On the Sigmoid Function as a Variable Permeability Model for Brinkman Equation

D.C. ROACH
Department of Engineering
University of New Brunswick
100 Tucker Park Road, Saint John, New Brunswick, E2L 4L5
CANADA

M.H. HAMDAN
Department of Mathematics and Statistics
University of New Brunswick
100 Tucker Park Road, Saint John, New Brunswick, E2L 4L5
CANADA

Abstract: - A collection of popular variable permeability functions is presented and discussed in this work. The functions have been used largely in Brinkman's equation which governs the flow through a porous domain in the presence of solid, macroscopic boundaries on which the no-slip condition is imposed, and has been used in transition layer modelling. A convenient classification of permeability functions is also provided. The sigmoid logistic function is presented in this work in a modified form that is suitable for variable permeability modelling, and is used in obtaining solution to Poiseuille flow through a Brinkman porous channel.

Key-Words: - Sigmoid function, Porous media, Transition layer, Variable permeability

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1 Introduction

In order to overcome some of the short-comings of Darcy's equation, the literature reports on a number of its modifications and generalizations, [1], [2], [3]. As is well known, the celebrated Darcy's equation is used in the study of seepage flow through porous media and does not account for microscopic inertial effects that arise due to tortuosity of the flow path. It is marked by the absence of a viscous shear term that is necessary to account for viscous shear effects. These effects arise when a macroscopic, solid boundary is encountered, and are useful in treating flows with special viscosities (*cf.* [1] ,[3] and the references therein) and with flows that involve dust particle settling (*cf.* [2] and the references therein). Darcy's equation may be valid in low permeability, low porosity media where variations at the microscopic, pore-level length scale are negligible at the macroscopic length scale. In its steady-state form, Darcy's equation can be written in the following form when permeability to the fluid is a non-tensorial quantity:

$$\frac{\mu}{k(\vec{x})}\vec{u} + \nabla p = \rho\vec{g} \tag{1}$$

wherein μ is the base-fluid constant viscosity, ρ is the density of the fluid, p is the pressure, $\vec{g}$ is the gravitational acceletarion, $k(\vec{x})$ is the medium permeability as a function of position $\vec{x}$, and $\vec{u}$ is the seepage velocity vector.

Nakshatrala and Rajagopal, [3], discussed other limitations of Darcy's equation and its short-comings in predicting important phenomena in flow through porous media, and emphasized that while Darcy's equation merely predicts fluid flux through the porous structure, its flux prediction is not accurate at high pressures and pressure gradients. Consequently, flows of fluids with variable viscosity, namely ones in which viscosity changes with pressure, require a generalization of Darcy's equation in order to better model functional dependence of viscosity on pressure and to quantify the drag force in this type of flow and its dependence on permeability variations.

Nakshatrala and Rajagopal, [3], provided a generalized form of Darcy's equation in which the drag coefficient depended on pressure. Chang *et.al.*, [4], reported Darcy's equation in the following generalized form:

$$\alpha(p,\vec{x})\vec{u} + \nabla p = \rho\vec{g} \quad (2)$$

with drag force $\alpha(p,\vec{x})$ given by:

$$\alpha(p,\vec{x}) = \frac{\mu_0}{k(\vec{x})}.\exp[\beta_B p] \quad (3)$$

wherein μ_0 is the fluid fixed viscosity, and β_B is the experimentally obtained Barus coefficient.

In order to overcome the limitation imposed by the absence of a viscous shear term in Darcy's equation, Brinkman, [5], modified Darcy's equation into the form

$$\frac{\mu}{k(\vec{x})}\vec{u} + \nabla p = \mu_{eff}\nabla^2\vec{u} + \rho\vec{g} \quad (4)$$

where μ_{eff} is the effective viscosity of the fluid in the porous medium and $\vec{u}$ is the superficial average velocity vector. The effective viscosity was shown to depend on porosity of the medium and the viscosity of the base fluid, [6].

Various authors have discussed validity and limitations of Brinkman's equation, [7], [8], [9], [10]. Rudraiah, [10], suggested that Brinkman's equation is the most appropriate model of flow through porous layers of finite depth, while Nield, [8], elegantly concluded that the use of Brinkman's viscous shear term requires a redefinition of the porosity near a solid boundary due to a process referred to as channelling. Sahraoui and Kaviany, [11], and Kaviany, [12], studied the case of flow through variable permeability media when using Brinkman's equation and emphasized the need for variable permeability near macroscopic boundaries, whether slip or no-slip conditions are applied. Hamdan and Barron, [13], showed numerically that the Laplacian in Brinkman's equation is significant in a thin layer near a solid boundary, but less significant in the core of the porous medium.

Brinkman's equation received considerable attention in the literature in the analysis of flow through porous layers with applications to heat and mass transfer. For uni-directional flow, equation (4) takes the following form, wherein $u(y)$ is the tangential velocity component and $G\ (=p_x)$ is the constant driving pressure gradient:

$$u'' - \frac{\mu}{\mu_{eff}k(y)}u = \frac{1}{\mu_{eff}}G \quad (5)$$

When permeability is constant, equation (5) admits the following solution, [14], in the flow through a channel with solid walls at $y = 0\ and\ y = D$, wherein $\beta = \sqrt{\frac{\mu}{k\ \mu_{eff}}}$:

$$u(y) = -\frac{1}{\mu_{eff}}G\{-1 + \cosh\beta y + \sinh\beta y\ (\coth\beta D - cosech\ \beta D)\} \quad (6)$$

While in many idealizations and analyses of uni-directional and two-dimensional flows through porous media, the use of constant permeability has been the rule rather than the exception, it has long been recognized that naturally occurring porous media possess variable permeability. A number of articles in the scientific literature have reported a shift towards variable permeability modelling. A number of reasons are behind this shift, some of which are summarized in what follows.

1- **Transition layer:** In the study of flow through channels over porous layers, the use of a constant permeabilitty in the porous layer results in a permeability discontinuity at the interface. To circumvent, Nield and Kuznetsov, [15], suggested the use of a transition layer between a constant permeability porous layer and free-space. Permeability in the transition layer is variable and ranges from the constant permeability at the intersection with the porous layer to its infinity value at the interface with the free-space channel. While Nield and Kuznetsov, [15], introduced one model to account for variations in permeability, various other models have been developed.
2- **Channeling effects:** In dealing with the problem of flow through a porous medium as governed by Brinkman's equation, Hamdan and Kamel, [14], emphasized the need for variable permeability modelling to be compatible with Brinkman's equation and to better handle the no-slip condition on macroscopic, solid boundaries and to alleviate channelling effects near a solid boundary.

3- **Influence on heat and mass transfer:** In many arising applications of flow through porous media, such as microfluidics and flows with variable viscosity fluids, the flow of polar fluids, [1], and the flow of dusty gases over porous layers, [2], it is imperative to account for permeability and porosity variations. These variations influence heat and mass transfer in the porous layers.

The above, and many other reasons, emphasize the need to develop robust variable permeability models. The main contributions in this area have recently been reviewed by Roach and Hamdan, [16], who also reported on the most recent models available for the transition layer. They provided a classification of the available models into five categories.

In this work, further comments on the available models and the solution to Brinkman's equation when Poiseuille flow is considered through a variable permeability porous channel bounded by solid walls have been provided. Additionally, a new variable permeability model that is compatible with Brinkman's equation is developed. The model is based on the sigmoid logistic function and can be used in the study of flow through porous layers with or without interfacial conditions, as well as in the study of flow of pressure-dependent viscosity fluids in porous channels and layers with variable permeability. The sigmoid function avoids the sudden variation in the permeability present in other approaches to modeling porous flows and can provide a more gradual, natural, variation of the permeability.

2 Overview of Permeability Functions

2.1 Direct and Inverse Models

In three-dimensional flow through naturally occurring porous media, permeability is a tensorial quantity. Idealizations of flow through one and two space dimensions, however, is important to our understanding of the flow phenomena. This idealization gives rise to variable permeability models that use algebraic functions of one of the space variables. Equation (5) describes fully-developed, uni-directional flow and involves two functions of the normal space variable: the tangential velocity function $u(y)$, and the medium permeability function, $k(y)$.

A determinate solution of (5) necessitates that one of the functions must be known. If the permeability function is specified, then solution of equation (5) is sought for $u(y)$ which then describes the velocity profile associated with the prescribed permeability distribution. This approach is referred to as the direct method and is more popular in the literature, where a number of variable permeability models are available and serve a spectrum of flow situations and flow domains of specific industrial applications (*c.f.* [14], [18], [19], [20], and the references therein). An important set of the available variable permeability models that received considerable attention in the literature are classified in the subsections to follow.

In inverse analysis, however, once can assume the form of $u(y)$ and solve equation (5) for $k(y)$ to provide a permeability distribution that produces the specified velocity profile.

The above ideas were implemented in devising a permeability function for Brinkman's equation by Hamdan and Kamel, [14]. They determined that, in Poiseuille flow through a porous channel, the permeability function is of the form

$$K(y) = 4(y - y^2) \tag{7}$$

and

$$u(y) = A(y^2 - y) \tag{8}$$

where

$A = -4\frac{D^2C}{8+H_a}$; D is the depth of a porous channel, $C = -\frac{1}{\mu_{eff}}p_x$ and H_a is a dimensionless number given by $H_a = \frac{D^2}{K_{max}}\frac{\mu}{\mu_{eff}}$, wherein K_{max} is the maximum (constant) permeability attained in the porous channel.

The idea above emphasizes that velocity and permeability functions can be taken proportional to each other. This concept was implemented by Abu Zaytoon et al, [21], in their analysis of flow of pressure-dependent viscosity fluid over an inclined variable permeability porous layer whose permeability function is of the form

$$K(y) = K_0 u(y) \tag{9}$$

where K_0 is a reference constant permeability.

Another permeability function that proved to be of utility in the study of flow of pressure-dependent viscosity fluids down an inclined plane was provided by Alzahrani *et.al.*, [22], and in which the permeability was taken as the square of the variable pressure function, given by:

$$K(y) = [p(y)]^2 \tag{10}$$

where $p(y) = c - \rho ghcos\vartheta\ y$, c is a constant, ρ is the density of the fluid, g is the gravitational acceleration and ϑ is the angle of inclination.

In obtaining analytic solutions to the Darcy-Lapwood-Brinkman equation with variable permeability, Alharbi *et.al.*, [23], found it convenient to introduce the following permeability function:

$$K(y) = \frac{\alpha Re}{y} \quad (11)$$

where $y \neq 0$, α is a parameter, Re is Reynolds number, defined as $Re = \rho UL/\mu$, wherein ρ and μ are density and viscosity of the fluid, respectively, and U and L are characteristic velocity and length, respectively.

2.2. Exponential Models

In their study of similarity solutions for buoyancy induced flows in a saturated porous medium adjacent to impermeable horizontal surfaces, Chandrasekhara *et al.*, [24] assumed that permeability varied exponentially from the wall and provided the model given by

$$K(y) = K_0(1 + de^{-y/\alpha}) \quad (12)$$

where K_0 is the value of the permeability at the edge of the boundary layer, α is a constant having dimensions of y, and d is a constant. A similar model was used by Hassanien *et al.*, [25], in their study of vortex instability of mixed convection flow.

Rees and Pop, [26], studied the free convection in a vertical porous medium with the exponential model of permeability

$$K(y) = K_\infty + (K_w - K_\infty)e^{-y/d} \quad (13)$$

where K_w is the permeability at the wall, K_∞ is the permeability of the ambient medium, and d is the length scale over which the permeability varies.

Alloui *et al.*, [27], analyzed convection in binary mixtures using the exponential model of the form

$$K(y) = e^{cy} \quad (14)$$

where c is a fitting parameter. When c is small, permeability function (7) behaves like the linear function

$$K(y) = 1 + \text{cy} \quad (15)$$

Abu Zaytoon *et.al.*, [18], considered an exponential permeability of the form

$$K(y) = \frac{1}{2e}(e - e^{-y}) \quad (16)$$

in their analysis of flow through composite porous layers, one of which was a Darcy layer.

Pillai *et.al.*, [28], considered the steady MHD flow in an inclined channel over a porous layer with a decaying exponential permeability that depends on the depth of the porous layer, using the following model due to Sinha and Chadda, [29]:

$$K(y) = K_0 e^{-cy} \quad (17)$$

while Silva- Zea *et.al.*, [30], studied MHD flow using a model of the form

$$K(y) = K_0 e^{c\left(\frac{y}{h}\right)} \quad (18)$$

where c is a positive number, and K_0 is the average permeability of the medium.

2.3. Polynomial Models

Other important models of one dimensional permeability variation used in soil mechanics include those found in Schiffman and Gibson, [31]; Mahmoud and Deresiewicz, [32], and Jang and Chen, [33]. Cheng, [17], used the model

$$K(y) = K_o(1 + \beta y)^\delta \quad (19)$$

where β and δ are parameters of curve fittings, and K_0 is the characteristic permeability of the medium. A typical value for δ is 2.

In their study of the effects of variable permeability on MHD flow in a porous channel of depth h, Narasimha Murthy and Feyen, [34], used the following form of equation (12):

$$K(y) = K_0(1 + \frac{y}{h})^2 \quad (20)$$

where K_0 is the permeability in the interior of the porous medium, while Srivastava and Deo, [35], employed the form:

$$K(y) = K_0(1 - \varepsilon y)^2 \quad (21)$$

where $0 \leq \varepsilon < 1$, in their analysis of Couette and Poiseuille MHD flow in a porous layer.

In their analysis of thermo-solutal convection in a heterogeneous porous layer enclosed in a rectangular cavity, Choukairy and Bennacer, [36], implemented a permeability function of the following form, wherein *n* is a parameter.

$$K(y) = 1 + 4(2y)^n \quad (22)$$

2.4. Periodic Models

Mathew, [37], employed the following periodic variable peermeability model in the study of two-dimensional MHD convective heat transfer through a vertical porous channel:

$$K(y) = K_0[1 + b\cos(\pi y)]^{-1} \quad (23)$$

where b is a constant and K_0 is a constant reference permeability.

2.5. Transition Layer Models

Some of the recent advances in the study of flow through a free-space channel over porous layers include imbedding a Brinkman transition layer of variable permeability between the channel and a constant permeability porous layer. This is depicted in **Fig. 1**, if we take Layer 1 to be a semi-infinite Darcy layer, Layer 2 a Brinkman variable permeability layer and Layer 3 a free-space fluid layer.

At the interface with the channel, permeability of the transition layer approaches infinity, while at the interface with the porous layer, it assumes the value of the constant permeability of the porous layer. Nield and Kuznetsov, [15], introduced a permeability function in their analysis of the transition layer, in such a way that the reciprocal of the permeability varies linearly across the layer, according to

$$K(y) = aK_0H/y \quad (24)$$

where H is the overall porous layer thickness, a is related to the thickness of the transition layer, K is the transition layer permeability, and K_0 is the constant permeability of the underlying porous layer. This choice of permeability distribution reduces Brinkman's equation in the transition layer to an Airy's inhomogeneous ordinary differential equation.

The Nield and Kuznetsov approach has been successfully implemented in obtaining solutions to flow through composite porous layers of variable permeability by Abu Zaytoon *et.al.*, [19]. They, [19] also provided analysis suggesting that the permeability distribution in the transition layer can be modelled in a way that reduces the governing Brinkman's equation to the generalized inhomogeneous Airy's equation, of which the Nield and Kuznetsov, [15], model is a special case. This is depicted in **Fig. 1**, if Layer 1 is taken to be a free-space fluid layer, Layer 2 a Brinkman variable permeability layer and Layer 3 a Brinkman constant permeability layer. Permeability function in this case has been described as:

$$K(y) = \frac{K_0[(\xi-\eta)H]^n}{(y-\eta H)^n}; \eta H < y < \xi H \quad (25)$$

Although Airy's and generalized Airy's equations are useful in the analysis of the transition layer, there are situations that lead to other special differential equations. One such flow configuration is where a porous layer of variable permeability is immersed in a free-space channel, [38].

If, in **Fig. 1**, Layer 1 is a free-space fluid layer, Layer 2 is a Brinkman variable permeability layer and Layer 3 is a free-space fluid layer, then permeability distribution across the Brinkman porous layer, with interfaces at $y = \eta H$ and $y = \xi H$, can be modelled using the following function:

$$K(y) = \frac{-\frac{H^2K_0}{c^2}}{(y-cH)^2 + H^2(\eta\xi - c^2)}; \eta H < y < \xi H \quad (26)$$

where K_0 is a reference constant permeability, and c is an adjustable parameter such that $\xi H + \eta H = 2cH \leq H$, or $\xi + \eta = 2c \leq 1$. The resulting porous layer thickness is $\xi H - \eta H$. Equation (26) produces both symmetric and non-symmetric regions and can be used in a number of ways, one of which is selecting the widths of the lower and upper fluid layer then determining ξ, η and c. It should be noted that this choice of permeability function reduces Brinkman's equation to a Weber inhomogeneous differential equation whose solution has been discussed in details by Abu Zaytoon and Hamdan, [38].

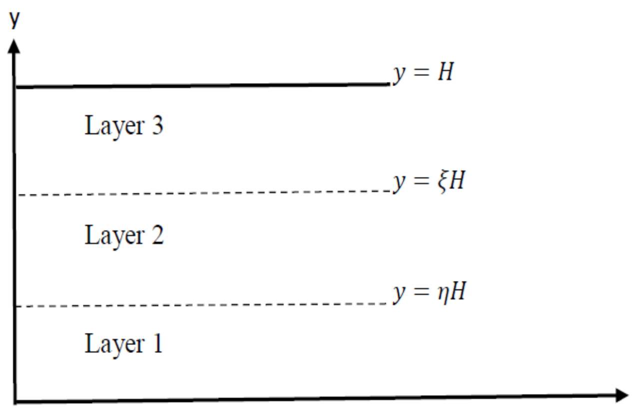

Fig. 1. Representative Sketch

3 Sigmoid Function

A function that has received considerable attention in neural networks and deep learning is the sigmoid function, [39], [40], [41], [42]. This function possesses a characteristic S-shaped curve and maps the number line onto a finite-length subinterval, such as (0,1).

Three popular sigmoid functions are the logistic function, the arctangent and hyperbolic tangent. In its most common form, the sigmoid logistic function function is given by:

$$S(x) = \frac{e^x}{1+e^x} = 1 - S(-x) \quad (27)$$

and has horizontal asymptotes at $x = \mp\infty$ with

$$\lim_{x\to+\infty} S(x) = 1 \text{ and } \lim_{x\to-\infty} S(x) = 0 \quad (28)$$

Its first two derivatives are given by

$$S'(x) = \frac{e^x}{(1+e^x)^2} \quad (29)$$

$$S''(x) = \frac{e^x(1-e^x)}{(1+e^x)^3} \quad (30)$$

$S(x)$ is increasing on its domain and has a point of inflection at $x = 0$. It represents the solution to the initial value problem

$$y' - y + y^2 = 0; y(0) = \frac{1}{2} \quad (31)$$

and its integral given by

$$\int S(x)dx = \ln(1 + e^x) + C \quad (32)$$

where C is a constant.

The sigmoid function enjoys various applications in deep learning and neural networks as it serves as an activation function (*cf.* [40], [41] and the references therein). Since this smoothly-increasing function has not made it to the porous media literature, the intention here is to present it as a candidate for modelling variations in permeability across a porous layer. The *S-shaped* graph of the sigmoid function, [42], makes it appealing in the study of transition layers. . In what follows, a modification of $S(x)$ is provided and used in the Poiseuille flow through a Brinkman porous layer of variable permeability. The sigmoid function is used here to create continuously varying permeability between relatively constant permeability regions. This approach, which treats the flow domain as one region with variable permeability, seems to replicate the flow in layered media.

Consider the unidirectional, fully-developed flow through the porous domain described by $\{(x, y)|-\infty < x < +\infty, 0 < y < h\}$. The porous layer is bounded by solid, impermeable walls at $y = 0$ and $y = h$, and the flow is governed by Brinkman's equation (5). On the macroscopic solid walls, the no-slip condition $u(0) = u(h) = 0$ is used. It is assumed that the permeability in the layer varies in the lateral direction, y, and permeability variations are governed by the following modified form of the sigmoid function:

$$k(y) = k_1 + (k_2 - k_1)\left(1 + e^{-(y-y_s)/\delta}\right) \quad (33)$$

where $y_s = h/2$, $\delta = h/10^n$, and n is adjusted to change the rate of transition between k_1 and k_2.

In this work, varying widths of permeability transition are simulated and the different parameters in (33) investigated. Graphs of $k(y)$ are shown in **Fig. 2** for *h=0.1m*, $k_1 = 0.0001m^2$, $k_2 = 0.001m^2$, and various values of *n*. With increasing *n* and the creation of very narrow variable permeability region, a stepwise change in permeability is observed. For *n=0* and *n=-1*, permeability varies approximately linearly, with a minimum amount of change when *n < 0*. This is quite significant and implies that the thickness of the variable permeability layer (region) can be controlled as it varies smoothly between two constant permeability layers (regions) without having to resort to interfacial conditions.

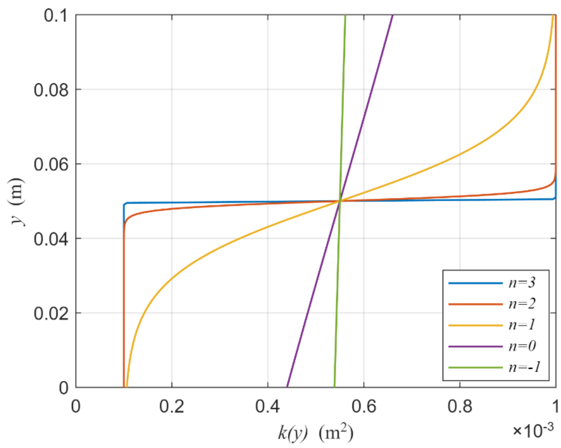

Fig. 2. Variable Permeability as a Function of Parameter *n*

It should be noted that the permeability on the solid bounding walls is zero. This creates a jump in the permeability as we move away from the solid walls into the porous layer. However, this situation is no different than the usual practice of analyzing the flow through a constant permeability layer bounded by solid walls, where there is a jump in permeability as it starts with a value of zero on the boundary and takes on a non-zero constant value in the porous layer.

Equation (5), with $k(y)$ given by (33), was solved numerically using fifth order algorithm in MATLAB with relative error tolerance of 10^{-6} and absolute error tolerance of 10^{-6}. All quantities and parameters have been left in dimensional form for ease of physical interpretation of results. Results were generated for the following dimensional quantities:

- $h = 0.1\,m$
- $G = -10\,Pa/m$
- $\mu = \mu_{eff} = 0.001\,N \cdot s/m^2$ (water)
- $k_1 = 0.0001\,m^2$
- $k_2 = 0.001\,m^2$
- $n = -1, 0, 1, 2, 3$

Velocity profiles for various values of n are illustrated in **Fig. 3**, which shows the faster flow regiments in the upper parts of the channel with increasing n. In these regions, permeability is higher, thus resulting in faster flow. For the cases of *n=0* and *n=-1*, the velocity profiles resemble those of Brinkman flow through constant permeability layers. This is an expected behaviour as the permeability variations across the layer are linear with small variations when *n=0* and *n=-1*. Furthermore, inflection in the velocity profiles is found to increase in strength as the transition region narrows with increasing n.

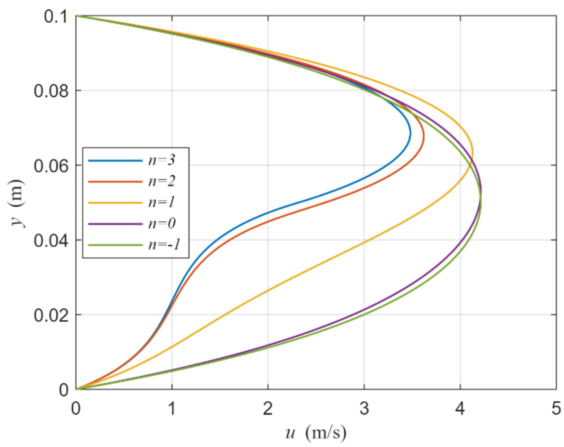

Fig. 3. Velocity Profiles for Various Values of *n*

Darcy friction, or Darcy resistance in Brinkman's equation, is given by the quantity $-\frac{\mu}{k(y)}u$. This has been calculated for various values of *n* and illustrated in **Fig. 4**, which shows the effects of variable permeability on resistance offered to the flow across the layer, and the thickness of variable permeability region that is impacted (in particular, for *n = 1,2,3*). As the transition layer thickness decreases with increasing *n*, the flow approaches a step-change in permeability and the peak of the Darcy friction increases dramatically. For *n=0* and *n=-1*, the near constant permeability across the channel results in a parabolic Darcy resistance profile, as there are no spikes or sudden changes in permeability.

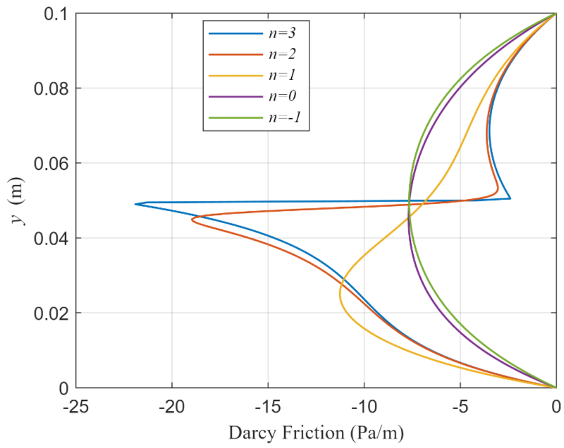

Fig. 4. Darcy Friction for Various Values of *n*

Viscous shear stress, $\tau(y)$, across the porous layer is given by the expression: $\tau = \mu_{eff}\left(\frac{\partial u}{\partial y}\right)$. **Fig. 5** illustrates the effects of the employed permeability model and the values of *n* in the Sigmoid function on shear stress across the channel. For a given effective viscosity coefficient, $\mu_{eff} = \mu$, the term $\frac{\partial u}{\partial y}$ gives the instantaneous change in velocity with respect to the lateral variable. In regions where the transition across the variable permeability layer is rapid (i.e. a thin transition layer), these changes are abrupt, as shown for the cases of *n=2, 3.*

For the case of *n=1*, changes are smooth, and for *n=0, -1*, the shear stress profiles are closer to linearity. Since the corresponding velocity profiles are almost parabolic, the viscous shear profiles should be almost linear. The reason that they are not linear could be ascribed to the fact that a no-slip condition was used on the solid walls, which resulted in a jump in the permeability as one moves into the porous channel. It is expected that the viscous shear profiles would be different in cases of a slip condition associated with a non-zero permeability on a porous boundary.

The above behaviour also influences, and explains, the behaviour of the net viscous shear profiles, given in **Fig. 6** for various *n*. The net viscous shear stress across a fluid element is given by the net instantaneous change of the shear stress term with respect to the lateral direction, namely,

$$\left(\frac{\partial \tau}{\partial y}\right) = \mu_{eff}\frac{\partial^2 u}{\partial y^2} = -G + \frac{\mu}{k}u \qquad (34)$$

As the value of n increases and the permeability approaches a step-change, the net viscous shear across a fluid element must also adjust abruptly to maintain the force balance modelled by equation (5).

As can be seen in **Fig. 6** when comparing the net viscous shear profiles of rapidly varying permeability to those of near constant permeability, there are substantial deviations to the net viscous shear force. For example in the cases of $n=0, -1$, the net viscous shear is always negative while for the positive values of n there are regions of positive net viscous shear. This signifies a profound physical change in the physics governing the flow across the layer as the direction of the net viscous shear force changes from impeding the flow (negative net viscous shear) to enhancing the velocity (positive net viscous shear).

The direction change of the net viscous shear force acting on a fluid element that can be seen for the cases of $n=1, 2, 3$ results from the development of the inflection in the velocity profiles, which themselves are produced by the rapidly varying permeability. In these regions of inflection, regions of faster flow attempt to accelerate the adjacent slower regions in the direction of flow. Furthermore, in the cases of $n=2, 3$ it is seen that there is abrupt sign change in the net viscous shear, the location of which correlates to the region of rapid variation of the permeability.

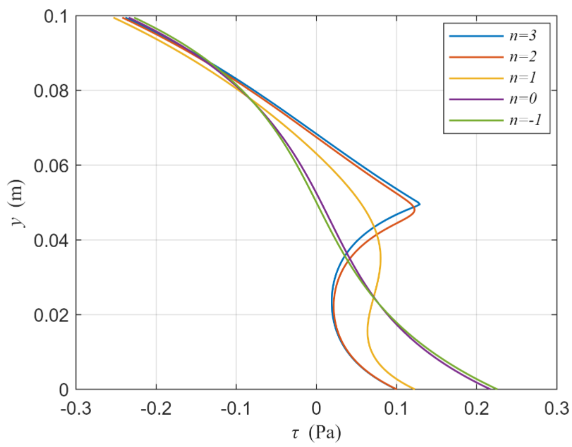

Fig. 5. Viscous Shear Stress for Various Values of n

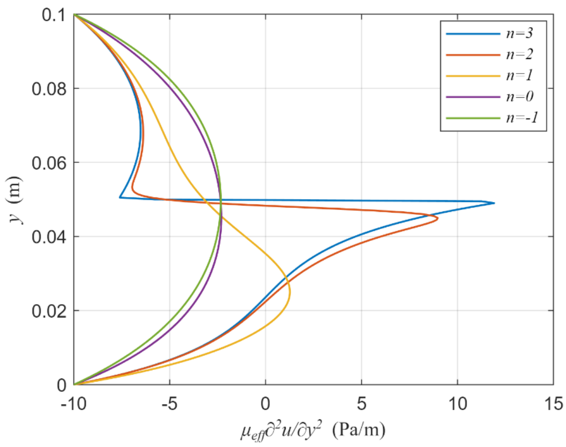

Fig. 6. Net Viscous Shear Stress for Various n

4 Conclusion

In this work, a listing and a classification of variable permeability models that have been reported and used frequently in the literature on porous media, have been provided. The classification provided groups the models into five different sub-classes for ease of reference.

A variable permeability model that is based on the sigmoid logistic function, which was modified to describe smooth transition between two regions of constant permeability in a single domain without the need for interfacial conditions, has been implemented. The sigmoid function approach is believed to be promising in the analysis of flow through variable permeability layers and in the simulation of flow in the transition layer. Furthermore, the effect of imposing step-wise changes to the permeability on the various terms governing flow in porous media has been examined.

Future aspects of this work is to compare the different classes using a number of flow situations of interest and of practical significance, and to extend the models to the study of pressure-dependent viscosity fluid flows.

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Conflicts of Interest

The authors have no conflicts of interest to declare .

Contribution of individual authors

Both authors carried out the literature survey, problem formulation, result analysis, and manuscript preparation.
D.C. Roach formulated the sigmoid model and carried out the simulation.

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