Study of the Ground Reaction Force Induced by Swaying or Bobbing People and its Dynamic Effect on Loaded Structures

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Abstract: - The core of the submitted article is in the modeling of bobbing people for describing the action of qualified vandals, who try to achieve an excessive level of vibration by their periodic sway in the knees while they are not losing contact between the footbridge deck and their feet. The DLF models, which already exist, provide particular coefficients only for specific pacing frequencies. On the other hand, our study presents DLF coefficients as a continuous function for frequencies in the range of 1 Hz – 3 Hz. The newly presented DLF model is based on the measurement of 15 random people and compared with the experimental data. Each of these people was measured by a force plate in the frequency range of 1 Hz – 3 Hz. Since we know who exactly was present during the experiment, we also monitored the contact forces produced by these people at frequencies identical to some natural frequencies of the footbridge according to the experimental setup. These measured forces were used directly as the input into the calculation process and compared with the experiment too. Subsequent dynamic calculations of the forced vibration were carried out by Modal Decomposition Method. This method requires a mode shape as one of the inputs, these mode shapes were calculated by the Subspace Iteration Method using commercial software Dlubal RFEM 5.03. Numerical integration of the equations of motion (forced vibration analysis) was done by self-written MATLAB codes and routines. At the end of the article, we summarize the results of theoretical dynamic analysis obtained by theoretical modeling of these vandals. The main outcomes are in the determination of the continuous functions for DLFs and their phase angles based on the experimental results. These values are crucial e.g. for designers, who need to compute the response of a footbridge or a grandstand which could be excited by swaying or bobbing vandals or spectators. Obtained and evaluated continuous functions for DLFs were compared by literature where researchers presented some DLFs for discrete sets of frequencies, which produced a good level of accordance.

Key-Words: - Footbridge dynamics, Structural dynamics, Human induced vibration, DLF model for bobbing, Qualified vandalism, Experimental dynamics

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1 Introduction

Since the accident events connected with the opening day of the Millennium Bridge in London in 2000, structural engineers have developed many advanced procedures for modeling the human-induced vibration with or without direct interaction with the vibrating structure to prevent excessive vibrations of the structure originating from the resonance behavior among the dynamic forces produced by pedestrians and vandals and particular natural frequency of the footbridge associated with some global structural mode shape. Between these approaches, we can name for instance the Dynamic Load Factor technique (DLF) based on the approximation of the Fourier Series and approximation based on the biodynamic models (1DOF or MDOF) or some advanced kinematic models, [1], [2]. While the core of DLF techniques lies in the direct modeling of Ground Reaction Force (GRF), the biodynamic and kinematic models allow us to consider the direct interaction, which means that the vibrating structure affects the trajectory of the pedestrian's center of mass and vice versa.

DLF models, which are based on the theory of the Fourier Series, were provided by a huge number of authors. One of the well-known and most quoted masterpieces can be found in [3], [4]. The author also provided the coefficients for various rhythmical activities such as walking, skipping, jumping, dancing, and hand clapping. These models describe the human (pedestrian/runner/vandal) body as a mass point placed in the human's body centroid. In [5], the authors provided a DLF model with equivalent coefficients α_i for loads induced by a jumping crowd. An overview of coefficients α_i for vertical walking force defined by many authors can be seen in [6]. While some investigators supplied these coefficients for exact frequency value (e.g., [7], [8], [9]), other researchers (e.g., [10], [11], [12], [13], [14]) offered continuous functions for each harmonic member. The number of harmonic components differs for each author.

More advanced models, called GRF procedures, enable to model independently each foot of the pedestrian. Time behavior GRFs based on real measurements can be found e.g., in [15], [16], [17]. The biggest difference can be seen in the time behavior of the loading. While DLF factors change the point of action in each time step, the GRF models remain at the same point for the time instance, which is adequate for the stance phase of an individual foot. The solution of the oscillating beam induced by periodic movable force is quite a demanding procedure. Results of such type of excitation can be found in [18], where the author solved relevant partial differential equations of motion by Laplace Transform and provided the time behavior of deflection in arbitrary section x in the closed form.

2 Description of the Investigated Footbridge

The footbridge, which was subjected to the experimental and theoretical investigation of the response is located in Lužec nad Vltavou (Czech Republic) at a distance of 35 km from Prague in Central Bohemia, see Figure 1 and Figure 2. Since 1907, when a lateral canal was built between the municipalities of Hořín and Vraňany, it has been the only municipality in the Czech Republic whose entire territory lies on an island. The island is also the largest island on the Vltava River. This structure connects two banks of the Vltava River between villages Lužec nad Vltavou and Bukol over the impassable course of the river and serves pedestrians and cyclists as a part of the cycling routes CT 7 and EuroVelo 7, which leads from Sweden to Sicily. It was designed as a cable-stayed continuous bridge with spans of 99.18 + 39.9 m, see Figure 3 and Figure 4.

The load-bearing structure was made of UHPFRC (ultra-high performance fiber reinforced concrete) C 110/130 prefabricated segments, which were post-tensioned by tendons made of prestressing steel St 1640/1860 MPa. The seventeen pairs of Redaelli hanger cables were anchored to the steel pylon on one side and to the concrete anchor block on the side of the footbridge deck. The width of the footbridge deck is 4.5 m with a transverse inclination of 1 %. The trajectory of the grade line is guided in an elevated arc with a radius of 777 m. A new road is designed and implemented in a width arrangement defined by the width of the thoroughfare of the walking space between the frames of 3 m. The crossing angle between the axis of the road and the theoretical axis of the Vltava River flow is approximately 83˚.

Fig. 1: Location of the footbridge structure in the scope of the Czech Republic

Fig. 2: Location of the footbridge structure in the scope of the Lužec nad Vltavou – Bukol

Fig. 3: Side view of the entire structure

Fig. 4: View of the steel pylon

The location of the footbridge structure can be seen in Figure 1 and Figure 2. Basic dimensions in the longitudinal and crosswise directions are apparent from Figure 5, which depicts the longitudinal and cross sections of the structure, which was subjected to the experimental and theoretical dynamic analysis.

3 Theoretical Analysis of the Dynamic Behavior of the Footbridge

For the purpose of the theoretical modal analysis of the dynamic behavior of the footbridge, the 3D model was created. The model was created in Dlubal RFEM software. This software includes a huge number of packages appropriate for a wide spectrum of civil engineering problems and belongs to the diversified family of commercial software dealing with finite element modeling. In the Graphical User Interface (GUI), the qualified user can select from the library of elements containing beam, plate, and solid elements.

3.1 Computational 3D Model of the Footbridge

The geometry of the structure was imported from AutoCAD 2019 into Dlubal RFEM 5.30 respecting the height curvature of the footbridge grade line. For structure modeling, beam and slab elements were used. Some beams with complex cross-sections were simplified under the presumption, that the cross-sectional areas will be preserved.

The individual components of the footbridge deck were modeled as a system of longitudinal and crossbeams, which were simplified by the beam elements. This system of longitudinal and crossbeams support the concrete slab, which was modeled by the plate elements. The external prestressing cables, the pylon, and the abutments were all simulated by the beam elements. The stays were modeled by beam elements with a neglected bending stiffness. All the chosen materials in the model are in accordance with the description in the realization documentation of the structure, [19]. The structure was modeled without the stiffness parameters of the substructure, only the theoretical supports were used. It means that the real stiffness of the abutments and pillars was not projected into the theoretical model. The support on the O1 abutment was modeled as sliding support (left support in Figure 6). The support of the pylon and the O3 abutment (right side in Figure 6) were modeled as fixed supports. The prestressing of the stays was involved by including the equivalent normal forces in cables to modal analysis. The first, third, and fourth computed mode shapes may be observed in Figure 7, Figure 8, and Figure 9. The blue line (in Figure 7, Figure 8, Figure 9) denotes the footbridge deck, which was covered by the network of measured points denoted as blue points. The green points stand for the placement of the accelerometers for the forced vibration measurements. The red point denotes the location, where vandals have been placed to oscillate the structure.

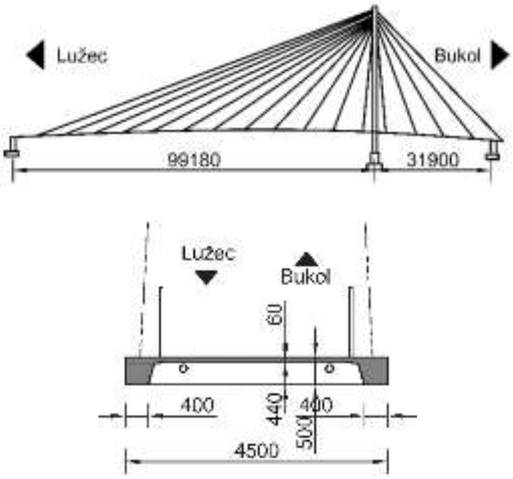

Fig. 5: Longitudinal section (top) and cross-section (bottom) of the investigated structure

Fig. 6: The 3D computational model of the investigated footbridge in Lužec nad Vltavou

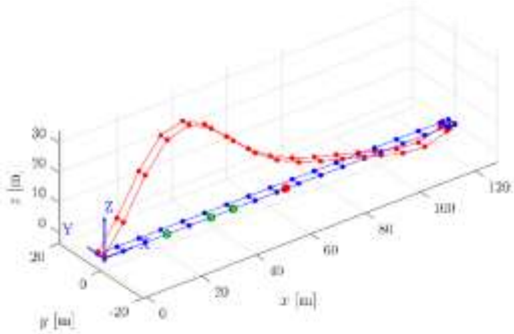

Fig. 7: An example of the 1st calculated mode shape ($f = 0.75$ Hz)

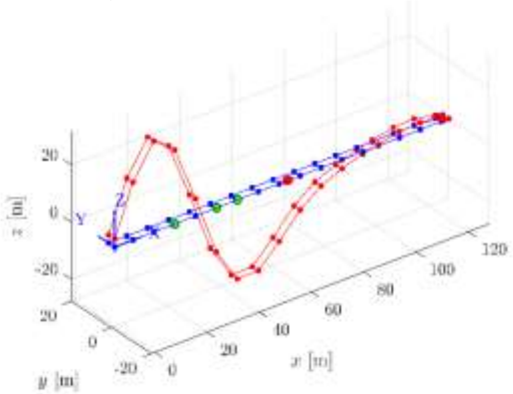

Fig. 8: An example of the 3rd calculated mode shape ($f = 1.15$ Hz)

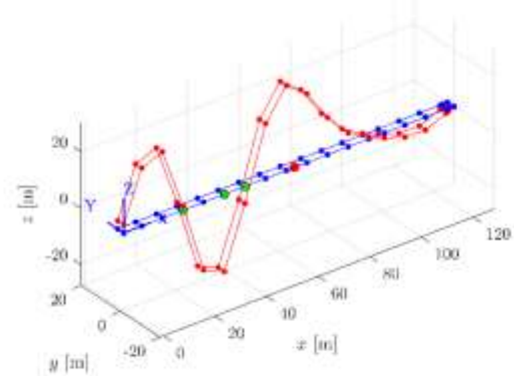

Fig. 9: An example of the 4th calculated mode shape ($f = 1.62$ Hz)

4 Qualified Vandalism

Vandalism as an independent word, which is stated without any wider context, has a very negative connotation sensed by the unqualified public. Usually, it is perceived as someone, who destroys historical objects for instance, or sprays on facades. In the scope of structural dynamics, especially dynamics of footbridges, qualified vandalism can be characterized as an intentional periodic motion of a person or a synchronous periodic motion of a group of people with the aim to achieve an abnormal excitation of the structure. The motion considered vandalism can be swaying/bobbing in the knees or jumping. Swaying/bobbing in the knees or jumping are described for dynamic analysis by the frequency, the place of the action, and the time function of the dynamic load.

Due to the combination of the slenderness and the largest span of the footbridges, the natural frequency frequently belongs to the range, which is typical and natural for forces connected with human-induced excitation such as walking, running, swaying, bobbing, and skipping. While the walking and running modeling of a standardized group of humans is required by Eurocodes for theoretical and experimental dynamic analysis, vandalism is not a part of the unitized forces, which influence the serviceability of the investigated structure even if this type of loading oftentimes reaches the greatest values of acceleration.

In the context of vandalism, we should also mention the grandstands at football or hockey stadiums, which are directly associated with this phenomenon. As we can see from the past, the mass of synchronized spectators and fans can cause an excessive level of vibration of the whole grandstand or its part or it can directly cause its collapse. For example, the large opening of cracks may be seen at the Boca Juniors stadium La Bombonera, in Bueno Aires Argentina and part of the load-bearing structure of the grandstand girder collapsed at the Goffertstadion in Nijmegen Netherlands under the bouncing crowd.

5 Theoretical Modeling of Dynamic Forces Induced by Human Activities

The most frequently used approach for dynamic forces induced by human activities is the model according to the equivalent dynamic load factor (DLF model). This type of theoretical model uses sine functions with dominant harmonics to simplify the real-time behavior of the measured GRF. A big advantage of this model is without a shadow of a doubt its simplicity and the possibility to define this model for both vertical and lateral excitation. Resolution for the vertical and lateral excitation is ensured by characterization of the DLFs typical for a specific direction. The value of these coefficients depends on both the direction of the acting force and the type of the activity (walking, running, swaying, bobbing, [20], [21], [22], bouncing [22], where DLFs discrete frequencies were estimated, etc.). In [22], the authors presented their data based on the measurement of 8 jumping, bobbing, and bouncing people (4 women and 4 men) for 1, 2, 3, and 4 Hz. Difference between bouncing and bobbing is in the fact, that during bobbing our subjects remained in the contact with the force plate while bouncing lead to the short-time contact loss. The study of a swaying/bobbing group of people, who were mathematically considered in interaction with an oscillating structure (HSDI model), was presented in [23]. To the authors' knowledge, DLF coefficients for bobbing people as a continuous function of excitation frequency have not been published yet. In [24], the researchers studied horizontal GRFs, which arise during excitation in a vertical direction induced by swaying and jumping people.

5.1 Ground Reaction Force based on the Real Measurement

In the submitted paper, we dealt with the definition of the DLF coefficients for swaying activity by measuring a group of 15 people. These subjects were rhythmically bending their knees on the force plate to produce the ground reaction force with a predefined frequency in the range from 1 Hz up to 3 Hz.

Fig. 10: Two examples of the GRF measurement (top left and top right) and a utilized force plate (bottom middle)

Figure 10 represents the two tested subjects for GRF measuring as well as the force plate, which was used during these experiments. This force plate is educational equipment from Vernier company with sizes 28 × 32 × 5 cm and a weight of almost 5 kg. The measurement range is 4500 N with declared sensitivity of 1.2 N.

The investigated footbridge was excited by several types of load states with different pacing and actuating frequency such as synchronous walkers, joggers, and vandals. Since this paper is aimed at the study of the effect of swaying and bobbing people, we were focused only on vandalism. We have measured the forces induced by the people, who were directly present at the experiment to obtain real forces based on the factual mass and frequency of each vandal. In the second stage, we carried out a statistical evaluation of the results based on the GRFs measurement of 15 people. These results were used for determining the general model, where the particular DLFs are dependent on the pacing frequency f. It connotes that we measured both, the direct GRFs at the resonant frequencies identical with the arrangement of the experiment, which were used as direct input to the calculations, and GRFs induced by vandals in the frequency range 1 Hz – 3 Hz to obtain a more general mathematical model.

$$F(t) = G \cdot \left[1 + \sum_{j=1}^{N=3} \alpha_j \sin\left(2\pi j f t + \varphi_j\right)\right] \quad (1)$$

where G denotes the static weight of the vandal, j is the natural number, f stands for excitation frequency, t is the independent parameter (time) and φ_j means the phase shift of the j-th member.

6 Dynamic Response Calculation

The dynamic response of the vibrating footbridge, loaded by the group of synchronous vandals, was calculated with the help of the commercial software Dlubal RFEM 5.03 and self-created MATLAB scripts and routines using the modal decomposition method. The main advantage of this approach is the decomposition of the equations of motion into a set of independent equations (in the case of the proportional damping model). In addition to the proportional damping model, we also assume the linear behavior of the oscillating structure, which means that nonlinear effects, such as the involvement of supports in oscillation, etc., were neglected. The commercial software Dlubal RFEM 5.03 was used for the theoretical modal analysis, where the global mode shapes and appropriate natural frequencies were calculated by Subspace Iteration Method with consistent mass matrix $[\mathbf{M}]$. The resulting mode shapes assembled into the modal matrix $[\mathbf{\Phi}]$ were normalized with respect to the mass matrix $[\mathbf{M}]$, which denotes that $[\mathbf{\Phi}]^{\boldsymbol{T}}[\mathbf{M}][\mathbf{\Phi}] = [\mathbf{E}]$, $[\mathbf{\Phi}]^{\boldsymbol{T}}[\mathbf{K}][\mathbf{\Phi}] = [\mathbf{\Omega}]^{\mathbf{2}}$, and $[\mathbf{\Phi}]^{\boldsymbol{T}}[\mathbf{C}][\mathbf{\Phi}] = 2\xi_i\omega_{0i}$, where $[\mathbf{E}]$ is the $N \times N$ unity matrix, $\mathbf{\Omega}$ is the $N \times N$ spectral matrix, ξ_i is the critical damping ratio of the i-th natural frequency and ω_{0i} denotes the i-th circular natural frequency. According to the character of the solved problem, the matrices $[\mathbf{M}]$, $[\mathbf{K}]$ (stiffness matrix) and $[\mathbf{C}]$ (damping matrix, see section 8) are real, symmetrical, and square matrices.

The spots of the acting forces were placed on the footbridge deck according to their positions during the in-situ experiment. Since the forces were not directly in the FE nodes, we had to transform these extra-nodal forces into nodal forces by the base functions for rectangular FE. This approach was included in self-programmed MATLAB routines during the calculation process. The equations of motion are described as a set of second-order ordinary differential equations with constant coefficients. These equations can be written in the matrix form as

$$[\mathbf{M}]\{\ddot{\mathbf{w}}\}_t + [\mathbf{C}]\{\dot{\mathbf{w}}\}_t + [\mathbf{K}]\{\mathbf{w}\}_t = \{\mathbf{p}\}_t \qquad (2)$$

where $[\mathbf{M}]$, $[\mathbf{C}]$, and $[\mathbf{K}]$ mean mass, damping, and stiffness matrices. $\{\ddot{\mathbf{w}}\}_t$, $\{\dot{\mathbf{w}}\}_t$, and $\{\mathbf{w}\}_t$ denote unknown acceleration, velocity, and deflection column vectors (time-dependent) and $\{\mathbf{p}\}_t$ is the right-hand side column vector of forces (time-dependent). If we apply the modal decomposition technique ($\{\mathbf{w}\}_t = [\mathbf{\Phi}]\{\mathbf{q}\}_t$) to Eq. (2), we can write equations of motion in the form

$$[\mathbf{E}]\{\ddot{\mathbf{q}}\}_t + [\mathbf{C}]_{\text{mod}}\{\dot{\mathbf{q}}\}_t + [\mathbf{\Omega}]^{\mathbf{2}}\{\mathbf{q}\}_t = [\mathbf{\Phi}]^{\boldsymbol{T}}\{\mathbf{p}\}_t \qquad (3)$$

where $\{\ddot{\mathbf{q}}\}_t$, $\{\dot{\mathbf{q}}\}_t$, and $\{\mathbf{q}\}_t$ stand for column vectors of acceleration, velocity, and deflection in the modal domain. $[\mathbf{C}]_{\text{mod}} = \begin{bmatrix} 2\xi_1\omega_{01} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & 2\xi_N\omega_{0N} \end{bmatrix}$ is the damping matrix in the modal domain and $[\mathbf{\Omega}]^{\mathbf{2}} = \begin{bmatrix} {\omega_{01}}^2 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & 0 & {\omega_{0N}}^2 \end{bmatrix}$.

After the modal decomposition, the subsequent decoupled equations of motion, Eq. (3), were solved by the implicit Newmark's integration method, see, [25]. This method must fulfill two requirements for time step Δt. We were setting the largest time step for calculations as $\Delta t = 0.03$ s.

- $\Delta t/T \leq \sqrt{3}/\pi = 0.55\text{: } 0.03/(2\pi \cdot 0.75) = 0.006 \ll 0.5$
- $\Delta t_{\text{max}} \approx \frac{T}{10} : \frac{2\pi \cdot 0.75}{10} = 0.47\ \text{s}$

where T stands for the shortest period of excitation or the shortest period of natural vibration. Finally, the original vector of unknowns was calculated by reverse transform ($\{\mathbf{q}\}_t = [\mathbf{\Phi}]^{\boldsymbol{T}}\{\mathbf{w}\}_t$).
All these procedures were carried out in the self-written MATLAB routines.

7 Experimental Part

The in-situ experiment was performed on 15th October 2021. We can divide this event into two independent parts. At first, we performed an experimental modal analysis, where the structure was excited by the effects of wind, Ambient Vibration Method (AVT). The footbridge deck was covered by a network of 19 spots at both edges of the footbridge cross-section. These points served for the placement of accelerometers, which were fixed on the steel weight unit in three independent directions (Cartesian coordinate system). The seismic accelerometers Brüel&Kjær, type 8344, were connected to the eight-channel vibration control station SIRIUSi 6ACC – 2ACC. These sensors are piezoelectric acceleration transducers a with working range of 0.2 Hz – 3 kHz and a sensitivity of approximately 2500 mV/g. Eight sensors were used during measurement in one profile, three of them at each side of the footbridge cross-section and two as reference. The fixed reference sensors were located in the spot with non-zero coordinates of all presumed global mode shapes.

Last but not least, the footbridge was subjected to the dynamic load test, where a group of synchronized vandals and pedestrians excited this structure to achieve its excessive vibration. The response was measured in three spots, labeled 72, 112, and 132. For example, the number 72 denotes the seventh profile on the right side of the footbridge deck in the stationing direction, see Figure 11.

8 Results

In this section, we present the results, which were achieved during the solution of the modal analysis and forced vibration analysis. Relevant results were compared with experimental data.

Tab. 1 refers to the measured and calculated frequencies, the description of the appropriate global mode shape, and the difference Δ calculated as $\Delta_j = \left(f_j - \widetilde{f_j}\right)/f_j \cdot 100$ % according to [26].

The evaluation of the bobbing/swaying group of people led us to the following equations, which describe the DLF coefficients $\alpha_j(f)$ for the first three members of the series described by Eq. 1. These relations were derived based on the Least Squares Method.

$$\alpha_1(f) = -0.087 \cdot f^2 + 0.632 \cdot f - 0.409 \tag{4}$$
$$\alpha_2(f) = -0.021 \cdot f^2 + 0.055 \cdot f + 0.162 \tag{5}$$
$$\alpha_3(f) = 0.016 \cdot f^2 - 0.086 \cdot f + 0.141 \tag{6}$$

where f is the frequency of bobbing or swaying. The relevant phase angles φ_j can be computed from the relations

$$\varphi_1(f) = 0 \tag{7}$$
$$\varphi_2(f) = -0.65 \cdot f^3 + 4.51 \cdot f^2 - 10.24 +5.88 \tag{8}$$
$$\varphi_3(f) = -0.41 \cdot f^2 + 1.15 \cdot f - 1.83 \tag{9}$$

The previous equations are valid and derived as the continuous functions for the frequency range $f = <1;3>$ Hz.

Figure 12 and Figure 13 present the comparison of α_1, α_2, and α_3 with the ones obtained from the continuous functions $\alpha_1(f)$, $\alpha_2(f)$, and $\alpha_3(f)$ defined by Eq. 4 – Eq. 6 for 2 Hz and 3 Hz. One can see a reasonable level of agreement between our and McDonald's study.

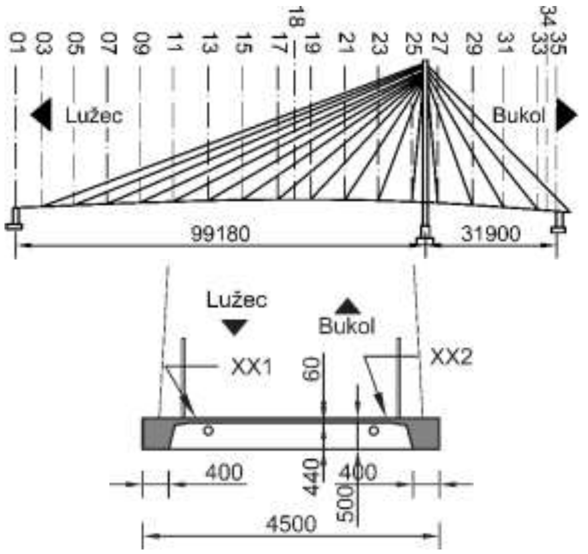

Fig. 11: Longitudinal (top) and cross-section (bottom) of the investigated structure

In addition to the measured natural frequencies, the critical damping ratios $\xi(f)$ were evaluated as results of the experimental modal analysis, see Figure 14. In this figure, we can see the experimentally obtained data (light blue circles), derived analytical curve based on the modeling of the damping matrix $\mathbf{C}$ as mass-proportional $\mathbf{C} = \alpha\,\mathbf{M}$ (blue dot-and-dash line) and optimized curve based on the Least Squares Method (red dashed line).

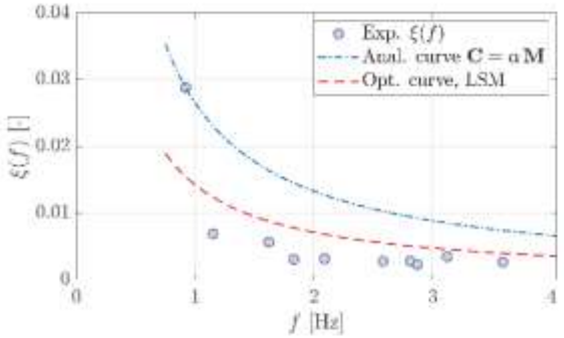

Fig. 12: Behavior of the critical damping ratio $\xi(f)$

The directly evaluated values of ξ from the in-situ measurements were used as the input to the calculation process because another two methods provided greater values of ξ and therefore inaccurate values of the response.

The next target of this paper was to provide a DLF model for bobbing/swaying modeling in the dependency on the frequency of the excitation. This model is valid only in the 1–3 Hz range and will be updated based on the larger group of tested subjects. We provide these values based on the measurement of the GRF of 15 people so far.

An example of the calculated response (acceleration) is depicted in Figure 15 as well as the computed time behavior of the RMS value. This value has been determined for 1 s intervals. According to Figure 16 and Figure 17, we can state that the directly measured force (of the people, who were directly present on the footbridge during the vandalism) provides rationally better results than the DLF force, where the coefficients were determined by their mean value. The biggest difference between the experimental and theoretical data was for the frequency 1.62 Hz (spot 72, see Figure 16), where the theoretical model has a significantly lower ordinate of the mode shape than the experimentally determined one. Other theoretical RMS acceleration values predicted higher values than the experiment showed, which is on the safe side for the footbridge design stage in terms of pedestrian comfort. The first nine measured and computed frequencies are presented in Table 1.

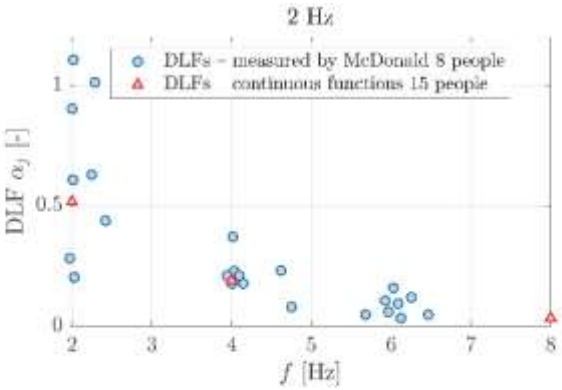

Fig. 13: Comparison of the α_{1-3} measured data provided by [22], for discrete frequency 2 Hz, with values evaluated from Eq. 4–Eq. 6

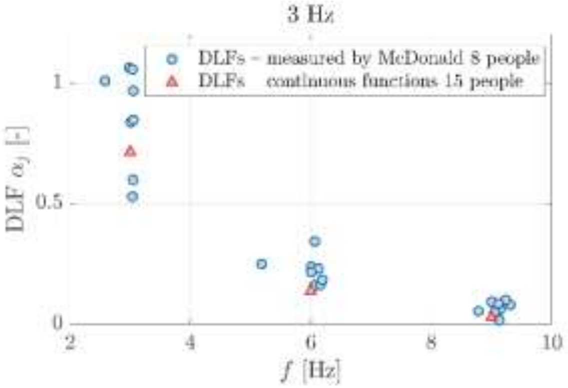

Fig. 14: Comparison of the α_{1-3} measured data provided by [22], for discrete frequency 3 Hz, with values evaluated from Eq. 4–Eq. 6

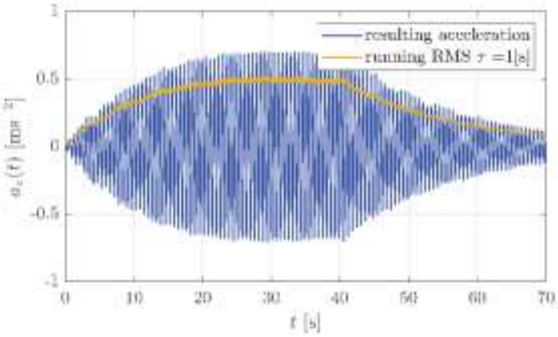

Fig. 15: Example of the calculated response (acceleration)

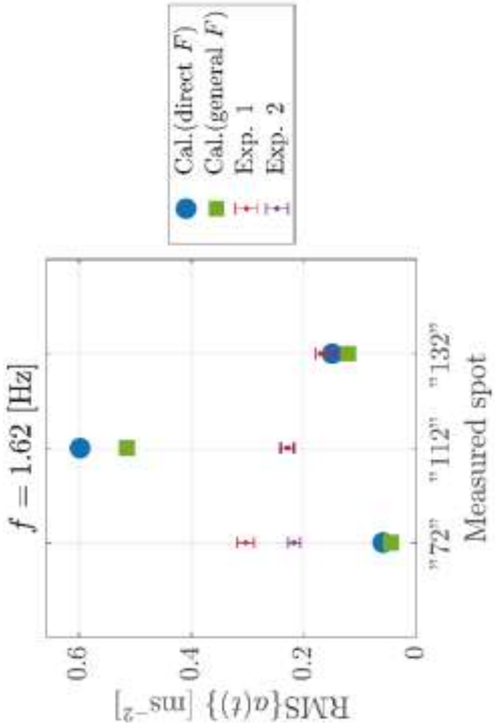

Fig. 16: Comparison of the theoretical and experimental RMS values of acceleration – excitation frequency $f = 1.62$ Hz

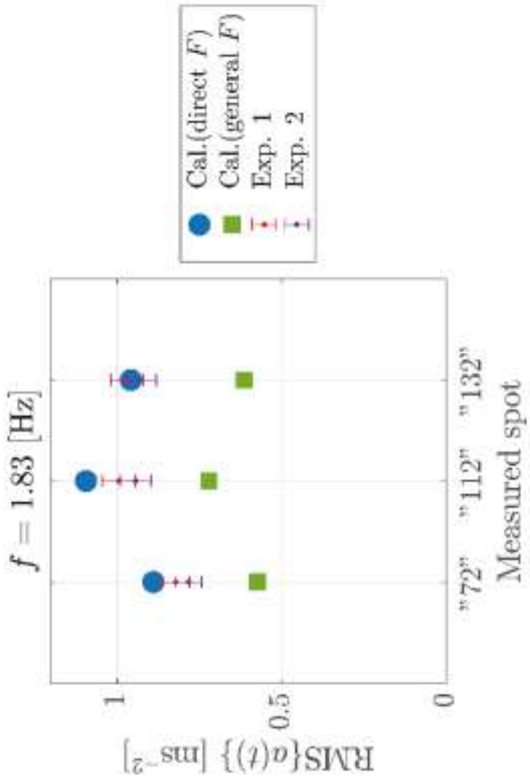

Fig. 17: Comparison of the theoretical and experimental RMS values of acceleration – excitation frequency $f = 1.83$ Hz

Table 1. The first nine measured and computed frequencies

# fr.	**Measured** [Hz]	**Computed** [Hz]	**Δ** [%]	**Description**
$f(1)$	0.72	0.75	4.0	Bending vertical
$f(2)$	0.92	0.92	0	Bending lateral
$f(3)$	1.19	1.15	-3.5	Bending vertical
$f(4)$	1.73	1.62	-6.8	Bending vertical
$f(5)$	2.11	1.83	-15.3	Torsional
$f(6)$	2.28	2.09	-9.1	Bending vertical
$f(7)$	2.77	2.58	-7.4	Bending vertical
$f(8)$	2.84	2.44	-16.4	Bending lateral
$f(9)$	2.98	2.88	-3.5	Bending lateral

9 Conclusion

In the presented paper, we were dealing with a dynamic analysis of the footbridge in Lužec nad Vltavou. Dynamic analysis was focused on both, theoretical modal analysis and theoretical forced vibration analysis, which was dealing with vandalism as a type of excitation. Individual vandals were simplified by DLFs, which were determined by the evaluation of the data provided by 15 bobbing people.

All of the theoretically obtained results were compared to the experimentally obtained ones. As can be seen, calculations provided great accordance with the experimental results.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Vladimír Šána accomplished dynamic calculations (modal forced vibration) and created a 3D theoretical calculation model.

-Tomáš Plachý performed the experimental dynamic analysis and its evaluation.

-Michal Polák carried out the experimental dynamic analysis and its evaluation.

-Magdaléna Boháčová was responsible for the experiment and computational model validation.

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Conflict of Interest

The authors have no conflicts of interest to declare.

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