On Nonlinear Spatial Vibrations of Rotating Drill Strings under the Effect of a Fluid Flow

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Abstract: - In this article, the development and subsequent numerical analysis of a nonlinear mathematical model of the drill-string dynamics taking into account the effect of a drilling fluid flow and the gravitational energy of the system is carried out. Spatial lateral vibrations of the drill string modeled as a rotating elastic rod are studied. The developed nonlinear model generalizes and refines the well-known linear models of rod vibrations with the considered effect. The obtained numerical results demonstrate the influence of geometric nonlinearity, the gravitational energy of the drilling system, additional Coriolis and centrifugal forces as well as the parameters of the fluid flow on spatial vibrations of the drill strings. It allows for giving some recommendations on the choice of the drilling system parameters for ensuring safe drilling operations.

Key-Words: - mathematical model, drill string, rod, nonlinear, vibration, fluid flow.

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1 Introduction

Modeling the motion of drill strings is a highly nonlinear and quite complex process from mathematical and physical points of view since their dynamics include all three main types of vibrations: lateral, longitudinal, and torsional ones, [1]. Amongst them, the lateral mode is said to be the most dangerous that causes breakdowns of drill strings and failure of drilling rigs, [2].

A number of works have been devoted to the problems of drill-string vibrations with a fluid flow, which is one of the main factors ensuring the efficiency of the entire process of drilling oil and gas wells. The impact of drilling fluid properties along with formation boundaries on the transmission of acoustic waves in a drill string was studied in, [3]. A slight linear decrease in the acoustic wave amplitude was observed when increasing the drilling fluid damping. In, [4], the authors examined the influence of the non-Newtonian drilling mud rheology on the vertical drill bit vibrations using the Herschel-Bulkley and Casson models and determined the parameters for the significant reduction of chatter. A stability map of the drill-string stick-slip vibrations based on its nonlinear distributed axial-torsional model with a rate-independent bit-rock interaction was developed in, [5]. The analysis allowed a better understanding of coupled self-excited vibrations in the drill string considered.

In, [6], the authors presented a comprehensive review of papers related to drill pipe failures indicating all metallurgical and mechanical aspects of these undesirable phenomena. Separate attention was paid to the corrosive and erosive behavior of drilling mud that might damage the internal coating of drill pipes. This problem was also investigated in, [7], [8]. In [7], the authors, in particular, considered the comparative design approach that normalized several factors inducing drill-string fatigue. An innovative approach for improving the directional well cleaning process by controlling drill-string buckling was proposed in [9]. In [10], the authors studied the stability of the lateral vibration of drill strings and found that the interaction between the drill string and the drilling fluid had a great impact on its buckling and dynamics. To solve the problem, the conventional Galerkin method was utilized.

In, [11], the authors suggested a method for estimation of the 3D static stress-strain state of a drill string and marine riser under their contact interaction. The stress-strain state of a rotating drill string affected by drilling mud and external forces for nonlinear and linear models was analyzed in [12], using the maximum stress intensity criterion. However, despite the existence of works in this direction, the problem of spatial vibrations of drill strings considering nonlinear effects and the influence of the drilling fluid flow is still insufficiently studied. In contrast to many articles (for instance, [13], [14]), where the linear models are utilized, the current research involves not only the effect of geometric nonlinearity along with the action of external forces but also the gravitational energy of the system.

Therefore, this work aims to study drill-string nonlinear spatial lateral vibrations taking into account the action of the drilling fluid flow along with the effect of the gravitational energy of the system. To demonstrate the significance of considering nonlinearity when modeling the drill-string dynamics, the comparative analysis of the developed nonlinear model with its linearized version is carried out. Moreover, the investigation of the impact of the fluid flow on the drill-string spatial vibrations allows for giving recommendations on the optimal choice of parameters of the drilling fluid for performing safe drilling operations.

2 Nonlinear Mathematical Model

The drill string is modeled as a homogeneous isotropic elastic rod of the constant cross-section with a length of *l* and rotating with an angular speed Ω. The upper end of the rod is subjected to a longitudinal compressive load $N(x_3,t)$ equal to the reaction of the lower end to the bottom of the well, and a torque $M(x_3,t)$ that causes the rod torsional deformation. An incompressible fluid flow moves along the internal tube of the drill string in the positive direction of the longitudinal axis x_3.

The design scheme of the drill string taking into account the mentioned effects is shown in Fig.1.

The hypothesis of plane sections and the main provisions of the V.V. Novozhilov nonlinear theory of elasticity, [15], are utilized. We use the second system of simplifications of the nonlinear theory, according to which the relative elongations, shears, and angles of rotation are assumed to be infinitesimal.

The rod displacement vector is decomposed into two components $u_1(x_3,t)$ and $u_2(x_3,t)$. The position vector of the rod and its velocity vector are determined from the following relations:

$$\mathbf{r}(x_3,t) = u_1(x_3,t)\mathbf{i_1} + u_2(x_3,t)\mathbf{i}_2 - \left(\frac{\partial u_1(x_3,t)}{\partial x_3}x_1 + \frac{\partial u_2(x_3,t)}{\partial x_3}x_2\right)\mathbf{i}_3, \quad (1)$$

$$\mathbf{v} = \frac{\partial \mathbf{r}}{\partial t} = \left(\frac{\partial u_1}{\partial t} - \Omega u_2\right)\mathbf{i_1} + \left(\frac{\partial u_2}{\partial t} + \Omega u_1\right)\mathbf{i}_2 - \left(\frac{\partial^2 u_1}{\partial x_3 \partial t}x_1 + \frac{\partial^2 u_2}{\partial x_3 \partial t}x_2\right)\mathbf{i}_3 \quad (2)$$

where $\mathbf{i_1} = \mathbf{i_1}(t), \mathbf{i}_2 = \mathbf{i}_2(t), \mathbf{i}_3$ are unit vectors.

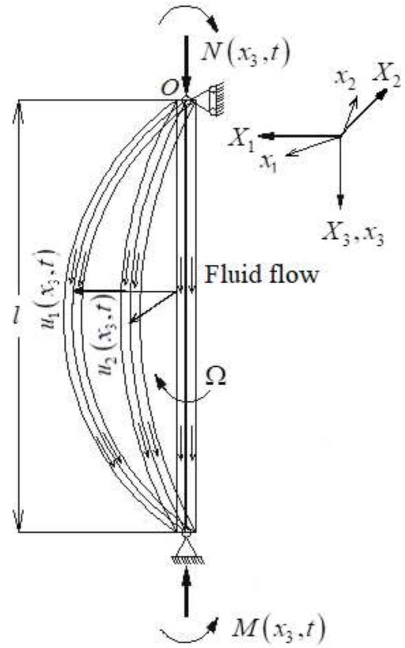


Fig. 1: The drill string scheme.

The strain components of the rod are given in the form:

$$\varepsilon_{11} \approx \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}\right)^2, \varepsilon_{12} \approx \frac{\partial u_1}{\partial x_3}\frac{\partial u_2}{\partial x_3}, \varepsilon_{22} \approx \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}\right)^2,$$
$$\varepsilon_{13} = \varepsilon_{23} = 0, \quad (3)$$
$$\varepsilon_{33} \approx -\frac{\partial^2 u_1}{\partial x_3^{\,2}}x_1 - \frac{\partial^2 u_2}{\partial x_3^{\,2}}x_2 + \frac{1}{2}\left(\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2\right),$$

using which the elastic potential Φ for the case of the rod vibrations in the considered planes, Ox_1x_3 and Ox_2x_3, is written as

$$\Phi = \frac{G+\lambda}{2}\left[\left(\left(\frac{\partial u_1}{\partial x_3}\right)^2 + \left(\frac{\partial u_2}{\partial x_3}\right)^2\right)^2 + \left(\left(\frac{\partial^2 u_1}{\partial x_3^{\,2}}\right)x_1 + \left(\frac{\partial^2 u_2}{\partial x_3^{\,2}}\right)x_2\right)^2 - 2\left(\frac{\partial^2 u_1}{\partial x_3^{\,2}}x_1 + \frac{\partial^2 u_2}{\partial x_3^{\,2}}x_2\right)\right.$$

$$\times\left(\left(\frac{\partial u_1}{\partial x_3}\right)^2+\left(\frac{\partial u_2}{\partial x_3}\right)^2\right)\Bigg]+\frac{G}{2}\Bigg[3\left(\frac{\partial u_1}{\partial x_3}\right)^2\left(\frac{\partial u_2}{\partial x_3}\right)^2$$
$$+\left(\left(\frac{\partial^2 u_1}{\partial x_3^2}\right)x_1+\left(\frac{\partial^2 u_2}{\partial x_3^2}\right)x_2\right)^2\Bigg], \quad (4)$$

where $G=\frac{E}{2(1+\nu)}, \lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}$ are Lame parameters.

The velocity vector of the drilling fluid in accordance with the work of [16], is defined as follows:

$$\mathbf{v_f}=\mathbf{v}+V_f\boldsymbol{\tau}, \quad (5)$$

where V_f is the fluid flow velocity, $\boldsymbol{\tau}=\frac{\partial \mathbf{r}}{\partial t}$ is the direction vector tangential to the rod axis.

To derive a nonlinear mathematical model, the Ostrogradsky-Hamilton variational principle is applied. According to this principle, the following condition must be satisfied:

$$\delta J=\delta\int_{t_1}^{t_2}\left(T-U_0+\Pi\right)dt=0, \quad (6)$$

where T and U_0 are kinetic and potential energies, Π is the potential of external forces, taking into account the effect of the longitudinal load, torque, as well as the gravitational energy of the drill string and the pressure of the fluid flow. The expressions for the energies and the potential can be found in [17].

Thus, the nonlinear mathematical model of the drill-string spatial lateral vibrations, allowing for the influence of the fluid flow and the effect of external loads is obtained in the form [12]:

$$EI_{x_2}\frac{\partial^4 u_1}{\partial x_3^4}-\rho I_{x_2}\frac{\partial^4 u_1}{\partial x_3^2\partial t^2}+\frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_2}{\partial x_3}\right)$$
$$+\frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_1}{\partial x_3}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^3$$
$$-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right)$$
$$+\left(\rho A+\rho_f A_f\right)\left(\frac{\partial^2 u_1}{\partial t^2}-2\Omega\frac{\partial u_2}{\partial t}-\Omega^2 u_1\right)$$
$$-\rho_f I_{x_2}\left(\frac{\partial^4 u_1}{\partial x_3^4}+2\frac{\partial^4 u_1}{\partial x_3^3\partial t}+\frac{\partial^4 u_1}{\partial x_3^2\partial t^2}\right)$$
$$+\rho_f A_f\left(V_f^2\frac{\partial^2 u_1}{\partial x_3^2}+2V_f\frac{\partial^2 u_1}{\partial x_3\partial t}-2V_f\Omega\frac{\partial u_2}{\partial x_3}\right)$$
$$+\left(\rho A+\rho_f A_f\right)g\left(\frac{\partial u_1}{\partial x_3}-(l-x_3)\frac{\partial^2 u_1}{\partial x_3^2}\right)=0, \quad (7)$$

$$EI_{x_1}\frac{\partial^4 u_2}{\partial x_3^4}-\rho I_{x_1}\frac{\partial^4 u_2}{\partial x_3^2\partial t^2}-\frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_1}{\partial x_3}\right)$$
$$+\frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_2}{\partial x_3}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^3$$
$$-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^2\right)$$
$$+\left(\rho A+\rho_f A_f\right)\left(\frac{\partial^2 u_2}{\partial t^2}+2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2\right)$$
$$-\rho_f I_{x_1}\left(\frac{\partial^4 u_2}{\partial x_3^4}+2\frac{\partial^4 u_2}{\partial x_3^3\partial t}+\frac{\partial^4 u_2}{\partial x_3^2\partial t^2}\right)$$
$$+\rho_f A_f\left(V_f^2\frac{\partial^2 u_2}{\partial x_3^2}+2V_f\frac{\partial^2 u_2}{\partial x_3\partial t}+2V_f\Omega\frac{\partial u_1}{\partial x_3}\right)$$
$$+\left(\rho A+\rho_f A_f\right)g\left(\frac{\partial u_2}{\partial x_3}-(l-x_3)\frac{\partial^2 u_2}{\partial x_3^2}\right)=0,$$

where E is Young's modulus, I_{x_1}, I_{x_2} axial inertia moments, ρ the drill string density, ν Poisson's ratio, A the cross-sectional area of the drill string, ρ_f the fluid density, A_f and the internal cross-sectional area of the drill string.

The boundary conditions corresponding to the simply supported rod are given as

$$u_1(x_3,t)=u_2(x_3,t)=0 \quad (x_3=0, x_3=l),$$
$$EI_{x_2}\frac{\partial^2 u_1(x_3,t)}{\partial x_3^2}=EI_{x_1}\frac{\partial^2 u_2(x_3,t)}{\partial x_3^2}=0 \quad (8)$$
$$(x_3=0, x_3=l).$$

The initial conditions are written as follows:

$$u_1(x_3,t)=u_2(x_3,t)=0 \quad (t=0),$$
$$\frac{\partial u_1(x_3,t)}{\partial t}=C_1, \quad \frac{\partial u_2(x_3,t)}{\partial t}=C_2 \quad (t=0), \quad (9)$$

where $C_i\ (i=1,2)$ are constants.

Neglecting the terms responsible for the contribution of geometric nonlinearity, the effect of the longitudinal load and the torque leads to a model obtained by [13], without considering the axial deployment of the rod.

3 Method of Solution

Direct integration of the obtained governing equations, complicated by geometric nonlinearity, causes great mathematical difficulties. Therefore, to solve the nonlinear model (7)-(9), the Bubnov-Galerkin method, which allows reducing partial differential equations (PDEs) to ordinary differential equations (ODEs), is utilized. According to this approach, the desired solutions are represented as expansions into finite series in terms of the basis functions:

$$u_1(x_3,t)=\sum_{i=1}^{n}\sin\left(\frac{i\pi x_3}{l}\right)f_i(t),$$
$$u_2(x_3,t)=\sum_{i=1}^{n}\sin\left(\frac{i\pi x_3}{l}\right)g_i(t). \quad (10)$$

It is assumed that the longitudinal load changes with time according to a sinusoidal law and is written in the following form:

$$N(x_3,t)=N_0+N_t\sin\left(\overline{\Omega}t\right), \quad (11)$$

where N_0 is the constant component of the external load, N_t is the variable one, and $\overline{\Omega}$ is the frequency of the load, which depends on the angular speed Ω. The action of the torque is assumed to be constant and distributed along the length of the rod, $M(x_3,t)=M$.

We also take into account that the axial moments of inertia are equal to each other; hence, the stiffness of the rod relative to the axes x_1, x_2 is constant, and due to the homogeneity of the material properties, does not change along the length of the drill string, $EI_{x_1}=EI_{x_2}=EI$.

Substituting the displacement projections $u_1(x_3,t)$ and $u_2(x_3,t)$, given in the form (10), into Eqs. (7) and considering the first three terms of the series $(n=3)$, a system of six second-order nonlinear ODEs with respect to the functions $f_i(t)$ $g_i(t), i=\overline{1,3}$ is obtained. Accounting for a larger number of terms in (10) gives only a slight refinement of the solution, requiring, at the same time, much more time for conducting the numerical experiment, which is inappropriate from the "implementation time–computational accuracy" viewpoint and practically unreasonable. For reasons of space, the nonlinear ODEs are not presented in this article but can be easily determined.

Then, the obtained ODE system is solved numerically using the stiffness switching method including the eighth-order explicit Runge-Kutta method and the linearly implicit Euler method. The stiffness switching method is applied since the studied equations belong to the class of stiff equations, the numerical solution of which is accompanied by the non-uniform change at different intervals. Thus, the solution to the problem includes so-called boundary (inner) layers characterized by very fast change of the solution on a small interval, followed by intervals of slow evolution (quasi-stationary mode). Other variations of the stiffness-switching method can be found in the work of [18]. Another reason for using the stiffness switching method is its high efficiency compared to other numerical methods when solving the considered type of stiff equations, [19].

4 Numerical Results and Discussions

The software package used to analyze the numerical results is Wolfram Mathematica.

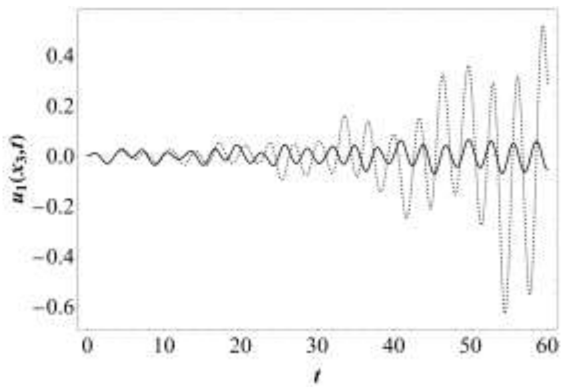
To demonstrate the need of considering nonlinear terms in model (7), the comparative analysis of the model with its linear analog is carried out (Fig.2).

The values of the main parameters of the drill string, external loads, and drilling fluid are given in Table 1. Spatial lateral vibrations of the steel rod in the cross-section $z=0.49l$ with $C_1=C_2=0.01\,\mathrm{m/s}$ are considered.

Table 1. Parameters of the drilling system

System parameter	**Value**
Drill string length, l	100m
The angular speed of rotation, Ω	$1.5\,\text{rad/s}$
Young's modulus, E	$2.1\times10^{11}\,\text{Pa}$
Drill string density, ρ	$7800\,\text{kg/m}^3$
Poisson's ratio, ν	0.28
The outer diameter of the drill string, D	$63.5\times10^{-3}\,\text{m}$
Wall thickness, h	$4.5\times10^{-3}\,\text{m}$
Drill string cross-sectional area, A	$0.834\times10^{-3}\,\text{m}^2$
Longitudinal load, constant part, N_0	$1.2\times10^{3}\,\text{N}$
Longitudinal load, variable part, N_t	$10^{3}\,\text{N}$
Torque, M	$10^{4}\,\text{N}\cdot\text{m}$
Fluid density, ρ_f	$1800\,\text{kg/m}^3$
Internal cross-sectional area, A_f	$2.33\times10^{-3}\,\text{m}^2$
Fluid flow speed, V_f	$3.5\,\text{m/s}$

As illustrated in Fig.2, the use of the linear model for modeling the drill string vibrations under the effect of the fluid flow results in the sharp rise of the vibration amplitude with time, whereas the application of the nonlinear model shows that the oscillatory process remains stable. It indicates the importance of accounting for nonlinear terms in mathematical models to obtain more accurate numerical results. It also confirms the results of, [12], where the stress-strain state of rotating drill strings with drilling mud was studied.



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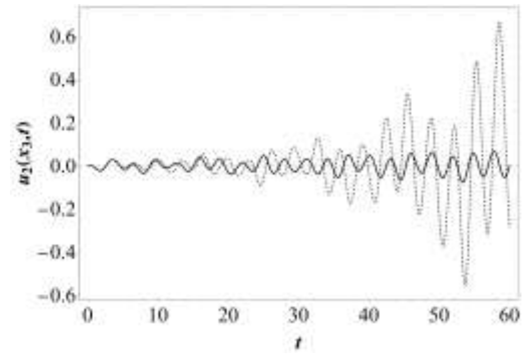

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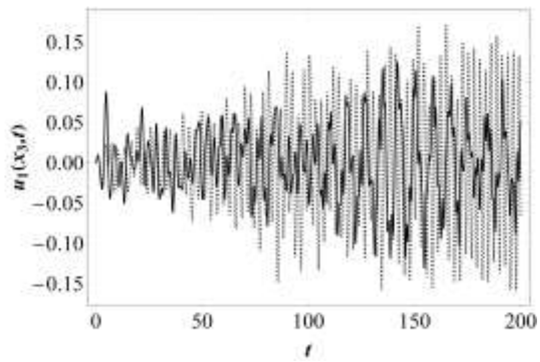
Fig. 2: Results of the comparative analysis for the linear (dotted) and nonlinear (solid) mathematical models in the a) Ox_1x_3 and b) Ox_2x_3 planes.

As stated before, the nonlinear model (7) includes the effect of the gravitational energy of the drill string and the fluid flow. Fig. 3 demonstrates the influence of the gravitational energy on the drill string lateral vibrations at the same values of the system parameters. As can be seen from the obtained graphs, taking into account the gravitational energy of the system leads to a noticeable increase in the amplitude of the drill string spatial lateral vibrations in both planes.



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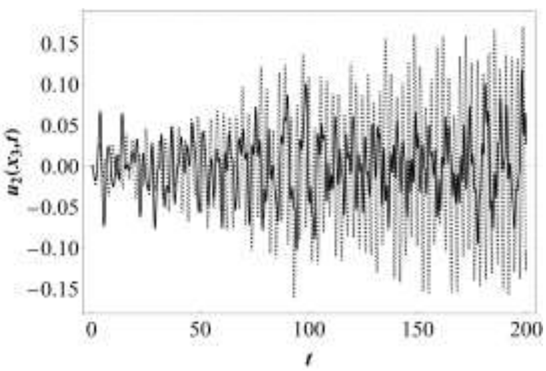
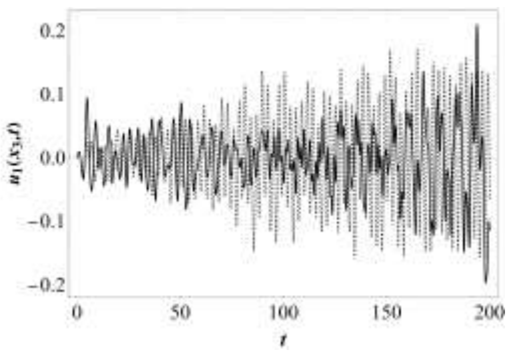


b)

Fig. 3: Drill string vibrations in the a) Ox_1x_3 and b) Ox_2x_3 planes with (dotted) and without (solid) gravitational energy of the system.

Also, the effect of the additional Coriolis and centrifugal forces on the drill string dynamics was studied (Fig.4). When these additional forces and the gravitational energy of the drill string and the fluid flow are not considered, the vibration amplitude over time becomes higher than that obtained by solving the nonlinear model (7). This finding provides further evidence for using the full nonlinear model for studying the drill string spatial vibrations with the fluid flow.

The form of bending of the drill string axis under the effect of the drilling fluid and the gravitational energy is presented in Fig.5. As the graph shows, the bending of the drill string axis is observed in its lower part, which is related to the effect of the gravitational energy of the system.



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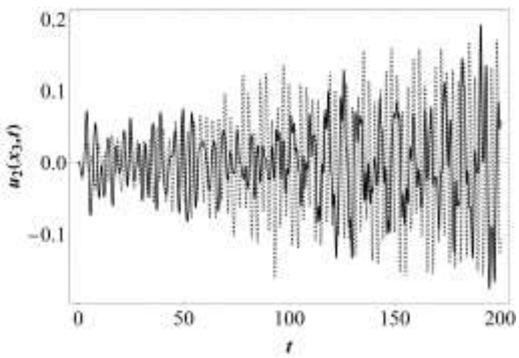
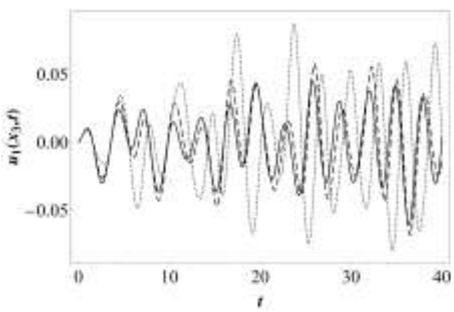


b)

Fig. 4: Comparative analysis of the nonlinear model (dotted) and the one without the effect of the gravitational energy and additional forces (solid) in the a) Ox_1x_3 and b) Ox_2x_3 planes.

Fig. 5: Drill string spatial bending at the time moment t=68s.

Further, the change in the drilling fluid parameters is investigated. The influence of the fluid flow speed on the drill string nonlinear lateral vibrations are illustrated in Fig.6. As can be seen from the graphs below, a three-fold increase in the fluid speed up to $V_f = 10.5\,\mathrm{m/s}$ has a minor impact on the vibration amplitude, whereas the further speed increase to $V_f = 21\,\mathrm{m/s}$ results in the sharp rise of the amplitude. It shows that the high speed of the drilling fluid may be one of the reasons leading to the collapse of the borehole walls and the curvature of the drill string axis and confirms the need to regulate the speed regimes of the borehole cleaning.



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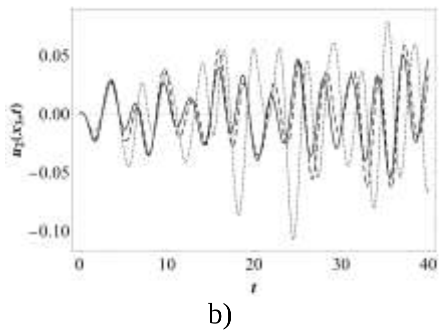
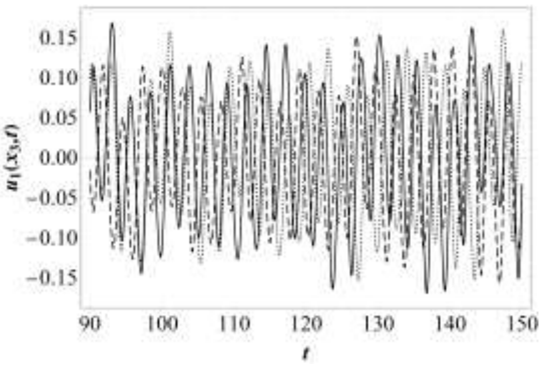


b)

Fig. 6: Influence of the fluid flow speed on the drill-string vibrations in the a) Ox_1x_3 and b) Ox_2x_3 planes: $V_f = 3.5\,\mathrm{m/s}$ (solid), $V_f = 10.5\,\mathrm{m/s}$ (dashed), $V_f = 21\,\mathrm{m/s}$ (dotted).

Figure 7 and Figure 8 demonstrate the effect of the drilling fluid density on the oscillatory process. At a relatively low speed of the fluid flow ($V_f = 3.5\,\mathrm{m/s}$), the comparative analysis of the drill string spatial vibrations at varying values of the fluid density from $\rho_f = 1000\,\mathrm{kg/m^3}$ to $\rho_f = 1600\,\mathrm{kg/m^3}$ and then to $\rho_f = 2200\,\mathrm{kg/m^3}$ shows that the use of industrial water with density $\rho_f = 1000\,\mathrm{kg/m^3}$ yields the maximum values of the vibration amplitude, slightly surpassing the amplitude of the drill string vibrations under the effect of the weighted fluid with density $\rho_f = 2200\,\mathrm{kg/m^3}$ (Figure 7).



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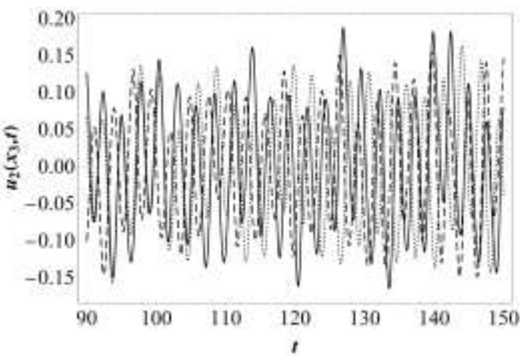

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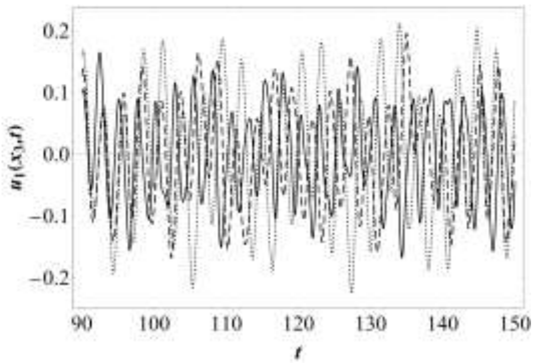
Fig. 7: Influence of the drilling fluid density on the drill-string vibrations in the a) Ox_1x_3 and b) Ox_2x_3 planes: $\rho_f = 1000\,\mathrm{kg/m^3}$ (solid), $\rho_f = 1600\,\mathrm{kg/m^3}$ (dashed), $\rho_f = 2200\,\mathrm{kg/m^3}$ (dotted).

On the other hand, when the flow speed increases up to $V_f = 20\,\mathrm{m/s}$, the utilization of the industrial water allows the performing of drilling operations with the smallest vibration amplitude compared to other drilling fluids (Figure 8). In turn, the largest amplitude of the lateral vibrations is observed when the weighted drilling fluid is applied. Another advantage of industrial water is its availability and low cost. Having a low viscosity, it successfully flushes the mud from the bottom of the well and cools down the bit.



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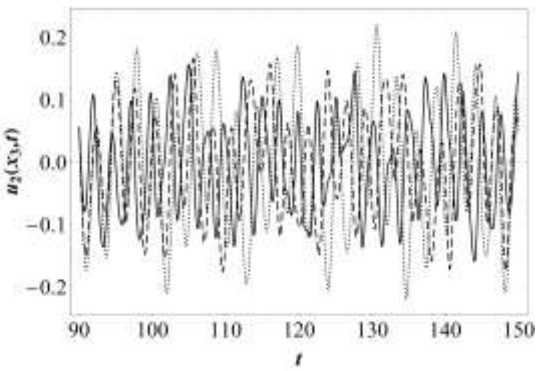

b)

Fig. 8: Influence of the drilling fluid density on the drill-string vibrations in the a) Ox_1x_3 and b) Ox_2x_3 planes when the flow speed increases, $V_f = 20\,\mathrm{m/s}$ $\rho_f = 1000\,\mathrm{kg/m^3}$ (solid), $\rho_f = 1600\,\mathrm{kg/m^3}$ (dashed), $\rho_f = 2200\,\mathrm{kg/m^3}$ (dotted).

5 Conclusion

In this work, spatial lateral vibrations of rotating drill strings under the effect of a drilling fluid flow were considered. Using the main provisions of the V.V. Novozhilov nonlinear theory of elasticity and the Ostrogradsky-Hamilton variational principle, a nonlinear mathematical model of the drill-string vibrations accounting for the effect of the fluid flow was developed.

The comparative analysis of the developed nonlinear model and its linear analog showed the importance of adding nonlinear terms to the mathematical models for obtaining more accurate numerical results. Also, the effect of the gravitational energy of the drill string and the fluid flow and the influence of additional Coriolis and centrifugal forces included in the model on the drill-string spatial lateral vibrations was analyzed. It was obtained that taking into account the gravitational energy of the system led to a noticeable increase in the drill-string vibration amplitude in both planes. When the additional forces and the gravitational energy were not considered, the research results demonstrated the increase in the amplitude of vibrations compared to that obtained by solving the full nonlinear model over time.

The study of the influence of the drilling fluid parameters on the drill string vibrations revealed that the high speed of the drilling fluid flow could be the reason for the vibration amplitude sharp rise and confirmed the need to regulate the speed regimes of the borehole cleaning. It was also shown that the industrial water, having a low density compared to other drilling fluids, allowed for conducting the drilling process with relatively small vibrations of the drill string at the high speed of the fluid flow.

Our work has some limitations. The most important one lies in the fact that the interaction of the drill string with the borehole wall was not considered. It would allow restricting the drill-string vibrations and make the problem closer to the real drilling process. Therefore, to further our research, we are going to study the drill-string nonlinear dynamics and its stability, taking into account the intermittent contact with the borehole, and also consider the effect of a two-directional fluid flow.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

-Askar K. Kudaibergenov developed the nonlinear mathematical model of the drill string vibrations.
-Askat K. Kudaibergenov conducted the numerical analysis of the drill string vibrations.
-L.A. Khajiyeva was responsible for the conceptualization and methodology of the studied problem.

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Conflict of Interest

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