

Theorem 8 Three first integrals (78), (81), (82) of the system (76), (77) are equivalent to three first integrals (73), (74), (75) of the system (69), (70).

Indeed, the couples of the first integrals (78), (73) and (82), (75) coincides, if we substitute $b = -b_*$. And finally, we need to identify the phase variables Z_k , $k = 1, 2$, for the system (76), (77) with the phase variables w_k , $k = 1, 2$, of the system (69), (70). Because of their cumbersome character, the similar arguments concerning of the couples of the first integrals (81), (74), we do not represent.

Thus, we have the following topological and mechanical analogies in the sense explained above.

(1) A motion of a fixed physical pendulum on a spherical hinge in a flying medium (nonconservative force fields)

(2) A spatial free motion of a rigid body in a nonconservative force field under a tracing force (in the presence of a nonintegrable constraint).

(3) A spatial composite motion of a rigid body rotating about its center of mass, which moves rectilinearly and uniformly, in a nonconservative force field

On more general topological analogues, see also [8], [9], [12], [13], [15], [16], [17].

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The author(s) contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflict of Interest

The author(s) declare no potential conflicts of interest concerning the research, authorship, or publication of this article.

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