Dynamic behavior of composite sandwich panel with CFRP outer layers

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Abstract: — A sandwich panel with laminate faces is used for free vibration analysis. The periodic microstructure and Mori-Tanaka model are used for homogenization of unidirectional fiber reinforced composite. The Shear Deformation Theory is considered for analytical and numerical analysis. FEM in ANSYS is used for numerical analysis. The effect of sandwich design parameters such as panel length, core thickness and fiber reinforced angle on vibration response is investigated. Natural frequencies of sandwich panel versus sandwich design parameters are presented in graphical form. From the results can be concluded that sandwich design parameters affect the natural frequencies of sandwich panels, and this effect is important for designing of sandwich panels under dynamic load.

Keywords: —frequency analysis, sandwich plate, CRFP faces, FEM analysis

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1.Introduction

Rapid growth in manufacturing industries has led to the need for the betterment of materials in terms of strength, stiffness, density, and lower cost with improved sustainability. Composite materials have emerged as one of the materials possessing such betterment in properties serving their potential in a variety of applications [1].

The motivation for sandwich composites is two-fold [2]. If a plate is bent, the maximum stresses occur at the top and the bottom surface. So, it makes sense using high strength materials only for the sheets and using low and lightweight materials in the middle. The resistance to bending of a rectangular cross-section is proportional to the cube of the thickness. Increasing the thickness by adding a core in the middle increases the resistance. The shear stresses have a maximum in the middle of a sandwich beam requiring the core to support the shear.

Sandwich panels are one of very important groups of laminated composites. They consist of two thin faces with thickness of $h^{(1)}$ and $h^{(3)}$ and a core with thickness of $h^{(2)}$. The faces are made of high strength materials having good properties under tension, such as fiber reinforced polymer matrix laminates used in this paper, while the core is made of lightweight materials such as foam. The advantage of weight and bending stiffness makes sandwich composites more attractive for some applications than other composite or conventional materials [3],[4],[5],[6],[7],[8].

The analysis of simple sandwich structures may be achieved by analytical methods, by adapting the classical tools of structural analysis on anisotropic elastic structure face elements [9]. For more complex structures, such as more general boundary conditions or loading, numerical methods such as Finite Element Method, Boundary Element Method, etc. are used [10],[11].

assumptions necessary The use of is to mathematical modeling of laminated composites. These include an elastic behavior of fibers and matrices, a perfect bonding between fibers matrices, and low mechanical characteristics of the variation of the individual fibers, uniform fiber diameters, their regular arrangement in the matrix, etc.

Taking into account the different size scales of mechanical modelling of structure elements composed of fiber reinforced composites, the micro, macro and structural modeling levels must be considered [12],[13],[14],[15],[16].

Free vibration analysis is a very important analysis. The lowest natural frequency is often referred to as the fundamental frequency, which is the most important parameter for design engineers as many of the systems are designed to operate below it [17],[18],[19],[20],[21], [22],[23].

For obtaining the material characteristics of composite laminated faces, the experimental tests were performed [24],[25],[26],[27],[28].

2. Materials and Methods

The motivation for sandwich composites is two-fold: If a beam is bent, the maximum stresses occur at the top and the bottom surface. So, it makes sense using high strength materials only for the sheets and using low and lightweight materials in the middle. The resistance to bending of a rectangular cross-sectional beam is proportional to the cube of the thickness. Increasing the thickness by adding a core in the middle increases the resistance. The shear stresses have a maximum in the middle of a sandwich beam requiring the core to support the shear. This advantage of weight and bending stiffness makes sandwich composites more attractive for some applications than other composite or conventional materials.

Composite materials consist of two or more constituents and the modelling, analysis and design of structures composed of composites are different from conventional materials such as steel. There are three levels of modelling. At the micromechanical level, the average properties of a single reinforced layer (a lamina or a ply) have to be determined from the individual properties of the constituents, the fibers and matrix. The average characteristics include the elastic moduli, the thermal and moisture expansion coefficients, etc. The micromechanics of a lamina does not consider the internal structure of the constituent elements but recognizes the heterogeneity of the ply. The micro-mechanics is based on some simplifying approximations. These concern the fiber geometry and packing arrangement, so that the constituent characteristics together with the volume fractions of the constituents yield the average characteristics of the lamina. Note that the average properties are derived by considering the lamina to be homogeneous. In the frame of this paper only the micro-mechanics of unidirectional reinforced laminates are considered. The calculated values of the average properties of a lamina provide the basis to predict the macrostructural properties. At the macro-mechanical level, only the averaged properties of a lamina are considered and the microstructure of the lamina is ignored. The properties along and perpendicular to the fiber direction, these are the principal directions of a lamina, are recognized and the so-called on-axis stress strain relations for a unidirectional lamina can be developed. A laminate is a stack of laminae. Each layer of fiber reinforcement can have various orientation and in principle each layer can be made of different materials. Knowing the macro-mechanics of a lamina, one develops the macro-mechanics of the laminate. Average stiffness, flexibility, strength, etc. can be determined for the whole laminate. The structure and orientation of the laminae in prescribed sequences to a laminate lead to significant advantages of composite materials when compared to a conventional monolithic material. In general, the mechanical response of laminates is anisotropic.

One very important group of laminated composites are sandwich composites. They consist of two thin faces (the skins or sheets) sandwiching a core. The faces are made of high strength materials having good properties under tension such as fiber reinforced laminates while the core is made of lightweight materials such as foam, resins with special fillers, called syntactic foam, having good properties under compression. Sandwich composites combine lightness and flexural stiffness.

When the micro- and macro-mechanical analysis for laminae and laminates are carried out, the global behavior of laminated composite materials is known. The last step is the modelling on the structure level and to analyze the global behavior of a structure made of composite material. By adapting the classical tools of structural analysis on anisotropic elastic structure elements the analysis of simple structures as beams or plates may be achieved by analytical methods, but for more general boundary conditions and/or loading and for complex structures, numerical methods are used.

3. First-Order Shear Deformation Theory (FSDT)

FSDT considers that transverse normals do not remain perpendicular to the midsurface after deformation. This theory is used for thicker plates or sandwiches taking into account the Reissner kinematics.

Transverse shear stresses are added to the state of plane stress for this reason. Shear deformations are written following the Figure 1:



Fig. 1. Geometry of shear deformation.

$$\begin{aligned} \gamma_{xz2} &= \left(\frac{u_{12} - u_{32}}{h^{(2)}} + \frac{\partial w}{\partial x}\right) = \left(\frac{u_1 - u_3}{h^{(2)}} + \frac{d}{h^{(2)}}\frac{\partial w}{\partial x}\right),\\ \gamma_{yz2} &= \left(\frac{v_{12} - v_{32}}{h^{(2)}} + \frac{\partial w}{\partial y}\right) = \left(\frac{v_1 - v_3}{h^{(2)}} + \frac{d}{h^{(2)}}\frac{\partial w}{\partial y}\right), \end{aligned} \tag{1}$$

where d is midplane distance of sandwich faces:

$$d = h^{(2)} + \frac{h^{(1)} + h^{(3)}}{2}$$
(2)

Internal forces are expressed by terms:

$$\begin{split} \mathbf{N} &= \int_{-\left(\frac{1}{2}h^{(2)}+h^{(1)}\right)}^{-\frac{1}{2}h^{(2)}} \boldsymbol{\sigma} dz + \int_{\frac{1}{2}h^{(2)}+h^{(3)}}^{\frac{1}{2}h^{(2)}+h^{(3)}} \boldsymbol{\sigma} dz, \\ \mathbf{M} &= \int_{-\left(\frac{1}{2}h^{(2)}+h^{(1)}\right)}^{-\frac{1}{2}h^{(2)}} \boldsymbol{\sigma} z dz + \int_{\frac{1}{2}h^{(2)}}^{\frac{1}{2}h^{(2)}+h^{(3)}} \boldsymbol{\sigma} z dz, \\ \mathbf{V} &= \int_{-\frac{1}{2}h^{(2)}}^{\frac{1}{2}h^{(2)}} \boldsymbol{\tau} dz. \end{split}$$
(3)

The constitutive equations for a sandwich are in the form:

$$\begin{pmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{V} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}^{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_{\mathrm{m}}^{0} \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{pmatrix}, \tag{4}$$

with stiffness coefficients:

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$$\begin{aligned} A_{ij} &= A_{ij}^{(1)} + A_{ij}^{(2)}, \\ B_{ij} &= \frac{1}{2} h^{(2)} \left(A_{ij}^{(2)} - A_{ij}^{(1)} \right) \\ C_{ij} &= C_{ij}^{(1)} + C_{ij}^{(2)}, \\ D_{ij} &= \frac{1}{2} h^{(2)} \left(C_{ij}^{(2)} - C_{ij}^{(1)} \right) \\ A_{ij}^{s} &= E_{ij}^{s} h^{(2)}; \ i, j = 4, 5, \end{aligned}$$
(5)

where A_{ij} , D_{ij} are coefficients of extension and bending stiffness matrix, B_{ij} , C_{ij} are coefficients of extension-bending coupling stiffness matrix, E_{ij}^s are the transverse shear moduli of the core.

4. Free Vibrations of Sandwich Plates

The equations to determine the natural frequencies of the symmetric sandwich panel are used [1]:

$$D_{11} \frac{\partial^2 \Phi_x}{\partial x^2} + D_{66} \frac{\partial^2 \Phi_x}{\partial y^2} + (D_{12} + D_{66}) \frac{\partial^2 \Phi_y}{\partial x \partial y} -k^s A_{55} \left(\Phi_x + \frac{\partial w_0}{\partial x} \right) - I_2 \frac{\partial^2 \Phi_x}{\partial t^2} = 0,$$

$$(D_{12} + D_{66})\frac{\partial^2 \Phi_x}{\partial x \partial y} + D_{66}\frac{\partial^2 \Phi_y}{\partial x^2} + D_{22}\frac{\partial^2 \Phi_y}{\partial y^2} -k^s A_{44}\left(\Phi_y + \frac{\partial w_0}{\partial y}\right) - I_2\frac{\partial^2 \Phi_y}{\partial t^2} = 0,$$
(6)

$$k^{s}A_{55}\left(\frac{\partial \Phi_{x}}{\partial x} + \frac{\partial^{2}w}{\partial x^{2}}\right) + k^{s}A_{44}\left(\frac{\partial \Phi_{y}}{\partial y} + \frac{\partial^{2}w_{0}}{\partial y^{2}}\right) - \rho_{m}h\frac{\partial^{2}w_{0}}{\partial t^{2}} = 0,$$

where k^{s} is the transverse shear deformation factor given by value 5/6.

$$\rho_m = \frac{1}{h} \sum_{k=1}^{N} \rho_k \left(z^{(k)} - z^{(k-1)} \right),$$

$$I = \frac{\rho_m h^3}{12} \frac{1}{3} \sum_{k=1}^{N} \rho_k \left(z^{(k)} \right)^3 - (z^{(k-1)})^3,$$
(7)

where ρ_k is the mass density of the kth layer. For the simply supported plate on each edge let: $w_0(x, y, t) = w_0^{\circ} e^{i\omega_{mn}t}$,

$$\Phi_{\chi}(x, y, t) = \Phi_{\chi}^{\circ} e^{i\omega_{mn}t}, \qquad (8)$$

$$\Phi_y(x, y, t) = \Phi_y^{\circ} e^{i\omega_{mn}t}$$

where *m*, *n* are integers only, ω_{mn} is natural angular velocity. After substituting of Eqs. (8) into the Eqs. (6) we get Eva Kormaníková, Kamila Kotrasová

$$\begin{bmatrix} \dot{L}_{11} & L_{12} & L_{13} \\ L_{12} & \dot{L}_{22} & L_{23} \\ L_{13} & L_{23} & \dot{L}_{33} \end{bmatrix} \begin{bmatrix} \dot{A}_{mn} \\ \dot{B}_{mn} \\ \dot{C}_{mn} \end{bmatrix} = \begin{cases} 0 \\ 0 \\ 0 \end{cases},$$
(9)

where:

$$\dot{L_{11}} = L_{11} - \frac{\rho_m h^a}{12} \omega_{mn}^2,$$

$$\dot{L_{22}} = L_{22} - \frac{\rho_m h^3}{12} \omega_{mn}^2,$$

$$\dot{L_{33}} = L_{33} - \rho_m h \omega_{mn}^2,$$

$$L_{11} = D_{11} \lambda_m^2 + D_{66} \lambda_n^2 + k^s A_{55},$$

$$L_{12} = (D_{12} + D_{66})\lambda_m\lambda_n,$$

$$L_{13} = k^s A_{55}\lambda_m,$$

$$L_{22} = D_{66}\lambda_m^2 + D_{22}\lambda_n^2 + k^s A_{44},$$

$$L_{23} = k^s A_{44}\lambda_n,$$

$$L_{33} = k^s A_{55}\lambda_m^2 + k^s A_{44}\lambda_n^2,$$
(10)

and

(6)

$$\lambda_m = \frac{m\pi}{a}, \qquad \lambda_n = \frac{n\pi}{b}.$$
 (11)

Natural angular velocity can we obtain from:

$$\omega_{mn}^{2} = \frac{QL_{33} + 2L_{12}L_{23}L_{13} - L_{22}L_{13}^{2} - L_{11}L_{23}^{2}}{\rho_{m}hQ},$$

where $Q = L_{11}L_{22} - L_{12}^{2}$. (12)

(7) where

5. Results and Discussion

The parametric study of free vibration has been solved for a simply supported panel on each edge. Length of panel *L* is varied from 2 m to 3.8 m, with step of 0.2 m, width b = 1 m and thickness *h* of the panel is varied from 0.04 m to 0.12 m, with step of 0.02 m.

The thickness of outer layers $h^{(1)} = 0.0004$ m and $h^{(3)} = 0.0006$ m. Outer layers are made of carbon/epoxy laminates with stacking sequence of layers [0/90/0], [0]₃.

Carbon reinforced fiber polymer (CRFP) was considered with the following characteristics:

$$E_f = 230$$
 GPa; $E_m = 3$ GPa; $v_f = 0.3$; $v_m = 0.3$; $\xi_f = 0.6$.

Mass density of the laminated composite is $\rho_{Lc} = 1508$ kg/m³ and $\rho_{Sc} = 43$ kg/m³ for the polyurethane foam as sandwich core.

The material characteristics versus fiber volume fraction are illustrated in Figure 2. The graphs show similar results for longitudinal direction and differences in the transversal directions for bigger fiber volume fraction.





Fig. 2. Material characteristics vs. fiber volume fraction.

FEM analysis in ANSYS was used for free vibration. The influence of the variable thickness and length of the panel with CFRP $[0]_3$ faces to the first ten natural frequencies is presented in Figure 3. For illustration (Figure 3a), a length of 2 m was selected for analytical and numerical solution of natural frequencies depending on changing thickness.



Fig. 3. a) Natural frequencies for sandwich with $[0]_3$ CFRP faces, b) Natural frequencies for sandwiches with $[0]_3$ CFRP faces, numerical solution.

Increasing panel thickness causes an increase in natural frequencies (also seen in Figure 3a (L = 2 m) and differences between analytical and numerical solution. To study frequencies, depending on the changing length, a panel of thickness 0.08 m is used (Figure 3b). Fundamental frequencies are considerably higher for the panel supported on all sides opposite the panel with two opposite supports in the width direction. The influence of the variable face layout [0]₃ and [0/90/0] to natural frequencies is presented in Figure 4a. The analytical and numerical analysis of the effect of fiber

orientation to the global coordinate system on the fundamental frequency and considering the variable thickness and length is shown graphically in Figure 4b and Figure 5. From Figures 4b and 5 can be seen that for the laminate layout $[0]_3$, the frequencies are lower compared to laminate [0/90/0]. Differences between analytical and numerical solution are greater for the [0/90/0] stacking sequence and the thicker panel (Figure 5). It is also possible to observe the increase in frequency with reducing the length of the panel.



Fig. 4. a) Natural frequencies for sandwich with [0]3 and [0/90/0] CFRP faces, numerical solution, b) Fundamental frequency versus thickness for sandwiches with [0]3 and [0/90/0] CFRP faces.



Fig. 5. Fundamental frequencies versus length for sandwich with $[0]_3$ and [0/90/0] CFRP faces a) h = 0.04 m b) h = 0.12 m.

6. Conclusion

The periodic microstructure and Mori-Tanaka model were used for homogenization of unidirectional fiber reinforced composite. It is seen that both models provide similar results, instead characteristics in the direction of the transverse axes.

The FSDT was considered for free vibration analysis. The influence of the variable length, thickness, and fiber orientation of CFRP faces with $[0]_3$ and [0/90/0] layout to natural frequencies was presented. It is also possible to observe the increase in frequency with increasing the thickness of the panel and by reducing the length of panel for both $[0]_3$ and [0/90/0] CFRP faces sandwich panels. The analytical and numerical analysis of the effect of fiber orientation to the global coordinate system on the fundamental frequency, taking into account the variable thickness and length was shown graphically.

From the results obtained by the presented work can be concluded that sandwich design parameters affect the natural frequencies of sandwich panels, and this effect has been taken into consideration for designing of sandwich panels under dynamic load.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The author(s) contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The author(s) declare no potential conflicts of interest concerning the research, authorship, or publication of this article.

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