Optimal Design of Electric Motorcycle Tubular Frame using Topology Optimization

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Abstract: - This paper proposes a methodology for designing motorcycle tubular frames using simulation software such as MATLAB/Simulink and ANSYS, which provides an efficient and cost-effective way to approximate loads acting on the structure and topology optimization to meet performance and safety requirements. Using these tools, the design process can be simplified and reduce the number of costly physical prototypes and tests. The multi-body model developed in MATLAB® Simscape was used to approximate the loads and boundary conditions on the frame, while the ANSYS software was used for topology optimization. The resulting motorcycle frame was found to weigh 9.48 kg. The simulation results also showed that the proposed frame design met the required safety and performance criteria. The methodology presented in this paper is not limited to electric motorcycle tubular frames and can be applied to other types of vehicle frames or structures. The use of simulations allows for the exploration of different design options and the identification of optimal solutions with minimal cost and effort. The combination of MATLAB® Simulink and ANSYS is a powerful tool for the design and optimization of complex structures, providing accurate results and saving valuable time and resources.

Key-Words: - Electric Vehicle, Motorcycle, Single-Track Vehicle, Frame, Finite Element Analysis, Topology Optimization, Design Space, Non - Design Space, Limiting Conditions

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1 Introduction

The growing emission of air pollutants is increasingly affecting global trends. Products and services that do not negatively affect ecosystems or limit this impact are perceived positively by recipients. Due to good public opinion, you can see a positive effect on the sale of such goods, while legislators in many countries offer legal and tax relief for entities using them. As a result, many industries are looking for new technologies through which environmental degradation can be minimized. The transport sector is no exception in this case, as shown by the research and forecasts of the International Energy Agency. According to the Electric Vehicle Initiative (EVI), a multi-governmental policy forum created in 2010 under the Ministry of Clean Energy Program (CEM), more than 26 million electric cars were on the road in 2022, up to 60% relative to 2021 about three times more than in 2020 and more than four times in 2019, [1], [2].

Electromobility trends also apply to two-wheelers, which, thanks to zero-emission drives with energy recovery systems, are perfect for both urban and rural areas. Therefore, the average range of electric motorcycles, which is around 100 km, guarantees, with a safe margin, an ecological alternative in the private transport sector. Low noise emissions and zero pollutant emissions increase the comfort of living in urban and rural areas. The growing demand for the product results in the development of the industry and the innovation of the solutions offered. The successes of racing series such as the ABB FIA Formula E or MotoE World Cup, which are a testing ground for solutions in the field of electric drives, show the interest in the technology of both the public and manufacturers developing the products offered. The new niche created in this way allowed newly established manufacturers, such as Tesla Inc. to quickly stabilize their position on the market. or Zero Motorcycles, which confirms the possibility of entering the market with a proprietary product.

According to, [3], [4], the construction of the motorcycle frame plays a key role in ensuring the optimal performance and safety of the vehicle. The frame of all single-track vehicles is a kind of skeleton to which other components are mounted. Its main tasks are to ensure the required rigidity and strength of the entire vehicle structure, as well as shock and impact resistance to the vehicle. Due to this, it protects the driver and the most important elements of the vehicle. In, [5], the authors discussed studies that aim to centralize the load and reduce the load on the frame. This has been achieved by optimizing the weight distribution of the frame so that its center of gravity is below the rider's path. Also, in the article by, [6], a two-step process of designing a motorcycle frame is described. The authors showed how important the design process was to join with a numerical analysis of stereo mechanicals to obtain the optimal mass of the construction; a simultaneous fulfillment of the strength conditions was observed. In, [7], the authors focus on the chassis of an electric motorcycle. They analyzed both the different types of frames used for the motorcycle chassis and the different materials. Then they performed static analysis, modal analysis, side impact, and front impact analysis in CAE software for different loading conditions. They also showed that the use of topological optimization to minimize mass increases the efficiency of the entire construction process.

Optimization involves organizing activities and processes in a way that maximizes the effects while minimizing costs. Optimization has long been successfully applied in industry, increasing the efficiency of technological processes, and improving the quality of products. Using computer methods, optimization finds ever-widening applications in the design phase, allowing designers to improve the mechanical parameters of structures, reduce weight, and decrease project volume, resulting in more efficient use of resources. Before the use of computers, the optimization possibilities were limited, and finding the best solution often involved minimizing weight by reducing cross-sectional dimensions, using materials with better parameters, and limiting shape optimization.

With the progress of computing techniques, topology optimization has become a powerful tool for designing optimal structures that meet the criteria adopted by the decision-maker. In, [8], the authors presented the process of formulating an optimization task as a topology optimization model considering material strength, structural stiffness, and structure stability. On the other hand, the research by The study, [9], concerns the application of topology optimization to minimize the mass of various structures. Topology optimization in engineering applications most often consists of reducing the weight of the structure, considering the strength of the material, the stiffness of the structural structure, and the stability of the entire structure. This type of optimization combined with the design process significantly reduces costs and eliminates design failures. In the case of vehicle design, the use of topological optimization results in a weight reduction and increased stiffness of the structure, with positive effects on many other aspects, such as reduced energy consumption, improved traction, and mechanical durability, as well as improved dynamic properties. Therefore, it is desirable to subject designs to optimization processes that lead to products with better parameters. In, [10], the authors describe their research that involves a comparative analysis of three types of frames: a conventional lattice frame, a generative frame, and a topology optimization one. In, [11], the authors presented the formulation of the quad bike frame topology optimization problem, also discussing the specific properties of the design space and the specific cases of its loads. Typical design and manufacturing framework that involves the combination of topology optimization and 3D printing are presented in, [12]. The authors discussed this process in the example of a scooter frame.

This study aims to expedite the design process of an electric motorcycle frame by harnessing the potential of topology optimization. Traditional design methods for vehicle frames can be time-consuming and labor-intensive, often requiring numerous iterations and physical prototypes to achieve an optimal design. In contrast, topology optimization offers a more efficient approach by automating the search for the most effective material distribution within the frame. By utilizing topology optimization in the design of the electric motorcycle frame, the study seeks to reduce the number of trial-and-error design cycles and accelerate the identification of an optimal structure. The software-driven optimization process analyzes the load distribution, boundary conditions, and performance objectives, aiming to find the most efficient material arrangement that meets safety, performance, and weight requirements. The study endeavors to demonstrate the effectiveness of topology optimization in streamlining the design process while maintaining or even enhancing the structural integrity and mechanical performance of the frame. By identifying the ideal material layout, it becomes possible to create lightweight and mechanically robust frames, leading to improved energy efficiency and overall performance of the electric motorcycle. Through a combination of numerical simulations and experimental testing, the performance of the optimized frame was evaluated and compared to conventional motorcycle frame designs. The results of this research can significantly accelerate the process of designing motorcycle frames and provide information on the possibilities of using topology optimization in other engineering applications.

2 Formulation of the Topology Optimization Problem

In, [13], the authors discussed the theory concerning both the idea of topology optimization and various methods for the optimal design of topology, shape, and material for mechanical structures. This book also describes many practical applications of topology optimization to optimize the size and shape of mechanical components.

Topology optimization is a process whose task is to optimize the shape of a mechanical structure in a predefined 2D or 3D, [14], [15], [16], geometrical design space (domain Ω in $\mathbb{R}^2$ or $\mathbb{R}^3$), with boundary conditions imposed by the designer. The optimized structure is subjected to external loads and must satisfy many conditions defined by the designer, such as strength, stiffness, and stability. The main idea of this optimization method is the percentage distribution of the initial mass of the optimized structure in a predefined design space/domain such that a global measure takes a minimum. Topology optimization is a numerical method that involves modifying the shape (topology) of a designed part in such a way that areas that do not transfer loads are removed.

Two groups can be distinguished in topology optimization:

- General shape optimization is applied in the case of details with a specified working area, in which, during the optimization process, areas filled with material that transfer loads are separated from areas that do not transfer loads and are therefore being emptied.
- System optimization is used in the case of beam structures, where the solution consists of the truss system and cross-sections of individual beams.

Conventional topology optimization uses finite element analysis to evaluate design performance and create a mechanical structure that meets the imposed conditions. The predefined geometrical design space is also discretized using the finite element method, which allows to representation of the distribution of the structure material and simulates its deformations under the influence of applied loads. A discussion of the different methodologies used by many researchers in topological optimization can be found in, [17].

The subject of the optimization presented in the article is the motorcycle tabular frame, which we treat as a solid body. The deformation of such a body model under certain load conditions can be represented using the linear elasticity mathematical notation. To apply the linear theory of elasticity, [18], [19], small deformations (or strains) and linear relationships between the stress and strain components are assumed. The equations for the linear elastic boundary value problem are based on the equilibrium equations, stress-strain relations, and stress-displacement relations.

The equilibrium equations have the form:

$$\sigma_{ij,j} + f_i = 0 \quad \text{in } V \tag{1}$$

where:
σ_{ij} – the independent stress tensor,
f_i – body force,
V – the volume of the elastic body.

For linear elasticity, the stress-strain relationship takes the form:

$$\sigma_{ij} = a_{ijkl}\varepsilon_{kl} \quad \text{in } V \tag{2}$$

or

$$\varepsilon_{ij} = ab_{ijkl}\sigma_{kl} \quad \text{in } V \tag{3}$$

where:
ε_{ij} – the independent strain tensor,
a_{ijkl} – elastic constant,
b_{ijkl} – compliance constant.

The strain energy density A and complementary B we can define as:

$$A = \int_0^{\varepsilon} \sigma_{ij} d\varepsilon_{ij}, \tag{4}$$

$$B = \int_0^{\sigma} \varepsilon_{ij} ds_{ij}, \tag{5}$$

which satisfies the following energy identity:

$$A + B = \varepsilon_{ij}\sigma_{ij}. \tag{6}$$

For linear elasticity, the stress-displacement relationship takes the form:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad \text{in } S_u \tag{7}$$

where:
u_i – the displacement component,
S_u – indicates that the surface integrals are to be taken over that part of the surface only where the appropriate displacement is prescribed.

The boundary conditions for the considered optimization task can be presented as follows:

- For given surface displacement:

$$u_i = \bar{u}_i \text{ in } S_u \tag{8}$$

where:

$\bar{u}_i$ – displacement component on the boundary S_u

- For given external load on boundary surface:

$$\sigma_{ij} n_j = \bar{p}_i \text{ in } S_\sigma \tag{9}$$

where:

S_σ – indicates that the surface integrals are to be taken over that part of the surface only where the appropriate surface stress is prescribed
$\bar{p}_i$ – external force on the boundary S_σ

- the total boundary surface S equals:

$$S = S_u + S_\sigma. \tag{10}$$

The authors used the Hu-Washizu variational principle, in which, according to, [19], [20], [21], three types of variables are independent of each other, and the strain-stress relationships are variational constraints:

$$\Pi_{HW}(u_i, \varepsilon_{ij}, \sigma_{ij}) = \iiint_V \left(A - f_i u_i - \sigma_{ij}\left(\varepsilon_{ij} - \frac{1}{2}(u_{i,j} + u_{j,i})\right)\right) dV - \iint_{S_\sigma} \bar{p}_i u_i dS - \iint_{S_u} \sigma_{ij} n_j (u_i - \bar{u}_i) dS \tag{11}$$

To carry out the vulnerability (eq.11) minimization process, it is necessary to set constraints:

- The size of the available mass m_0 must satisfy the equation:

$$m_0 = \int_V \rho_h dV \tag{12}$$

where:

ρ_h- volume density V assuming homogeneity.

3 Problem Solution

To be able to perform the topological optimization of the motorcycle frame, it was necessary to start with the development of its CAD model. Figure 1 shows the assumed geometry of the supporting structure.

Fig. 1: CAD model of the motorcycle with the assumed geometry of the supporting structure

The design domain, also known as the analysis domain, is the spatial region where structural design optimization is performed. It defines the geometric space where material distribution can be modified to achieve an optimal design. The shape and size of the design area are important factors that have a significant impact on the optimization process. A well-defined design area is essential to ensure that the optimized structure resulting from it meets the performance requirements and requirements required. In topological optimization, the material distribution in the design domain is represented using continuous fields that indicate the presence of the material at each point. This is achieved by material interpolation techniques, using numerical functions (usually called design variables or density fields as discussed in eq.12) to assign values between 0 and 1 at each point in the domain. A density value of 1 corresponds to a solid, 0 to an empty space. The intermediate values indicate the presence of partial materials, allowing a gradual transition between solid and void regions. With the use of the FEA tools the continuous material is discretized, assigning the values at the nodes of the model. The design variable field acts as a key control variable in the optimization process, allowing the algorithm to shape and evolve the structural design to obtain the best solution.

3.1 Static Analysis

To optimize the topology, it is necessary to determine the state of stress in the analyzed motorcycle frame construction.

Static analysis requires the definition of material properties, discretization of the geometric model of the tested structure, which is shown in Figure 2 and the determination of boundary conditions and loads (Figure 3).

The motorcycle frame has six degrees of freedom, and the balance in a standing position while riding is maintained by the gyroscopic effect resulting from the principle of conservation of the angular momentum of rotating masses, which are the wheels of the motorcycle. However, considering the case of a simple load-bearing structure of a two-wheeler without suspension, it can be concluded that the wheel axles are the places of frame restraints, which was proved by the authors in, [22]. According to the studies, it was decided to fix the frame in the place where the front suspension and the rear swingarm were attached.

All these data are also used in the topology optimization process. The ANSYS software was used for static analysis using the finite element method.

Three types of steel were used as materials for the motorcycle frame: SSAB Docol R8, S355J2, and S460N. All steels are defined as isotropic and linear materials, meaning that their simplified stress-strain curve is a straight line. This approach is not recommended for calculations where the tested structure is subjected to heavy loads causing large deformations. However, when the expected strains are within the range of Hooke's law, this simplification has a negligible effect on the results and significantly reduces the calculation time. Material properties are listed in Table 1.

Table 1. Material properties

	SSAB Docol R8	S355J2	S460N
Density [kg/m^3]	7900	7850	7850
Young's modulus [GPa]	215	210	210
Poisson's ratio	0.3	0.29	0.29
Yield strength [MPa]	690	355	460
Tensile strength [MPa]	800	490	540

To determine the value of loads acting on the supporting structure of the motorcycle, its multi-body model was developed in MATLAB® Simscape. Table 2 and Figure 3 show the determined load values.

Table 2. The load-supporting structure of the motorcycle

The maximum force acting on the front suspension in one axis, in N	14453
The maximum force acting on the rear suspension in one axis, in N	25324
The maximum force acting on pull rod suspension in one axis, in N	39614
The maximum force acting on the rear swing arm in one axis, in N	40457

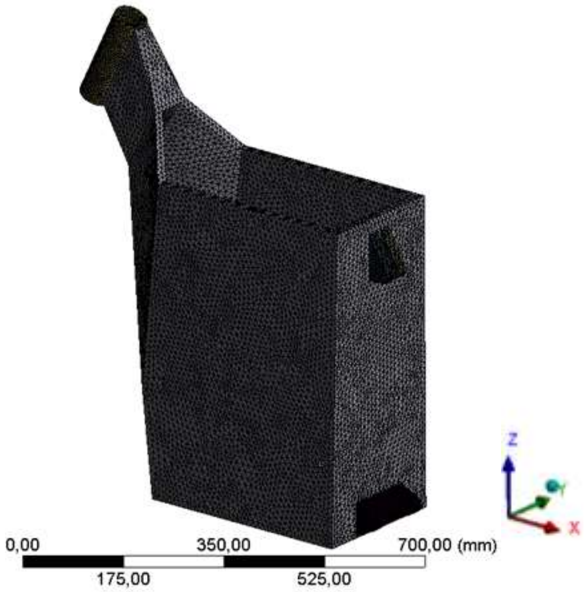

Fig. 2: Discreet model of the motorcycle frame

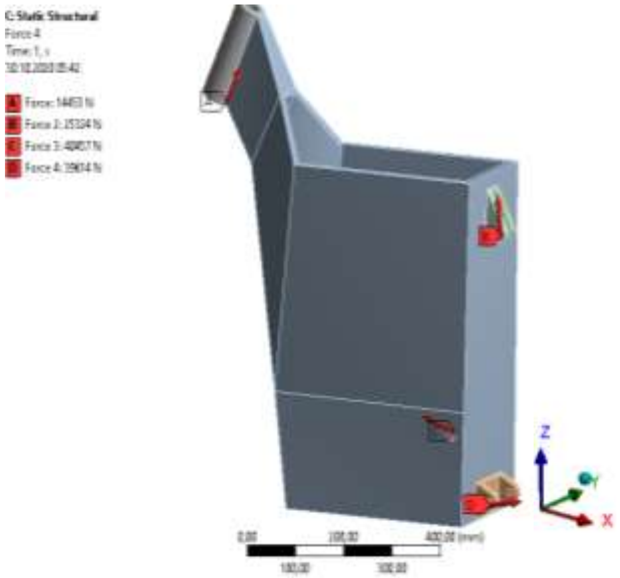

Fig. 3: Presentation of the loads acting on the model

The motorcycle frame has six degrees of freedom, and the balance in a standing position while driving is maintained by the gyroscopic effect resulting from the principle of conservation of the angular momentum of rotating masses, which are the motorcycle wheels. However, considering the case of a simple load-bearing structure of a two-wheeler without suspension, it can be assumed that the wheel axles are the places of frame restraints. Scientific publications, [23], [24], [25], describing a given problem confirm the validity of this assumption. The only difference in the considerations of individual scientists is the number of received degrees of freedom in the places where the model is fixed. Based on the literature review, it was decided to fasten in place of frontal suspension and the independent suspension arm.

During the static analysis, two variants of the restraint were considered:

– The variant I- takes away all degrees of freedom.
– The Variant II - the rear control arm mounting has one degree of freedom allowing for its rotation around its axis. In both variants, the front suspension fixing point has two degrees of freedom allowing for rotation around and along its axis.

After defining the boundary conditions and determining the loads acting on the motorcycle frame structure, static analysis was carried out in the ANSYS Mechanical program. Figure 4 and Figure 5 are the stress diagram, the total deformation diagram under the load condition, and the I variant of the boundary conditions. The same diagrams for the II variant of the boundary conditions are shown in Figure 6 and Figure 7.

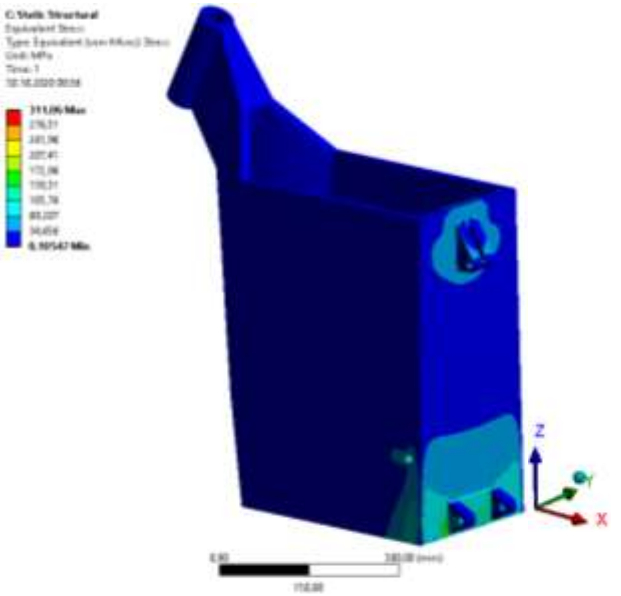

Fig. 4: Equivalent Huber-Mises stress diagram – the I variant of the boundary condition.

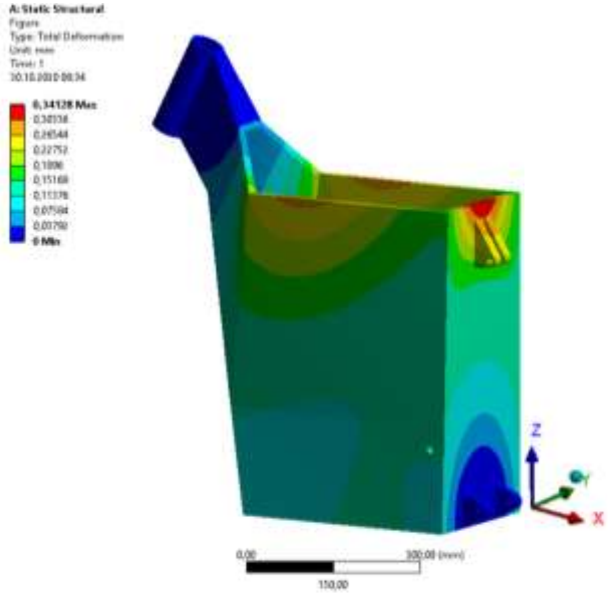

Fig. 5: Total deformation diagram – the I variant of the boundary condition.

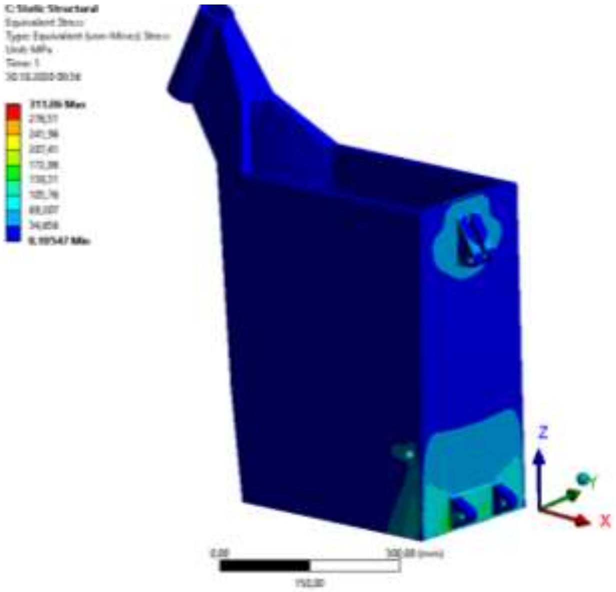

Fig. 6: Equivalent Huber-Mises stress diagram – the II variant of the boundary condition.

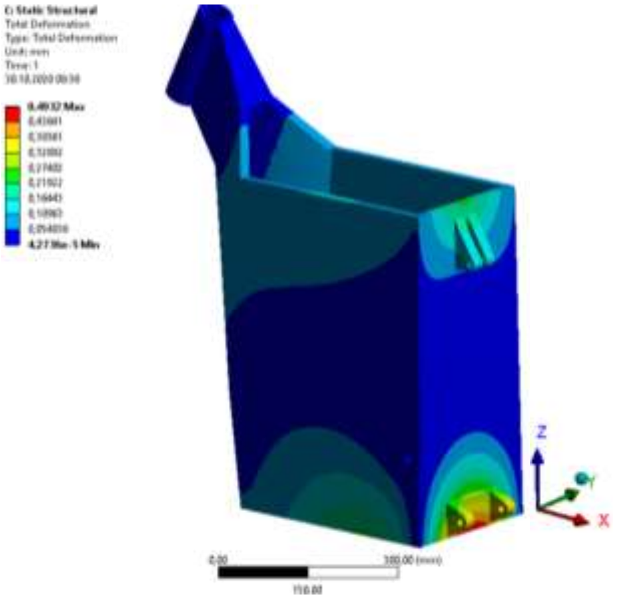

Fig. 7: Total deformation diagram – the II variant of the boundary condition

Comparing the results obtained for both variants of boundary conditions, it can be seen that the maximum value of equivalent stress decreased by 7% for the II variant, while the maximum value of total deformation for this variant increased by as much as 45% compared to the I variant.

3.2 Topology Optimization

In the context of ANSYS software, topology optimization is a powerful engineering tool that leverages the capabilities of the software to optimize the material distribution within a given design domain. ANSYS provides a comprehensive suite of tools and algorithms to perform topology optimization efficiently and accurately. The process begins by importing the geometry of the structure into ANSYS and defining the design domain and relevant boundary conditions. Users can specify loading conditions, constraints, and design objectives, such as maximizing stiffness, minimizing weight, or optimizing for other performance criteria. ANSYS employs advanced optimization algorithms, like the SIMP (Solid Isotropic Material with Penalization) method, to iteratively redistribute material within the design domain. This iterative process gradually removes less essential material while preserving critical load-bearing elements. The material density of the design domain is updated in each iteration, and stress analyses are performed to assess structural performance.

Before commencing topology optimization, it is essential to define its parameters, which include design space and non-design space, objective function, and limiting conditions. In this study, non-design space was defined by surfaces where boundary conditions and loads were set, representing the parts of the frame that need to meet the specific dimensions and shape to accommodate suspension and bearings. The limiting condition was defined as the final mass limit of 15% of the initial value, which was 110.15 kg. The objective function was to achieve the consistency of the results of the static analysis after optimization with the results before optimization. By default, the software defines the objective function to minimize compliance and, therefore maximize the rigidity of the structure. Since the manufacturing methods were not a limiting factor in this particular case there were no additional conditions considered. Those can play a major role in the final shape of many parts that are made by milling for example.

The resulting geometry of the tubular frame is shown in Figure 8. Plots of the objective function and constraint values during each iteration of the optimization process are shown in Figure 9. The method utilized a convergence criterion-based objective function (purple line in the upper plot) which assumed a certain ratio for the initial and final model energy during analysis. For the present case, the program determined the final objective function value to be at the maximum level of 0.15843, with a value of 0.0316 obtained after the last iteration (blue line in the upper plot). The constraint was a maximum of 15% of the initial mass (blue line in the lower plot).

Topology optimization has been carried out by using ANSYS.

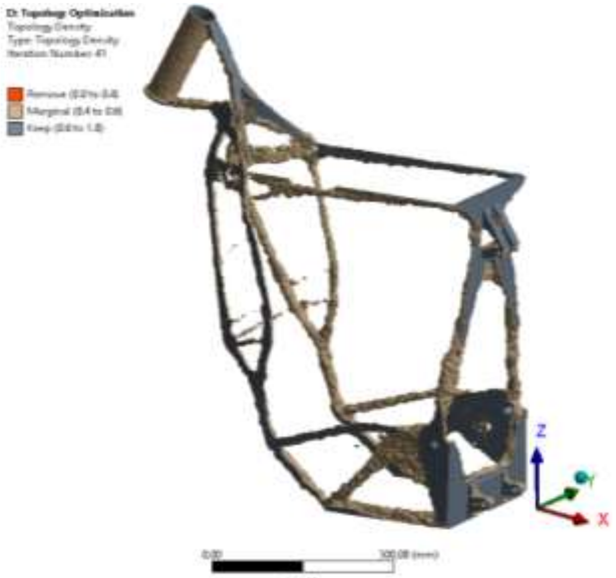

Fig. 8: Results geometry of the tubular frame

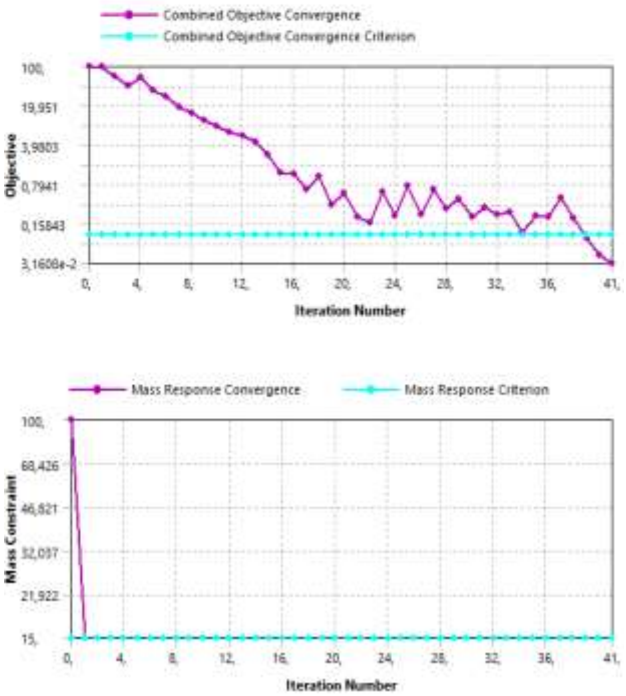

Fig. 9: The plots of the objective function and the constraint limit for the iterations of the optimization process

Fig. 10: Tubular frame geometry based on topology optimization result

The obtained geometry was then imported into CAD software, where its shape was replicated while ensuring the manufacturability of the structure. The resulting tubular frame geometry is shown in Figure 10, while the comparison of both geometries is shown in Figure 11.

The optimized construction of the motorcycle tubular frame weighs 9.48 kg. To verify its strength, a static analysis was carried out. The boundary conditions and loads were the same as those in Table 2, as shown in Figure 12.

The results of the numerical simulations carried out are shown in Figure 13 and Figure 14.

Fig. 11: Comparison of the structure design with the result of topology optimization

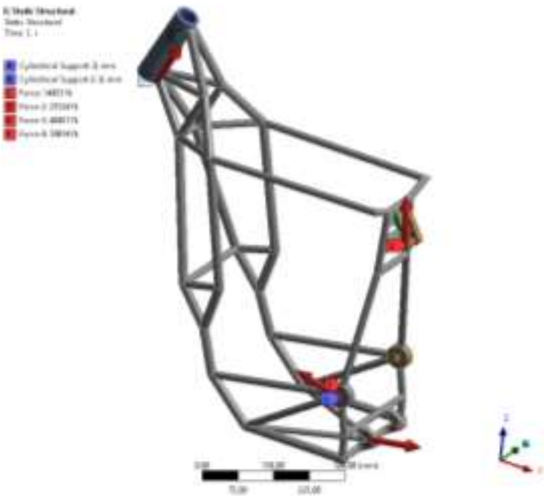

Fig. 12: The boundary conditions and loads acting on the model

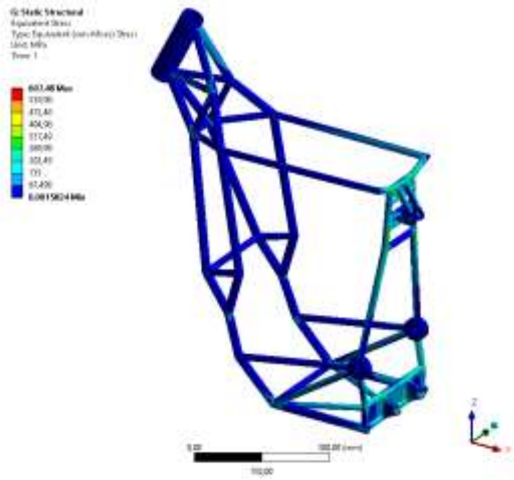

Fig. 13: Equivalent Huber-Mises stress diagram

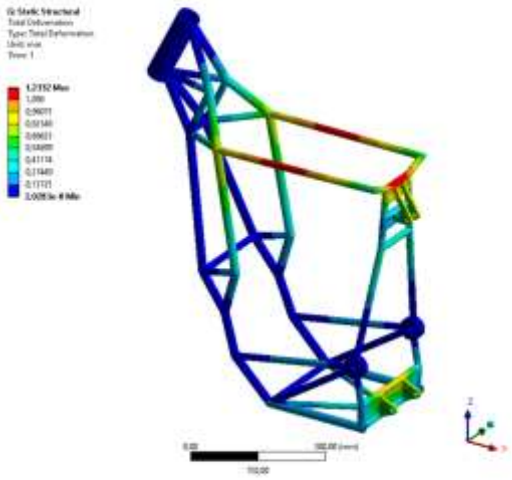

Fig. 14: Total deformation diagram

Based on the obtained results, it can be concluded that the optimization goal has been achieved. Compatibility of the results of the static analysis of the frame structure after optimization with the results before optimization was achieved. Limiting conditions were also met - the weight of the optimized structure of the load-bearing frame does not exceed 15% of the initial weight. It is 9.48kg which is 8.6% of the initial weight.

4 Conclusion

The proposed methodology presented in this paper allows for cost and time savings, provided that sufficient computational power is available. The use of MATLAB Simulink software for early approximation of loads acting on structures enables the quick acquisition of realistic results. The results obtained in this way make it possible to reduce costly tests and prototypes and enable an early application of topology optimization in the design phase. After defining the workspace, the majority of project iterations are performed by computer software, which, with access to adequate computational power, allows for obtaining geometry that closely approximates the final design in a short time. Furthermore, with an appropriate formulation of optimization parameters, it enables the preservation of safety coefficients, structural stiffness, and required mass right from the beginning. This ensures that the proposed design is optimally feasible within the given constraints.

In the examined case, a successful optimization process resulted in a load-bearing structure design with a mass of less than ten kilograms. For comparison purposes, the first frame design created by the authors using comparable materials for a two-wheeler with similar dimensions weighed 12.5 kg in its final version. This demonstrates the possibilities arising from design using optimization algorithms, which are increasingly applied in the industry.

The presented methodology of the procedures illustrates a scheme for conducting design work. The examined case of optimization of the motorcycle tubular frame can be expanded with additional simulations imitating component usage, which would provide further information for optimization. The topology optimization process itself should consist of as many iterations as possible. Based on the experience gained during the research, the authors also conclude that it is essential to reproduce the obtained geometry faithfully. Otherwise, there is a risk of increased stresses occurring in the structure.

The clear limitation of the usage of topology optimization during the design process is the need to design the part manually even after optimization. Despite the possibility of inclusion of manufacturing constraints, the commercial software is still not capable of creating a simple and easy-to-manufacture shape of a structure. In some cases, topology optimization may produce designs with intricate material distributions that are difficult to interpret or manufacture. While these designs can be highly efficient in terms of performance metrics, they may not be practical for real-world implementation.

The improvement seen by authors in the researched example, as well as future direction would be adding parametric optimization on top of the results of the topology optimization. With a definition of commercially available circular steel tubes, the whole structure would be optimized to an even greater extent. The algorithms responsible for the topology optimization in ANSYS software are also a subject of constant development, so the latest software should be monitored for new possibilities during the whole process.

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Conflict of Interest

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