equation are:

RF in Newton, clasp dimensions (W, Th, L. in mm), while A in degrees.

By replacing the values in formula (2) for each ring clasp, starting with ring 1:

$$20.028 = K \cdot 1.51 \cdot 1.05 \cdot A / 13.83$$

$$20.028 = K \cdot 0.11027 \cdot A \dots$$

$$K = 20.028 / 0.11027 \cdot A \dots$$

$$K = 181.63 / A$$

$$\text{Therefore, } K = 181.63 / A$$

for Ring clasp 1.

The A (angle of cervical convergence) is measured using photographic technique (Figure 5).

Depending on the molar configuration, the angle of cervical convergence varies between 20 and 25 angular degrees.

In this study, A = 20 degrees was used as a fixed angle to find the K value for the four-ring clasps.

The K value is now calculated =  $181.63 / 20 = 9.0815$ .

If the K value is replaced inside the equation, then the RF can be calculated for any sample of ring 1 clasp during RPD frame design.



If the average RF is used for each group of ring clasp types, then the K average value can be calculated using the same method.

$$RF = K \cdot W \cdot Th \cdot A / L$$

$$17.425 = K \cdot 1.51 \cdot 1.05 \cdot 20 / 13.83$$

$$17.425 = K \cdot 31.71 / 13.83 = K \cdot 2.268$$

$$\text{Therefore, } K = 17.425 / 2.268 = 7.6829 \approx 7.7$$

The K value is displayed for the four-ring types in Table 3.

Table 3. The K value of the four-ring clasps

C. type	Ring 1	Ring 2	Ring3	Ring 4
K value	7.7	7.4	5.46	4.74

C. type; clasp type, R1; classical ring, R2; one continuous rest ring, R3; one rest ring, R4; classical ring with a strut ends at the middle of the tooth

The same procedures may be applied for other clasps to find the RF when designing an RPD. We think this method provides a simple solution to select the clasp according to its RF and consequently secure nondestructive force to the abutment teeth while preventing dislodgement of RPD during function. Therefore, iatrogenic forces on periodontal ligaments are reduced to a minimum limit.

The size of the retentive arm was mathematically calculated for the four types of ring clasp and then correlated to their measured force of retention using Pearson correlation ( $R^2$ ), Table 4, and Table 5.

Table 4. The average retentive arm size and its corresponding average pulling force.

CT	N	L	MTh	MW	MCS	MPF
R1	6	13.83	1.05	1.51	34.45	17.43
R2	6	13.83	1.1	1.49	34.95	17.52
R3	6	13.91	1.05	1.5	34.43	12.35
R4	6	13.75	1.07	1.5	34.68	11.15

CT; clasp type, N; number, MTh; mean retentive arm thickness, MW; mean retentive arm width, MCS; mean clasp size, MPF; mean pulling force, R1; classical ring, R2; one continuous rest ring, R3; one rest ring, R4; classical ring with a strut ends at the middle of the tooth

### 3.4 Correlation between Retentive Arm Size and Pulling Force

Pearson correlation was used to estimate the linear correlation between the retentive arm size and the pulling force used to force it out of the tooth. The result was a negative weak correlation value ( $r = -0.2985$ ,  $DF = 46$ ,  $p = .03967$ ) between the two variables. Therefore, no further investigation regarding the linear regression analysis was performed.

### 3.5 The Difference between the Actual and the Calculated Pullout Force of Ring Clasps

Using the developed formula, the retentive force of the various ring clasps was determined, and a t-test was used to compare the actual results obtained by using UTM. The outcome of the difference was not significant at  $p < .05$  ( $t\text{-value} = -0.14572$ ,  $p\text{-value} = .442391$ ) (Table 5, Figure 10).

Table 5. The averages of true and calculated pulling-out forces

PF	True F	Calculated F
Average	14.61	14.75
St.D	3.714	2.909
No.	24	24
$t\text{-value} = -0.14572$ , $p\text{-value} = .442391$ , $p < .05$		

PF; pulling out force, True F; true force, Calculated F; calculated force, St.D; standard deviation, No.; number

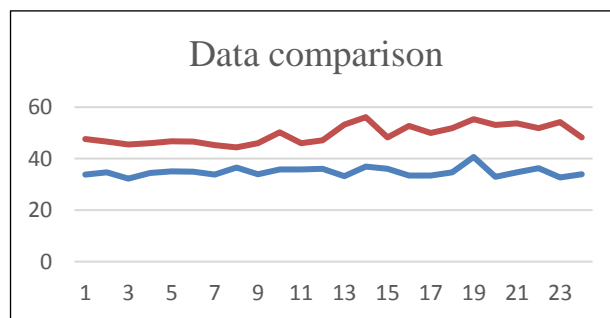


Fig. 10: The comparison of true records (red color) with calculated ones (blue color)

### 3.6 Discussion

RPD is an affordable prosthetic replacement of lost teeth and still constitutes a main treatment in partially edentulous patients; hence, dental implants are still considered an expensive and aggressive treatment. Therefore, enhancing the materials, design, and fabrication should continue actively, [7]. The mathematical approach in dentistry was used in many situations to predict and find certain values

like the setting of the teeth on the edentulous arches, [8], the location of the hinge axis of the mandible, [9], the amount of bone resorption, the vertical dimension in edentulous patients, [10], and the face form, [11]. The current work is another example.

Some critics may arise in this experiment regarding the standardization of clasp fabrication and retentive force recording. The perfect reproduction of the clasp sample is impracticable with the current technology of casting in dentistry; therefore, the size and retentive forces may vary. This is a true phenomenon when manual manufacturing is used, as in this situation. As a result, the values might vary accordingly. However, if small changes in the initial data for the differential equation produce correspondingly small changes in the subsequent approximations, the equation stability can be acceptable and the output variability is minor. In addition, the RPD resides inside a variable environment with biological structures and products that modify the physical and mechanical quality of the prosthesis instantaneously and continuously. Even though other methods may be used to study the features of stress distribution in the constructed components, like photoelastic resin models, model simulation, and infinite element analysis, the time and simplicity of procedures are always a major requirement and concern in applied dental practice, [12]. However, the current essay is a simple way to help dentists and prosthodontists design the RPD metal frame as an alternative to the use of sophisticated instruments. The resulting formula is integrated with the RPD design software to enhance its automation features, [3].

#### 4 Conclusion

The retentive or pulling force of four ring clasp types was calculated using a derived equation. The constant factor (K) was estimated for each type of clasp and a formula was derived;

$$RF = K \cdot W \cdot Th \cdot A / L$$

No statistical difference was estimated between the actual and the calculated retentive forces using the formula.

Another finding was the presence of a weak negative correlation between the retentive arm size and the pulling force.

New clasps will be studied and analyzed, and the force prediction will be counted using the formulae that will be displayed in a future study. The automated software used to design RPDs will be linked to the generated formulas. As a result, the treatment procedures will be more precise, fast, and

standardized by the dental community (students and dentists).

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### **Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)**

Laith Al-samawi wrote the manuscript, planned the test and prepared the sample and data for the experiment.

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