Hybrid damage prediction procedure for composite laminates submitted to spectra loading

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Abstract: - In this document, a hybrid procedure is constructed in order to predict the damage of a composite unidirectional laminate under random loading. This procedure is based on two pillars: a stiffness degradation model (SD-M) combined with an energy approach taking into account the effect of load ratio in addition to a system of equations generated by SSDQM method (Space State Differential Quadrature Method) which we have solved with a novel technic. The outputs of SSDQM method, previously serving for free vibration behavior analysis of composite structures, are used with those of SD-M model to predict damage failure of a composite laminate subjected to spectra loading. The results obtained correlate very well with experimental ones and an extensive comparison with other models validate the accuracy and convergence characteristics of this hybrid procedure.

Key-Words: - Composite laminate; Spectrum loading; Damage energy; Lifetime prediction; Stiffness degradation; Load ratio.

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1 Introduction

The life of a structure can come to a sudden end or last longer, but only for a limited period. This latter case is usually accompanied by a reduction in yield, known as aging. Under a high load, a structure or component can deteriorate in one fell swoop, while it can actually withstand lower loads. On the other hand, the same structure or component can also be ruined under lower loads if they are applied over longer delays, either constant amplitude (static) or variable amplitude (fatigue). These loadings sustained by the mechanical structures are induced by external stresses (forces, thermal, accelerations, etc.). The phenomenon of degradation of the properties of a material due to the application of loads that fluctuate over time is called fatigue and the resulting ruin is called fatigue failure.

Due to the complexity of fatigue damage process in composite materials, prediction of their fatigue life is of vital importance. But, a proper modeling of the damage evolution is the foundation for predicting the fatigue life of composite structures which enables an appropriate evaluating of structure's performances in its early cycles of life and prevents catastrophic failures. Some authors were based on residual strength or stiffness, Yao and Himmel [**1**] predict residual strength caused by fatigue damage in glass and carbon fiber reinforced plastics. To predict and investigate the effect of high-stress peaks on fatigue life of carbon fiber reinforced plastics, Aghazadeh and Majidi [**2**] applied a residual strength. Another stiffness-based model for life prediction of Wu and Yao researchers [**3**] is also considered quite well model to predict residual fatigue life of composites. However, predictions of these models are noticeably divergent to experimental values and mostly yield a high percent of error in fatigue life prediction.

In last decades, stiffness-based models [**4-7**] were another range of models that have been developed. The damage degree is quantified by measuring young's modulus of the material. But, most of these models present especially two major deficiencies, firstly is a high number of parameters which requiring extensive experimental data to calculate them. While the second deficiency is their inability to simulate accurately the damage progress in its well-known three stages [**7-9**]. Additionally to the aforementioned shortcoming, most of these models are validated for a specific typed of composite and are not evaluated in a wide range of loading levels [**1**, **10**, **11**].

On the other hand, vibration analysis of composite structures is also a big area of research encouraging researchers to ensure the usability, durability, and safety during composite structure's lifetime. Many works were conducted in this trend [**12-22**], a number of models and methods have been developed. Among them stands out state space method combined with differential quadrature method briefly noted SSDQM.

In this paper we aimed to resolve the discussed limitations of stiffness-based models throughout a hybrid damage prediction procedure. It consists of coupling a stiffness-based model with a SSDQM method while relying both on an energy approach for predicting damage rupture [**23**] and a well-known Palmgreen Miner rule [**24**]. In the first place, the SSDQM is solved by a new proposed technic differently to ones found in the literature. Then a coupling algorithm is developed to survey damage progress of the composite laminate and consequently predict their damage rupture. Numerical validation of the hybrid procedure demonstrates that most of the predicted lifetimes lead to quantitatively better estimations.

2 New technic for solving the SSDQM method

In their works [**14, 15**, **21**], authors combined state space method (SSM) [**18-20**] with differential quadrature method (DQM) [**12-14**] to establish an equation system (1) and each one proceeds in his own way to solve it.

$$\frac{\mathrm{d}\Delta}{\mathrm{dz}} = \mathrm{M}^{(\mathrm{k})}\,\Delta \tag{1}$$

Where:

$$\Delta = \begin{bmatrix} Z & U & V & W & T_{xz} & T_{yz} \end{bmatrix}^T,$$

$$\mathrm{Z} = [Z_1 \;\; Z_2 \;\; \ldots \;\; Z_N]^T\ ,$$

$$M^{(k)} = \begin{bmatrix} 0 & M_1^{(k)} \\ M_2^{(k)} & 0 \end{bmatrix},$$

$$\mathrm{M}_1^{(\mathrm{k})} = \begin{bmatrix} -\rho\omega^2 I & -\mathrm{g}^{(1)} & \lambda_\mathrm{b} I \\ -\mathrm{g}^{(1)} & \mathrm{c}_7 I & 0 \\ -\lambda_\mathrm{b} I & 0 & \mathrm{c}_8 I \end{bmatrix},$$

$$\mathrm{M}_2^{(\mathrm{k})} = \begin{bmatrix} \mathrm{c}_9 I & \mathrm{c}_1\mathrm{g}^{(1)} & -\mathrm{c}_5\lambda_\mathrm{b} I \\ \mathrm{c}_1\mathrm{g}^{(1)} & \left(\mathrm{c}_6\lambda_\mathrm{b}^2 - \rho\omega^2\right)\mathrm{I} - \mathrm{c}_2\mathrm{g}^{(2)} & (\mathrm{c}_3+\mathrm{c}_6)\lambda_\mathrm{b}\mathrm{g}^{(1)} \\ \mathrm{c}_5\lambda_\mathrm{b} I & -(\mathrm{c}_3+\mathrm{c}_6)\lambda_\mathrm{b}\mathrm{g}^{(1)} & \left(\mathrm{c}_4\lambda_\mathrm{b}^2 - \rho\omega^2\right)\mathrm{I} - \mathrm{c}_6\mathrm{g}^{(2)} \end{bmatrix}$$

The components of the matrix Δ are vectors defined as the state variables vector Z. N is the discretization number, k significant the k^{th} ply of the laminate, I is the identity matrix and $g_{ij}^{(n)}$ are the weighting coefficients [**16**] dependent on Chebyshev-Gauss-Lobatto points xi [**17**]:

$$\mathrm{x_i} = \frac{\mathrm{a}}{2}\left[1 - \cos\frac{(\mathrm{i}-1)\pi}{\mathrm{N}-1}\right], \quad \mathrm{i} = 1, 2, \ldots, \mathrm{N},$$

The coefficients c_i are defined and given in reference [**17**] which depends on the elastic material constants, ρ is the mass density and ω is a circular frequency. While λ_b is a constant parameter depending on an arbitrary positive integer n and is expressed as following: $\lambda_b = \frac{\pi n}{b}$.

For a specific problem, the boundary conditions at edges (x=0 and x=a) of the studied plate must be taken into consideration so that we can have a unique solution of the equation (1). By applying boundary conditions we add a subscript 'q' to equation (1) to indicate it:

$$\frac{\mathrm{d}}{\mathrm{dz}}\Delta_\mathrm{q} = \mathrm{M}_\mathrm{q}^{(\mathrm{k})}\Delta_\mathrm{q} \tag{2}$$

Explicit expressions of matrix $M_{1q}^{(k)}$ and $M_{2q}^{(k)}$ for each boundary condition case are given in appendix A.

Many methods are envisaged in the literature to solve this system (2), Xu and Ding [**22**] used algebra rules and Cayley-Hamilton theorem to solve it. Direct use of global transfer matrix is one of the methods found in the literature [**15-17**] to solve this system. In this work, the global transfer matrix is used also to solve the system (2) but in combination with a Coupling Joint matrix proposed and noted J_C.

The novel technic developed here to solve the expression (2) is consisted on following steps:

- First, the vector of state variables for the ply k is written as:

$$\Delta_i^{(k)} = \begin{Bmatrix} Z_i^{(k)} \\ U_i^{(k)} \\ V_i^{(k)} \\ W_i^{(k)} \\ T_{xz_i}^{(k)} \\ T_{yz_i}^{(k)} \end{Bmatrix} \quad (3)$$

Where i take '0' (inferior face of the ply) or '1' (superior face of the ply),

- Second, the following formula is supposed to assure the continuity condition between two adjacent plies: $J_C.\begin{Bmatrix} \Delta_1^{(k)} \\ \Delta_0^{(k+1)} \end{Bmatrix} = 0$ where $J_C = [I \ -I]$ is named the Coupling Joint matrix. Noting that ''I'' is identity matrix with the same dimension as the length of the vector Δ.
- Third, the loading conditions at the superior interface and the inferior one are expressed respectively like: $J_{sup}.\Delta_1^m = f_{sup}$ and $J_{inf}.\Delta_0^1 = f_{inf}$. The inferior face doesn't submit any mechanical forces where the vector force (stresses) f_{inf} is zero and consequently the matrix J_{inf} is equal to zero. On the other side, the superior face laminate's is submitted to a bending loading where the vector force f_{sup} and the matrix J_{sup} are written as following:

$$f_{sup} = \begin{Bmatrix} q_{sup} \\ 0 \\ 0 \end{Bmatrix} \quad (4)$$

$$J_{sup} = \begin{bmatrix} i_1 0 & 0 & 0 & 0 & 0 \\ 0 0 & 0 & 0 & i_5 & 0 \\ 0 0 & 0 & 0 & 0 & i_6 \end{bmatrix} \quad (5)$$

Where i_1, i_5 and i_6 are the matrix identities having dimensions adequate to the state variables vectors lengths Z, T_{xz} and T_{yz} respectively. We note also that J_{inf} matrix dimension is the same as matrix J_{sup}.

The gathering of all above expressions of joint coupling matrix conduct to the general formula (6):

$$J.\Delta = f \quad (6)$$

With:

$J = \text{diag}\,[J_{inf}\, J_{C_1} J_{C_2} \cdots J_{C_m} J_{sup}]$;

$f = [f_{inf}^T 0_1\ 0_2 \ldots 0_m f_{sup}^T]$ where 0_i is a zero vector of the i^{th} ply;

$$\Delta = [\left(\Delta_0^{(1)}\right)^T \begin{pmatrix} \Delta_1^{(1)} \\ \Delta_0^{(2)} \end{pmatrix}^T \ldots \begin{pmatrix} \Delta_1^{(m-1)} \\ \Delta_0^{(m)} \end{pmatrix}^T \left(\Delta_1^{(m)}\right)^T] ;$$

For any ply k of a composite laminate, the solution proposed of the matrix system (2) is written as follow:

$$\begin{Bmatrix} \Delta_0^{(k)} \\ \Delta_1^{(k)} \end{Bmatrix} = M_q^{(k)}.\Delta_0^{(k)} \quad (7)$$

Where

$M_q^{(k)} = \begin{bmatrix} I \\ T_k \end{bmatrix}$;

$T_k = \exp(\frac{h_k}{h}.M_q^{(k)})$;

The assembling of all plies of the laminate structure gives:

$$\Delta = \mathrm{M}.\Delta_0 \tag{8}$$

With

$M = diag[M_1\ M_2\ \dots\ M_{m-1}\ M_m]$;

$\Delta_0 = [(\Delta_0^{(1)})^T\ (\Delta_0^{(2)})^T\ \dots\ (\Delta_0^{(m-1)})^T (\Delta_0^{(m)})^T]^T$;

By substituting the equation (8) into the equation (6) the system bellow (9) is obtained:

$$\mathrm{J.M.}\Delta_0 = \mathrm{f} \tag{9}$$

Finally, the resolution of this system (9) gives all state variables vectors in both superior and inferior faces of each laminate's ply.

3 Lifetime assessment procedure

Together to the outputs given by the resolution of a system matrix developed in the previous section, the procedure that we aim to construct in this section is based also on both, a stiffness degradation model (noted SD-M) [**25**] and an algorithm used for an energy damage prediction model [**23**].

Stiffness degradation models category is one of the most popular manor to predict damage of structures [**4-7**, **25-26**] which quantify the extent of damage by measuring the Young's modulus of the material. The formula of (10) is used to construct the present procedure:

$$\frac{\mathrm{k_i}}{\mathrm{k_0}} = \mathrm{a_2} - \mathrm{a_1}\ln\left(\frac{\frac{\mathrm{n_i}}{\mathrm{N}}}{1 - \frac{\mathrm{n_i}}{\mathrm{N}}}\right) \tag{10}$$

Where a_1 and a_2 are material parameters depending on the ultimate static force F_r, the stress ratio r_i and the minimal force F_u for which the failure is not reached.

The second amount on which this procedure depends is an energy approach [**23**] used to determine two material parameters $\emptyset$ and α of the formula (11) in addition to lifetime at rupture N_{max}.

$$\mathrm{N} = \frac{\mathrm{N_{max}}}{1 + \mathrm{e}^{-\emptyset(\Psi+\alpha)}} \tag{11}$$

Based on this amounts, the algorithm developed of the procedure is scheduled as follow:

- Initially, a Ergodic, Gaussian, Stationary and random loading (noted EGSR) is considered and thanks to algorithm rainflow [**23**, **25**] we obtain for each cycle 'i' the mean value $F_{m,i}$ and the amplitude $F_{a,i}$. These values obtained for an elementary cycle 'i' are used as inputs of the SSDQM to calculate the six components of displacements and stresses on each point of the discretized laminate. Then, the maximum deflection $\delta_{max,i}$ resulted on the midle of the laminate is used to assess initial stiffness $k_{0,i}$ through expression (12), consequently, the minimal deformation energy of this cycle 'i' is deduced via expression (13).

$$\mathrm{Fa,i} = \mathrm{k0,i}\ \delta_{\mathrm{max,i}} \tag{12}$$

$$\Psi_{\mathrm{min,i}} = \frac{\mathrm{F_{a,i}^2}}{2\mathrm{k_{0,i}}} \tag{13}$$

- In the second step, SD-M model (10) is used to find final stiffness $k_{f,i}$ for each cycle 'i' which is used to determine the maximal deformation energy (14) of the considered cycle 'i'.

$$\Psi_{\mathrm{max,i}} = \frac{\mathrm{F_{a,i}^2}}{2\mathrm{k_{f,i}}} \tag{14}$$

- Thirdly, we calculate the loading parameter μ_i of each elementary cycle 'i' via expression (15). Therefore, by using the energy approach [**23**] the material parameters $\emptyset_i$ and α_i as well as the rupture lifetime N_i.

$$\mu_{\mathrm{i}} = \mathrm{r_{,i}} * \frac{\mathrm{F_{m,i}}}{\mathrm{F_{max,i}}} \tag{15}$$

- Finally, the well-known Palmgreen-Miner rule [**24**] is adopted to predict lifetime of the composite laminate examined:

$$T = \frac{1}{\sum \frac{n_i}{N_i}} \tag{16}$$

To sum-up, figure (Fig. 1) show clearly the steps of the procedure developed above.

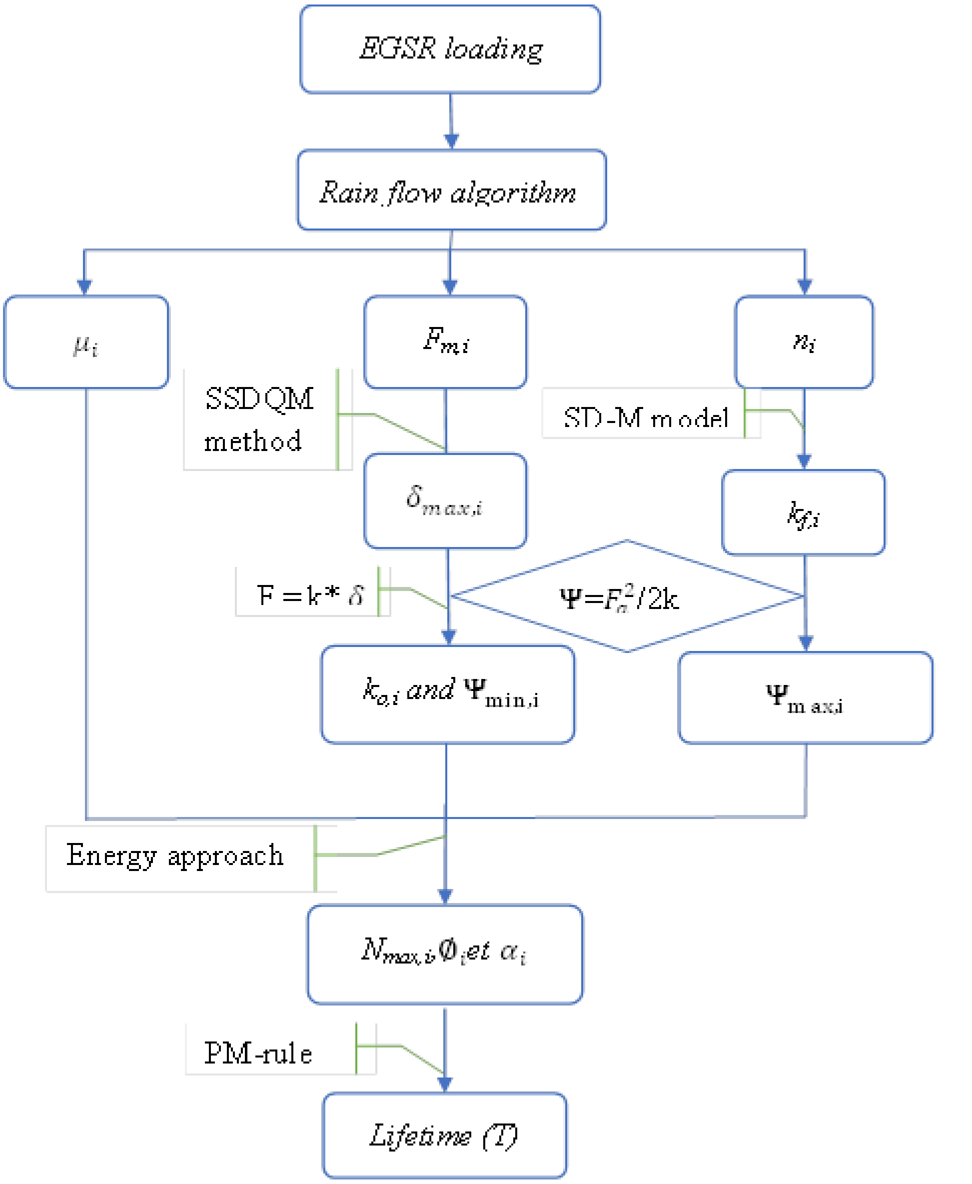

Fig. 1. *Hybrid procedure for damage prediction of a composite laminate.*

4 Results and discussions

The material used for the experimental study is a quasi-isotropic graphite/epoxy composite laminate $[45/0/90]_{3S}$ and the tests on which we based were performed by MTS 810 servo-hydraulic machine [**25**, **27**]. Two variable amplitude fatigue experiments are conducted where four specimens are tested for each experimental case defined by the mean value F_m and the standard deviation σ of the loading.

In order to validate the procedure developed in this work, two EGSR loadings were simulated identically to the experimental ones [**25**, **27**]. In the first case, the mean value of a loading is Fm = 2000 N and its standard deviation is σ = 500 N, while in the second case the mean value and the standard deviation taken are F_m = 1500 N and σ = 350 N respectively. Experimental lifetimes are given in Table 1 together with our model predictions and some model results of the literature [**23**, **25**, **27-28**].

Table 1 Comparison between hybrid model predictions, experimental lifetimes and some literature's models.

Lifetime (in cycles)	*Loading 1:* F_m=2000 (N) ; σ =500 (N)	*Loading 2:* F_m=1500 (N) ; σ =350 (N)
*Experimental test [**25**]*	*4410*	*54517,5*
*Energy model [**23**]*	*6504,9*	*57909*
*Statistical model [**27**]*	*5900*	*58700*
*Stiffness degradation model [**25**]*	*5100*	*52300*
*Niesloney's first model [**28**]*	*1800*	*3000*
*Niesloney's second model [**28**]*	*3700*	*38000*
Proposed hybrid procedure	*6454,8*	*57883*

Hybrid procedure predictions are quite identical to energy model [**23**] results and are very close to both, statistical model [**27**] and stiffness degradation model [**25**] ones. Niesloney's models [**28**] are too divergent in comparison with experimental tests [**25**] and with all models presented in the table 1.

On the other side, we remark that the difference rate between our procedure predictions and experimental lifetimes is very acceptable, especially for the second experiment which doesn't exceed 5 percent whereas in the first experiment this rate is quite higher. So despite that, all other models are also in the same order of magnitude as our hybrid model, but this difference can be explained from the fact that all these models used a linear cumulative damage rule to assess lifetimes which doesn't take into account the load sequence and interaction effects.

5 Conclusion

Space state differential quadrature method is solved with a new technic, a series of stresses and deformations are obtained for each load's cycle. The SSDQM is usually used to analyze free vibration behavior of different composite structures, in this work it was exploited in conjunction with one damage prediction model namely stiffness degradation model (SD-M model) and an energetic approach to predict damage rupture of a composite laminate.

A satisfactory convergence of this hybrid procedure is verified through numerical comparison with other numerical models and also versus experimental tests. Hence, this procedure presents an ambition solution to monitor and predict damage evolution inside the laminate.

Appendix A:

- Clamped - Clamped (CC) :

$$\mathrm{M}_{1\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}-\rho\omega^2 I_{N-2}-\frac{1}{c_7}f_{CC} & -g_{CC}^{(1)} & \lambda_b I_{N-2}\\ -g_{CC}^{(1)} & c_7 I_{N-2} & 0\\ -\lambda_b I_{N-2} & 0 & c_8 I_{N-2}\end{bmatrix},$$

$$\mathrm{M}_{2\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}c_9 I_{N-2} & c_1 g_{CC}^{(1)}(c_6\lambda_b^2-\rho\omega^2)I_{N-2} & -c_5\lambda_b I_{N-2}\\ c_1 g_{CC}^{(1)} & -c_2 g_{CC}^{(2)}-\frac{c_1^2}{c_9}f_{CC} & (c_3+c_6)\lambda_b g_{CC}^{(1)}\\ c_5\lambda_b I_{N-2} & -(c_3+c_6)\lambda_b g_{CC}^{(1)} & \left(c_4\lambda_b^2-\rho\omega^2\right)I_{N-2}-c_6 g_{CC}^{(2)}\end{bmatrix},$$

The elements f_{CC}, $g_{CC}^{(1)}$ and $g_{CC}^{(2)}$ are expressed as following:

$$f_{CCij}=g_{i1}^{(1)}g_{1j}^{(1)}+g_{iN}^{(1)}g_{Nj}^{(1)};$$

$g_{CCij}^{(r)}=w_{ij}^{(r)}$ (i, j = 2, 3, …, N-1) and (r=0 or 1).

- Clamped – Simply support (CS) :

$$\mathrm{M}_{1\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}-\rho\omega^2 I_{N-2}-\frac{1}{c_7}f_{1CS} & -g_{CS}^{(1)} & \lambda_b I_{N-2}\\ -[g_{CS}^{(1)}]^T & c_7 I_{N-1} & 0\\ -\lambda_b I_{N-2} & 0 & c_8 I_{N-2}\end{bmatrix},$$

$$\mathrm{M}_{2\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}c_9 I_{N-2} & c_1 g_{CS}^{(1)}(c_6\lambda_b^2-\rho\omega^2)I_{N-1} & -c_5\lambda_b I_{N-2}\\ c_1[g_{CS}^{(1)}]^T & c_2 h-\frac{c_1^2}{c_9}f'_{1CS} & (c_3+c_6)\lambda_b[g_{CS}^{(1)}]^T\\ c_5\lambda_b I_{N-2} & -(c_3+c_6)\lambda_b g_{CS}^{(1)} & \left(c_4\lambda_b^2-\rho\omega^2\right)I_{N-2}-c_6 g_{CS}^{(2)}\end{bmatrix},$$

Where :

$$h=f'_{NCS}-g'^{(2)}_{CS}$$

$$f_{1CSij}=g_{i1}^{(1)}g_{1j}^{(1)}\qquad(i,j=2,3,\dots,N-1),$$

$$g_{CS}^{(2)}=g_{CC}^{(2)},$$

$$f'_{NCSij}=g_{iN}^{(1)}g_{Nj}^{(1)},\qquad g'^{(2)}_{CSij}=w_{ij}^{(2)},$$

$$f'_{1CSij}=g_{i1}^{(1)}g_{1j}^{(1)}\qquad(i,j=2,3,\dots,N),$$

$$g_{CSij}^{(1)}=w_{ij}^{(1)}(i=2,3,\dots,N-1\ and\ \ j=2,\dots,N).$$

- Simply support – Simply support (SS) :

$$\mathrm{M}_{1\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}-\rho\omega^2 I_{N-2} & -g_{SS}^{(1)} & \lambda_b I_{N-2}\\ -[g_{SS}^{(1)}]^T & c_7 I_{N-2} & 0\\ -\lambda_b I_{N-2} & 0 & c_8 I_{N-2}\end{bmatrix},$$

$$\mathrm{M}_{2\mathrm{q}}^{(\mathrm{k})}=\begin{bmatrix}c_9 I_{N-2} & c_1 g_{SS}^{(1)} & -c_5\lambda_b I_{N-2}\\ c_1[g_{SS}^{(1)}]^T & \left(c_6\lambda_b^2-\rho\omega^2\right)I+c_2\left(f-g^{(2)}\right) & (c_3+c_6)\lambda_b[g_{SS}^{(1)}]^T\\ c_5\lambda_b I_{N-2} & -(c_3+c_6)\lambda_b g_{SS}^{(1)} & \left(c_4\lambda_b^2-\rho\omega^2\right)I_{N-2}-c_6 g_{SS}^{(2)}\end{bmatrix},$$

Where

$$f_{SSij}=g_{i1}^{(1)}g_{1j}^{(1)}+g_{iN}^{(1)}g_{Nj}^{(1)}\qquad(i,j=1,2,\dots,N),$$

$$g_{SSij}^{(1)}=w_{ij}^{(1)}\ \ (i=2,3,\dots,N-1\ and\ j=1,\dots,N),$$

$$g_{SS}^{(2)}=g_{CC}^{(2)},$$

- Clamped – Free (CF) :

$$\mathrm{M}_{1q}^{(k)} = \begin{bmatrix} -\rho\omega^2 E_1 - f_{1CF} & -g_{1CF}^{(1)} & \lambda_b E_1 \\ -g_{2CF}^{(1)} & c_7 I_{N-2} & 0 \\ -\lambda_b I_{N-1} & 0 & c_8 I_{N-1} \end{bmatrix},$$

$$\mathrm{M}_{2q}^{(k)} = \begin{bmatrix} c_9 E_2 & M_{2,12} & M_{2,13} \\ c_1 [g_{1CF}^{(1)}]^T & M_{2,22} & M_{2,23} \\ c_5 \lambda_b E_2 & M_{2,32} & M_{2,33} \end{bmatrix},$$

Where

$$M_{2,12} = c_1 \left[g_{2CF}^{(1)}\right]^T + \frac{c_2 c_9}{c_1} E_3,$$

$$M_{2,13} = -\frac{c_1}{\lambda_b} f_{NCF} - c_5 \lambda_b I_{N-1} - \frac{c_2 c_9}{\lambda_b c_1} E_4,$$

$$M_{2,22} = \left(c_6 \lambda_b^2 - \rho\omega^2\right) I_{N-2} + c_2 (f_{CF} - g_{CF}^{(2)}),$$

$$M_{2,23} = (c_3 + c_6) \lambda_b g_{2CF}^{(1)} + \frac{c_2}{\lambda_b} (\bar{f}_{NCF} - f_{NCF}^{*}),$$

$$M_{2,32} = -(c_3 + c_6) \lambda_b [g_{2CF}^{(1)}]^T + \frac{c_2 c_5}{c_1} \lambda_b E_3,$$

$$M_{2,33} = (c_3 + c_6) f_{NCF} + \left(c_4 \lambda_b^2 - \rho\omega^2\right) I_{N-1} - c_6 {g'}_{CF}^{(2)} - \frac{c_2 c_5}{c_1} E_4.$$

$$E_1 = \begin{bmatrix} 0 & 0 \\ I_{N-2} & 0 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0 & I_{N-2} \\ 0 & 0 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 0_{(N-2)x(N-2)} \\ \alpha_g \end{bmatrix}, \qquad E_4 = \begin{bmatrix} 0_{(N-2)x(N-1)} \\ \alpha_{fN} \end{bmatrix},$$

$$f_{1CFij} = g_{i1}^{(1)} g_{1j}^{(1)} \quad (i = 1, 2\ \dots, N-1\ and\ j = 2,3, \dots, N),$$

$$g_{1CFij}^{(1)} = w_{ij}^{(1)} \qquad (i = 1, 2, \dots, N-1\ and\ j = 2,3, \dots, N-1),$$

$$g_{2CFij}^{(1)} = w_{ij}^{(1)}, \qquad \bar{f}_{Nij} = g_{N1}^{(2)} g_{Nj}^{(1)},$$

$$f_{Nij}^{*} = g_{iN}^{(1)} g_{NN}^{(1)} g_{Nj}^{(1)}\ (i = 2, \dots, N-1\ and\ j = 2,3, \dots, N),$$

$$f_{NCFij} = g_{iN}^{(1)} g_{Nj}^{(1)} \qquad (i, j = 2,3, \dots, N),$$

$$\alpha_g = \left[g_{N2}^{(1)}\ g_{N3}^{(1)} \dots g_{N(N-1)}^{(1)}\right],$$

$$\alpha_{fN} = \left[g_{N2}^{(1)}\ g_{N3}^{(1)} \dots g_{NN}^{(1)}\right],$$

$$f_{CF} = f_{CC}, \qquad g_{CF}^{(2)} = g_{CC}^{(2)},$$

$$f_{NCF} = {f'}_{NCS}, \qquad {g'}_{CF}^{(2)} = {g'}_{CS}^{(2)}.$$

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Contribution of individual authors to the creation of a scientific article

Mohammed BOUSFIA: software, formal analysis, methodology and investigation.
Mohamed ABOUSSALEH: supervision, project administration and methodology.
Brahim OUHBI: validation and visualization.

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