Improved delay-dependent robust exponential stability for uncertain linear systems with mixed delays and nonlinear perturbations

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Abstract: In this article, we discuss a class of delay-dependent robust stability analysis for uncertain linear systems with mixed delays and nonlinear perturbations. Based on Lyapunov-Krasovskii functional, combined with descriptor model transformation, Leibniz-Newton formula, integral inequalities, Wirtinger-based integral inequality, Peng-Park's integral inequality and utilization of zero equation have been adopted to study. Improved delay-dependent robust stability criteria for uncertain time-delay systems are established in terms of linear matrix inequalities (LMIs). Finally, numerical examples suggest that the results given to illustrate the effectiveness and improvement over some existing methods.

Key–Words: Delay-dependent, Robust exponential stability, Time-varying delay, Nonlinear perturbations, Linear matrix inequalities (LMIs)

1 Introduction

Over the past decades, the problem of stability for linear differential systems, which delays in both its state and the derivatives of its states, has been widely investigated by many researchers, especially in the last decade. It is well known nonlinearities, as time delays, may cause instability and poor performance of practical systems such as engineering, biology, economics, and so on [1]. The problem of various stability and stabilization for dynamical systems have been intensively studied in the past years by many mathematics and control communities researchers [2], [3], [4], [5], [6], [7]. The linear delay systems constitute a more general class than those of the retarded type. Stability of these system proves to be more complex issue because the system involves the derivative of the delayed state [8], [9], [10], [11].

Many researchers have studied the problem of stability for time-delay systems with nonlinear perturbations for instance [12] considers the robust stability criteria for systems with interval time-varying delay and nonlinear perturbations. On the basis of the estimation and by utilizing free-weighting matrices, new delay-range-dependent stability criteria are established in terms of linear matrix inequalities (LMIs). In [13], exponential stability of time-delay systems with nonlinear uncertainties is studied. Based on the Lyapunov theory approach and the approaches of decomposing the matrix, a new exponential stability criterion is derived in terms of LMI. In [14], they propose a new delay-dependent stability criterion in terms of linear matrix inequality for dynamic systems with time-varying delays and nonlinear perturbations by using Lyapunov theory. Moreover, a descriptor model transformation and a corresponding Lyapunov-Krasovskii functionals have been introduced for stability analysis of systems with delays in [4], [7].

This study focuses on the problem of exponential stability for linear differential system with multiple delays. Based on combination of Leibniz-Newton formula, model transformation, linear matrix inequalities, the use of suitable Lyapunov-Krasovskii functional, new delay-dependent exponential stability criteria will be obtained in terms of LMIs. Finally, a numerical example will be given to show the effectiveness of the obtained results [15], [16], [17].

2 Problem formulation and preliminaries

We introduce some notations, definition and lemmas that will be used throughout the paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the vector norm $\|\cdot\|$;

$\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n\times r}$ denotes the set $n \times r$ real matrices; A^T denotes the transpose of the matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; matrix A is called semi-positive definite ($A \geq 0$) if $x^T Ax \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $x^T Ax > 0$ for all $x \neq 0$; matrix B is called semi-negative definite ($B \leq 0$) if $x^T Bx \leq 0$, for all $x \in R^n$; B is negative definite ($B < 0$) if $x^T Bx < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $A \geq B$ means $A - B \geq 0$; $C([-h_2, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-h_2, 0]$ into R^n; $*$ represents the elements below the main diagonal of a symmetric matrix. Consider the following uncertain linear system with multiple delays and nonlinear perturbations

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)x(t - h(t)) \\ \qquad + C(t)x(t - d) + f(t, x(t)) \\ \qquad + g(t, x(t - h(t))) \\ \qquad + w(t, x(t - d)), \qquad t \geq 0, \\ x(t_0 + t) = \phi(t), \quad \forall t \in [-h_2, 0], \\ A(t) = [A + \Delta A(t)], \quad B(t) = [B + \Delta B(t)], \\ C(t) = [C + \Delta C(t)], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable. $\phi(t)$ is the initial condition of system (1). A, B and C are real constant matrices with appropriate dimensions. The delay d is positive real constant and time-varying delay $h(t)$ is time-varying continuous function which satisfy

$$0 < d, \tag{2}$$
$$0 \leq h_1 \leq h(t) \leq h_2, \tag{3}$$

where d, h_1 and h_2 are positive real constants. The uncertain matrices $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ are norm bounded and can be described as

$$\begin{bmatrix}\Delta A(t) & \Delta B(t) & \Delta C(t)\end{bmatrix} = M\Delta(t)\begin{bmatrix}G_1 & G_2 & G_3\end{bmatrix}, \tag{4}$$

where M, G_1, G_2, and G_3 are real constant matrices with appropriate dimensions. The uncertain matrix $\Delta(t)$ satisfies

$$\Delta(t) = F(t)[I - JF(t)]^{-1}, \quad I - JJ^T > 0. \tag{5}$$

The uncertain matrix $F(t)$ satisfies

$$F(t)^T F(t) \leq I. \tag{6}$$

The uncertainties $f(.)$, $w(.)$ and $g(.)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delay state $x(t-d)$ and the delayed state $x(t - h(t))$, respectively, and are bounded in magnitude of the form

$$f^T(t, x(t))f(t, x(t)) \leq \eta^2 x^T(t)x(t), \tag{7}$$
$$w^T(t, x(t-d))w(t, x(t-d)) \leq \gamma^2 x^T(t-d)x(t-d), \tag{8}$$
$$g^T(t, x(t-h(t)))g(t, x(t-h(t))) \leq \rho^2 x^T(t-h(t))x(t-h(t)), \tag{9}$$

where η, γ and ρ are known positive real constants.

Definition 1 The system (1) is exponentially stable, if there exist positive real constants α and M such that, for each $\phi(t) \in C([-h_2, 0], R^n)$, the solution $x(t, \phi)$ of the system (1) satisfies

$$\|x(t, \phi,)\| \leq M\|\phi\|e^{-\alpha t}, \quad \forall t \in R^+.$$

Lemma 2($Jensen's\ inequality$) [1] For any constant matrix $Q \in R^{n\times n}, Q = Q^T > 0$, scalar $h_2 > 0$, vector function $\dot{x} : [-h_2, 0] \to R^n$ such that the integrations concerned are well defined, then

$$-h_2 \int_{-h_2}^{0} \dot{x}^T(s+t)Q\dot{x}(s+t)ds \leq -\Big(\int_{-h_2}^{0} \dot{x}(s+t)ds\Big)^T Q\Big(\int_{-h_2}^{0} \dot{x}(s+t)ds\Big).$$

Rearranging the term $\int_{-h_2}^{0} \dot{x}(s+t)ds$ with $x(t) - x(t - h_2)$, one can yield the following inequality:

$$-h_2 \int_{-h_2}^{0} \dot{x}^T(s+t)Q\dot{x}(s+t)ds \leq \begin{bmatrix} x(t) \\ x(t-h_2)\end{bmatrix}^T \begin{bmatrix} -Q & Q \\ * & -Q\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h_2)\end{bmatrix}.$$

Lemma 3 [18] Suppose that $\Delta(t)$ is given by (5)-(6). Let M, S and N be real matrices of appropriate dimensions with $M = M^T$. Then, the inequality

$$M + S\Delta(t)N + N^T\Delta^T(t)S^T < 0,$$

holds, if and only if, for any scalar $\delta > 0$,

$$\begin{bmatrix} M & S & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I\end{bmatrix} < 0.$$

Lemma 4 [19] For any constant symmetric positive definite matrix $Q \in R^{n\times n}$, $h(t)$ is discrete time-varying delays with (2), vector function ω :

$[-h_2, 0] \to R^n$ such that the integrations concerned are well defined, then

$$-(h_2-h_1)\int_{-h_2}^{-h_1}\omega^T(s)Q\omega(s)ds \le -\int_{-h(t)}^{-h_1}\omega^T(s)dsQ\int_{-h(t)}^{-h_1}\omega(s)ds - \int_{-h_2}^{-h(t)}\omega^T(s)dsQ\int_{-h_2}^{-h(t)}\omega(s)ds.$$

Lemma 5 [19] For any constant matrices $Q_1, Q_2, Q_3 \in R^{n\times n}$, $Q_1 \ge 0, Q_3 > 0, \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \ge 0$, $h(t)$ is discrete time-varying delays with (2) and vector function $\dot{x} : [-h_2, 0] \to R^n$ such that the following integration is well defined, then

$$-(h_2-h_1)\int_{t-h_2}^{t-h_1}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}\begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds$$
$$\le \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}^T \times \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3-Q_3 & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}.$$

Lemma 6 [19] Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices $X, M_i \in R^{n\times n}, i = 1, 2, \ldots, 5$ and $h(t)$ is discrete time-varying delays with (2),

$$-\int_{t-h_2}^{t-h_1}\dot{x}^T(s)X\dot{x}(s)ds \le \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \times \begin{bmatrix} M_1+M_1^T & -M_1^T+M_2 & 0 \\ * & M_1+M_1^T-M_2-M_2^T & -M_1^T+M_2 \\ * & * & -M_2-M_2^T \end{bmatrix} \times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}$$
$$+(h_2-h_1)\begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3+M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \end{bmatrix},$$

where
$$\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \ge 0.$$

Lemma 7 [10] For any matrix $Z > 0$, the following inequality holds for all continuously different-able function $x : [h_1, h_2] \to R^n$:

$$-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)Z\dot{x}(s)ds \le \omega^T\Omega\omega,$$

where
$\omega = \left[x^T(t-h_1), x^T(t-h_2), \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x^T(s)ds\right]^T$
and $\Omega = \begin{bmatrix} -4Z & -2Z & 6Z \\ * & -4Z & 6Z \\ * & * & -12Z \end{bmatrix}$.

Lemma 8 ($Peng-Park's\ integral\ inequality$) [9] For any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \ge 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \to R^n$ such that the concerned integrations are well defined, then

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)Z\dot{x}(s)ds \le \omega^T(t)\Lambda\omega(t),$$

where $\omega(t) = \left[x^T(t), x^T(t-\tau(t)), x^T(t-\tau)\right]^T$ and
$$\Lambda = \begin{bmatrix} -Z & Z-S & S \\ * & -2Z+S+S^T & Z-S \\ * & * & -Z \end{bmatrix}.$$

Lemma 9 [20] For any constant matrix $Z = Z^T > 0$ and positive number h_1, h_2 such that the following integrals are well defined, then

$$
\begin{aligned}
&-\int_{-h_2}^{-h_1}\int_{t+s}^{t} x^T(\tau)Zx(\tau)d\tau ds \\
\leq\ & -\frac{2}{h_2^2-h_1^2}\Big(\int_{-h_2}^{-h_1}\int_{t+s}^{t} x(\tau)d\tau ds\Big)^T Z \\
&\Big(\int_{-h_2}^{-h_1}\int_{t+s}^{t} x(\tau)d\tau ds\Big).
\end{aligned}
$$

Lemma 10 [21] For any matrix $Z \in R^{n\times n}$, $Z = Z^T > 0$, a scalar $\tau > o$, and a vector-valued function $\dot{x} : [-\tau, 0] \to R^n$ such that the following integrations are well defined, then

$$
\begin{aligned}
&-\int_{-\tau}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)Z\dot{x}(\theta)d\theta ds \\
&\leq \omega^T(t)\begin{bmatrix} -2Z & 2Z \\ * & -2Z \end{bmatrix}\omega(t),
\end{aligned}
$$

where $\omega(t) = \big[x^T(t), \frac{1}{\tau}\int_{t-\tau}^{t} x^T(s)ds\big]^T$.

3 Exponential stability analysis

In this section, by using the combination of linear matrix inequalities (LMIs) technique and Lyapunov theory method. We introduce the following notations for later use:

$$
\sum = \Big[\Sigma_{i,j}\Big]_{23\times 23}, \tag{10}
$$

where $\sum_{i,j} = \sum_{j,i}^T, \quad i, j = 1, 2, 3..., 23, U = P_1G,$

$$
\begin{aligned}
\Sigma_{1,1} &= 2\alpha P_1 + U + U^T + Q_1 + Q_1^T + P_2 + P_3 \\
&+ h_1^2P_5 + h_2^2P_6 + Q_3^T(A + B) + (A^T + B^T)Q_3 \\
&+ (h_2 - h_1)^2P_7 + (h_2 - h_1)^2P_8 - e^{-2\alpha h_1}P_9 \\
&- e^{-2\alpha h_2}P_{11} + \epsilon_1\eta^2 I + e^{-2\alpha h_2}[N_1 + N_1^T] \\
&+ h_2e^{-2\alpha h_2}N_3 - 2e^{-4\alpha h_2}P_{18} + h_1^2R_1 + h_2^2R_4 \\
&- \tfrac{2(h_2-h_1)}{h_2+h_1}e^{-4\alpha h_2}P_{19} + (h_2 - h_1)^2R_7 + W_1 \\
&- e^{-2\alpha h_2}R_6 + L_1 + L_1^T + L_4 + L_4^T + L_7^T(A + B) \\
&+ (A^T + B^T)L_7 + d^2[W_2 + R_{10}] - e^{-2\alpha d}W_3,
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{1,2} &= -U - Q_1^T + e^{-2\alpha h_2}(-N_1^T + N_2 + h_2N_4 + R_6) \\
&- L_1^T + L_2 + (A^T + B^T)L_8,
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{1,3} &= -U - Q_1^T - (Q_3^T + L_7^T)B - L_1^T + L_3 + A^TL_9 \\
&+ B^TL_9,
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{1,4} &= P_1 + (A^T + B^T)Q_2 - Q_3^T + h_1^2R_2 + h_2^2R_5 - L_7^T \\
&+ (h_2 - h_1)^2R_8 + d^2R_{11},
\end{aligned}
$$

$\Sigma_{1,5} = e^{-2\alpha h_1}P_9 - L_4^T + L_5^T, \quad \Sigma_{1,6} = e^{-2\alpha h_2}P_{11},$

$\Sigma_{1,7} = 0, \quad \Sigma_{1,8} = 2e^{-4\alpha h_2}P_{18}, \quad \Sigma_{1,9} = \frac{2e^{-4\alpha h_2}}{h_2+h_1}P_{19},$

$\Sigma_{1,10} = \frac{2e^{-4\alpha h_2}}{h_2+h_1}P_{19}, \quad \Sigma_{1,11} = \Sigma_{1,12} = 0,$

$\Sigma_{1,13} = -L_4^T + L_6, \quad \Sigma_{1,14} = \Sigma_{1,15} = 0,$

$\Sigma_{1,16} = -e^{-2\alpha h_2}R_5^T, \quad \Sigma_{1,17} = Q_3^T + L_7^T,$

$\Sigma_{1,18} = Q_3^T + L_7^T,$

$\Sigma_{1,19} = (Q_3^T + L_7^T)C + e^{-2\alpha d}W_3,$

$\Sigma_{1,20} = \Sigma_{1,21} = 0,$

$\Sigma_{1,22} = Q_3^T + L_7^T, \qquad \Sigma_{1,23} = 0,$

$$
\begin{aligned}
\Sigma_{2,2} &= e^{-2\alpha h_2}\Big[N_1 + N_1^T - N_2 - N_2^T + h_2(N_3 + N_5) \\
&+ K_1 + K_1^T - K_2 - K_2^T + (h_2 - h_1)(K_3 + K_5) \\
&- 2R_6 - 2R_9 - 2P_{16} + S + S^T\Big] - L_2^T - L_2 \\
&+ \epsilon_2\rho^2 I,
\end{aligned}
$$

$\Sigma_{2,3} = -L_2^T - L_3 - L_8^TB, \qquad \Sigma_{2,4} = -L_8^T,$

$$
\begin{aligned}
\Sigma_{2,5} &= e^{-2\alpha h_2}\Big[- K_1 + K_2^T + R_9^T + (h_2 - h_1)K_4^T \\
&+ P_{16}^T - S^T\Big],
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{2,6} &= e^{-2\alpha h_2}\Big[- N_1^T + N_2 + h_2N_4 - K_1^T + K_2 + R_6 \\
&+ R_9 + (h_2 - h_1)K_4 + P_{16} - S\Big],
\end{aligned}
$$

$\Sigma_{2,7} = \Sigma_{2,8} = 0, \quad \Sigma_{2,9} = e^{-2\alpha h_2}R_8^T,$

$\Sigma_{2,10} = e^{-2\alpha h_2}(-R_5^T - R_8^T),$

$\Sigma_{2,11} = \Sigma_{2,12} = \Sigma_{2,13} = \Sigma_{2,14} = \Sigma_{2,15} = 0,$

$\Sigma_{2,16} = e^{-2\alpha h_2}R_5^T, \quad \Sigma_{2,17} = \Sigma_{2,18} = L_8^T,$

$\Sigma_{2,19} = L_8^TC, \quad \Sigma_{2,20} = \Sigma_{2,21} = 0, \quad \Sigma_{2,22} = L_8^T,$

$\Sigma_{2,23} = 0,$

$\Sigma_{3,3} = -e^{-2\alpha h_2}P_{13} - L_3 - L_3^T - L_9^TB - B^TL_9,$

$\Sigma_{3,4} = -B^TQ_2 - L_9^T, \quad \Sigma_{3,5} = \Sigma_{3,6} = 0,$

$\Sigma_{3,7} = \Sigma_{3,8} = \Sigma_{3,9} = \Sigma_{3,10} = \Sigma_{3,11} = 0,$

$\Sigma_{3,12} = \Sigma_{3,13} = \Sigma_{3,14} = \Sigma_{3,15} = \Sigma_{3,16} = 0,$

$\Sigma_{3,17} = \Sigma_{3,18} = L_9^T, \quad \Sigma_{3,19} = L_9^TC,$

$\Sigma_{3,20} = \Sigma_{3,21} = 0, \quad \Sigma_{3,22} = L_9^T, \quad \Sigma_{3,23} = 0,$

$$
\begin{aligned}
\Sigma_{4,4} &= -Q_2 - Q_2^T + h_1^2(P_9 + P_{10} + R_3) + h_2P_{14} \\
&+ h_2^2(P_{11} + P_{12} + P_{13} + R_6 + P_{18}) \\
&+ (h_2 - h_1)^2(P_{15} + P_{16} + R_9) \\
&+ (h_2 - h_1)(P_{17} + h_2P_{19}) \\
&+ d^2\Big[W_3 + W_4 + R_{12}\Big],
\end{aligned}
$$

$\Sigma_{4,5} = \Sigma_{4,6} = \Sigma_{4,7} = \Sigma_{4,8} = \Sigma_{4,9} = 0,$

$\Sigma_{4,10} = \Sigma_{4,11} = \Sigma_{4,12} = \Sigma_{4,13} = \Sigma_{4,14} = 0,$

$\Sigma_{4,15} = \Sigma_{4,16} = 0, \ \Sigma_{4,17} = Q_2^T, \quad \Sigma_{4,18} = Q_2^T,$

$\Sigma_{4,19} = Q_2^TC, \quad \Sigma_{4,20} = \Sigma_{4,21} = 0, \quad \Sigma_{4,22} = Q_2^T,$

$\Sigma_{4,23} = 0,$

$$
\begin{aligned}
\Sigma_{5,5} &= e^{-2\alpha h_1}(-P_2 + P_4 - P_9) - L_5 - L_5^T \\
&+ e^{-2\alpha h_2}\Big[K_1 + K_1^T - 4P_{15} + (h_2 - h_1)K_3 \\
&- R_9 - P_{16}\Big],
\end{aligned}
$$

$\Sigma_{5,6} = e^{-2\alpha h_2}(-2P_{15} + S), \quad \Sigma_{5,7} = \Sigma_{5,8} = 0,$

$\Sigma_{5,9} = -e^{-2\alpha h_2}R_8^T, \quad \Sigma_{5,10} = \Sigma_{5,11} = \Sigma_{5,12} = 0,$

$\Sigma_{5,13} = -L_5^T - L_6, \Sigma_{5,14} = 0, \Sigma_{5,15} = 6e^{-2\alpha h_2}P_{15},$

$\Sigma_{5,16} = \Sigma_{5,17} = \Sigma_{5,18} = \Sigma_{5,19} = \Sigma_{5,20} = 0,$

$\Sigma_{5,21} = 0, \quad \Sigma_{5,22} = \Sigma_{5,23} = 0,$

$\Sigma_{6,6} = -e^{-2\alpha h_2}\Big[P_3+P_4+R_6+R_9+P_{11}+N_2+N_2^T + K_2 + K_2^T - h_2N_5 + 4P_{15} - (h_2-h_1)K_5 + P_{16}\Big]$, $\Sigma_{6,7} = \Sigma_{6,8} = \Sigma_{6,9} = 0$,
$\Sigma_{6,10} = e^{-2\alpha h_2}(R_5^T + R_8^T)$, $\Sigma_{6,11} = \Sigma_{6,12} = 0$,
$\Sigma_{6,13} = \Sigma_{6,14} = 0, \Sigma_{6,15} = 6e^{-2\alpha h_2}P_{15}$, $\Sigma_{6,16} = 0$,
$\Sigma_{6,17} = \Sigma_{6,18} = \Sigma_{6,19} = \Sigma_{6,20} = \Sigma_{6,21} = 0$,
$\Sigma_{6,22} = \Sigma_{6,23} = 0$,
$\Sigma_{7,7} = -e^{-2\alpha h_1}(P_5 + h_1R_1)$, $\Sigma_{7,8} = \Sigma_{7,9} = 0$,
$\Sigma_{7,10} = \Sigma_{7,11} = \Sigma_{7,12} = 0$, $\Sigma_{7,13} = -e^{-2\alpha h_1}R_2$,
$\Sigma_{7,14} = \Sigma_{7,15} = \Sigma_{7,16} = \Sigma_{7,17} = \Sigma_{7,18} = 0$,
$\Sigma_{7,19} = \Sigma_{7,20} = \Sigma_{7,21} = \Sigma_{7,22} = \Sigma_{7,23} = 0$,
$\Sigma_{8,8} = -h_2^2e^{-2\alpha h_2}P_6 - 2e^{-4\alpha h_2}P_{18}$, $\Sigma_{8,9} = 0$,
$\Sigma_{8,10} = \Sigma_{8,11} = \Sigma_{8,12} = \Sigma_{8,13} = \Sigma_{8,14} = 0$,
$\Sigma_{8,15} = \Sigma_{8,16} = \Sigma_{8,17} = \Sigma_{8,18} = \Sigma_{8,19} = 0$,
$\Sigma_{8,20} = \Sigma_{8,21} = \Sigma_{8,22} = \Sigma_{8,23} = 0$,
$\Sigma_{9,9} = -e^{-2\alpha h_2}(P_7 + R_7) - \frac{2e^{-4\alpha h_2}}{h_2^2-h_1^2}P_{19}$,
$\Sigma_{9,10} = -\frac{2e^{-4\alpha h_2}}{h_2^2-h_1^2}P_{19}$, $\Sigma_{9,11} = \Sigma_{9,12} = \Sigma_{9,13} = 0$,
$\Sigma_{9,14} = \Sigma_{9,15} = \Sigma_{9,16} = \Sigma_{9,17} = \Sigma_{9,18} = 0$,
$\Sigma_{9,19} = \Sigma_{9,20} = \Sigma_{9,21} = \Sigma_{9,22} = \Sigma_{9,23} = 0$,
$\Sigma_{10,10} = -e^{-2\alpha h_2}(P_7 + R_4 + R_7) - \frac{2e^{-4\alpha h_2}}{h_2^2-h_1^2}P_{19}$,
$\Sigma_{10,11} = \Sigma_{10,12} = \Sigma_{10,13} = \Sigma_{10,14} = \Sigma_{10,15} = 0$,
$\Sigma_{10,16} = \Sigma_{10,17} = \Sigma_{10,18} = \Sigma_{10,19} = \Sigma_{10,20} = 0$,
$\Sigma_{10,21} = \Sigma_{10,22} = \Sigma_{10,23} = 0$,
$\Sigma_{11,11} = -(h_2 - h_1)^2e^{-2\alpha h_2}P_8$, $\Sigma_{11,12} = 0$,
$\Sigma_{11,13} = \Sigma_{11,14} = \Sigma_{11,15} = \Sigma_{11,16} = \Sigma_{11,17} = 0$,
$\Sigma_{11,18} = \Sigma_{11,19} = \Sigma_{11,20} = \Sigma_{11,21} = \Sigma_{11,22} = \Sigma_{11,23} = 0$,
$\Sigma_{12,12} = -e^{-2\alpha h_2}(h_2^2 - h_1^2)^2P_8$, $\Sigma_{12,13} = 0$,
$\Sigma_{12,14} = \Sigma_{12,15} = \Sigma_{12,16} = \Sigma_{12,17} = \Sigma_{12,18} = 0$,
$\Sigma_{12,19} = \Sigma_{12,20} = \Sigma_{12,21} = \Sigma_{12,22} = 0$,
$\Sigma_{12,23} = 0$,
$\Sigma_{13,13} = -e^{-2\alpha h_1}(P_{10} + h_1R_3) - L_6^T - L_6$,
$\Sigma_{13,14} = \Sigma_{13,15} = \Sigma_{13,16} = \Sigma_{13,17} = \Sigma_{13,18} = 0$,
$\Sigma_{13,19} = \Sigma_{13,20} = \Sigma_{13,21} = \Sigma_{13,22} = \Sigma_{13,23} = 0$,
$\Sigma_{14,14} = -h_2^2e^{-2\alpha h_2}P_{12}$, $\Sigma_{14,15} = \Sigma_{14,16} = 0$,
$\Sigma_{14,17} = \Sigma_{14,18} = \Sigma_{14,19} = \Sigma_{14,20} = \Sigma_{14,21} = 0$,
$\Sigma_{14,22} = \Sigma_{14,23} = 0$,
$\Sigma_{15,15} = -12e^{-2\alpha h_2}P_{15}$, $\Sigma_{15,16} = \Sigma_{15,17} = 0$,
$\Sigma_{15,18} = \Sigma_{15,19} = \Sigma_{15,20} = \Sigma_{15,21} = \Sigma_{15,22} = 0$,
$\Sigma_{15,23} = 0$,
$\Sigma_{16,16} = -e^{-2\alpha h_2}R_4$, $\Sigma_{16,17} = \Sigma_{16,18} = 0$,
$\Sigma_{16,19} = \Sigma_{16,20} = \Sigma_{16,21} = \Sigma_{16,22} = \Sigma_{16,23} = 0$,
$\Sigma_{17,17} = -\epsilon_1 I$, $\Sigma_{17,18} = \Sigma_{17,19} = \Sigma_{17,20} = 0$,
$\Sigma_{17,21} = \Sigma_{17,22} = \Sigma_{17,23} = 0$,
$\Sigma_{18,18} = -\epsilon_2 I$, $\Sigma_{18,19} = \Sigma_{18,20} = \Sigma_{18,21} = 0$,
$\Sigma_{18,22} = \Sigma_{18,23} = 0$,
$\Sigma_{19,19} = -e^{-2\alpha d}(W_1 + W_3) + \epsilon_3\gamma^2 I$,
$\Sigma_{19,20} = \Sigma_{19,21} = \Sigma_{19,22} = \Sigma_{19,23} = 0$,
$\Sigma_{20,20} = -e^{-2\alpha d}(W_2 + R_{10}), \Sigma_{20,21} = -e^{-2\alpha d}R_{11}$,
$\Sigma_{20,22} = \Sigma_{20,23} = 0$,
$\Sigma_{21,21} = -e^{-2\alpha d}(W_4+R_{12})$, $\Sigma_{21,22} = \Sigma_{21,23} = 0$,
$\Sigma_{22,22} = -\epsilon_3 I$, $\Sigma_{22,23} = 0$, $\Sigma_{23,23} = -e^{-2\alpha h_2}P_{13}$.

Then, we study the nominal system (1) which is defined to be

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bx(t-h(t)) + Cx(t-d) \\ &\quad +f(t,x(t)) + g(t,x(t-h(t))) \\ &\quad +w(t,x(t-d)), \end{aligned} \quad (11)$$

By utilizing the following zero equation formula, we get

$$0 = Gx(t) - Gx(t-h(t)) - G\int_{t-h(t)}^{t} z(s)ds. \quad (12)$$

where $G \in R^{n\times n}$ will be chosen to guarantee the exponential stability of the system (11). Rewrite the system (11) in the following descriptor model transformation and adjust the system (11), then

$$\dot{x}(t) = z(t), \quad (13)$$

$$\begin{aligned} 0 &= -z(t) + [A+B]x(t) \\ &\quad -B\int_{t-h(t)}^{t} z(s)ds + Cx(t-d) \\ &\quad +f(t,x(t)) + g(t,x(t-h(t))) \quad (14) \\ &\quad +w(t,x(t-d)). \end{aligned}$$

Theorem 1. For given positive real constants h_1, h_2, d, η, ρ and γ , the system (11) is exponentially stable, if there exist positive definite symmetric matrices P_i, $i = 1,2,\dots,19$, any appropriate dimensional matrices $G, S, Q_j, N_k, K_k, , R_m, L_n$, $j = 1,2,3$, $k = 1,2,...,5$, $m = 1,2,\dots,12$, $n = 1,2,\dots,9$, and positive real constants $\epsilon_i > 0$, $i = 1,2,3$ satisfying the following LMIs :

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \quad (15)$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \quad (16)$$

$$\begin{bmatrix} R_6 & R_7 \\ * & R_9 \end{bmatrix} > 0, \quad (17)$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \quad (18)$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \geq 0, \quad (19)$$

$$\begin{bmatrix} P_{17} & K_1 & K_2 \\ * & K_3 & K_4 \\ * & * & K_5 \end{bmatrix} \geq 0, \quad (20)$$

$$\sum < 0. \quad (21)$$

Moreover, the solution $x(t,\phi)$ satisfies the inequality

$$\|x(t,\phi\| \leq \sqrt{\frac{N}{\lambda_{\min}(P_1)}}\|\phi\|e^{-\alpha t}, \quad t \in R^+, \quad (22)$$

where

$$\begin{aligned} N &= \lambda_{\max}(P_1) + h_2\lambda_{max}(P_2+P_3+P_4) \\ &+ d\lambda_{\max}W_1 + h_2^3\lambda_{\max}(P_5+P_6+P_7+P_8) \\ &+ d^3\lambda_{\max}(W_2+W_3+W_4) \\ &+ h_2^3\lambda_{\max}(P_9+P_{10}+h_2P_{11}+h_2P_{12}) \\ &+ h_2^3\lambda_{\max}(h_2P_{13}+P_{14}) \\ &+ h_2^2\lambda_{\max}(h_2P_{15}+h_2P_{16}+P_{17}) \\ &+ h_2^3\lambda_{\max}(P_{18}+P_{19}) \\ &+ h_2^3\lambda_{\max}\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} + h_2^3\lambda_{\max}\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \\ &+ h_2^3\lambda_{\max}\begin{bmatrix} R_7 & R_8 \\ R_8^T & R_9 \end{bmatrix} + d^3\lambda_{max}\begin{bmatrix} R_{10} & R_{11} \\ R_{11}^T & R_{12} \end{bmatrix}. \end{aligned}$$

Proof. Construct the following LyapunovKrasovskii functional candidate for the system

$$V(t) = \sum_{i=1}^{6} V_i(t), \quad (23)$$

where,

$$\begin{aligned} V_1(t) &= \begin{bmatrix} x(t) \\ z(t) \\ x(t) \end{bmatrix}^T \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & 0 & 0 \\ 0 & 0 & 0 \\ Q_1 & Q_2 & Q_3 \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \\ x(t) \end{bmatrix}, \\ V_2(t) &= \int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)P_2x(s)ds \\ &+ \int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)P_3x(s)ds \\ &+ \int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}x^T(s)P_4x(s)ds \\ &+ \int_{t-d}^{t} e^{2\alpha(s-t)}x^T(s)W_1x(s)ds, \\ V_3(t) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)P_5x(\theta)d\theta ds \\ &+ h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)P_6x(\theta)d\theta ds \\ &+ (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)P_7x(\theta)d\theta ds \\ &+ (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)P_8x(\theta)d\theta ds \\ &+ d\int_{-d}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}x^T(\theta)W_2x(\theta)d\theta ds, \end{aligned}$$

$$\begin{aligned} V_4(t) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_9z(\theta)d\theta ds \\ &+ h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{10}z(\theta)d\theta ds \\ &+ h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{11}z(\theta)d\theta ds \\ &+ h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{12}z(\theta)d\theta ds \\ &+ h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{13}z(\theta)d\theta ds \\ &+ \int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{14}z(\theta)d\theta ds \\ &+ (h_2-h_1) \\ &\times \int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{15}z(\theta)d\theta ds \\ &+ (h_2-h_1) \\ &\times \int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{16}z(\theta)d\theta ds \\ &+ \int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)P_{17}z(\theta)d\theta ds \\ &+ d\int_{-d}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)W_3z(\theta)d\theta ds, \\ &+ d\int_{-d}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}z^T(\theta)W_4z(\theta)d\theta ds, \\ V_5(t) &= h_1\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \\ &\times \begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix} d\theta ds \\ &+ h_2\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix}^T \\ &\times \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix} d\theta ds \\ &+ (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix}^T \\ &\times \begin{bmatrix} R_7 & R_8 \\ * & R_9 \end{bmatrix}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix} d\theta ds \\ &+ d\int_{-d}^{0}\int_{t+s}^{t} e^{2\alpha(\theta-t)}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix}^T \\ &\times \begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix}\begin{bmatrix} x(\theta) \\ z(\theta) \end{bmatrix} d\theta ds, \end{aligned}$$

$$V_6(t) = \int_{-h_2}^{0}\int_{\theta}^{0}\int_{t+s}^{t} e^{2\alpha(\tau+s-t)} z^T(\tau)P_{18}z(\tau)d\tau d\theta ds + \int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{t+s}^{t} e^{2\alpha(\tau+s-t)} z^T(\tau)P_{19}z(\tau)d\tau d\theta ds.$$

The time derivative of $V(t)$ along the trajectory for the system (11) is given by

$$\dot{V}(t) = \sum_{i=1}^{6} V_i(t),$$

where

$$\begin{aligned}
\dot{V}_1(t) &= 2\begin{bmatrix} x(t) \\ z(t) \\ x(t) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 & Q_1^T \\ 0 & 0 & Q_2^T \\ 0 & 0 & Q_3^T \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \end{bmatrix}, \\
&= \begin{bmatrix} 2x^T(t)P_1 & 0 & 2x^T(t)Q_1^T + 2z^T(t)Q_2^T + 2x^T(t)Q_3^T \end{bmatrix} \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \end{bmatrix}, \\
&= 2x^T(t)P_1\Big[z(t) + Gx(t) - Gx(t-h(t)) \\
&\quad -G\int_{t-h(t)}^{t} z(s)ds\Big] \\
&\quad +2x^T(t)Q_1^T\Big[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} z(s)ds\Big] \\
&\quad +2z^T(t)Q_2^T\Big[-z(t) + (A+B)x(t) \\
&\quad -B\int_{t-h(t)}^{t} z(s)ds + Cx(t-d) \\
&\quad +f(t,x(t)) + g(t,x(t-h(t))) + w(t,x(t-d))\Big] \\
&\quad +2x^T(t)Q_3^T\Big[-z(t) + (A+B)x(t) \\
&\quad -B\int_{t-h(t)}^{t} z(s)ds + Cx(t-d) \\
&\quad +f(t,x(t)) + g(t,x(t-h(t))) + w(t,x(t-d))\Big] \\
&\quad +2\alpha x^T(t)P_1x(t) - 2\alpha V_1(t), \\
\dot{V}_2(t) &= \Big[x^T(t)P_2x(t) - e^{-2\alpha h_1}x^T(t-h_1)P_2x(t-h_1)\Big] \\
&\quad +[x^T(t)P_3x(t) - e^{-2\alpha h_2}x^T(t-h_2)P_3x(t-h_2)] \\
&\quad +\Big[e^{-2\alpha h_1}x^T(t-h_1)P_4x(t-h_1) \\
&\quad -e^{-2\alpha h_2}x^T(t-h_2)P_4x(t-h_2)\Big] + \Big[x^T(t)W_1x(t) \\
&\quad -e^{-2\alpha d}x^T(t-d)W_1x(t-d)\Big] - 2\alpha V_2(t).
\end{aligned}$$

It's from the Lemma 2 and Lemma 4 that we have

$$\begin{aligned}
\dot{V}_3(t) &\le x^T(t)[h_1^2P_5 + h_2^2P_6 \\
&\quad +(h_2-h_1)^2(P_7+P_8)]x(t) \\
&\quad -e^{-2\alpha h_1}\Big(\int_{t-h_1}^{t} x(s)ds\Big)^T P_5\Big(\int_{t-h_1}^{t} x(s)ds\Big) \\
&\quad -e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T \\
&\quad \times h_2^2P_6\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big) \\
&\quad -e^{-2\alpha h_2}\int_{t-h(t)}^{t-h_1} x^T(s)dsP_7\int_{t-h(t)}^{t-h_1} x(s)ds \\
&\quad -e^{-2\alpha h_2}\int_{t-h_2}^{t-h(t)} x^T(s)dsP_7\int_{t-h_2}^{t-h(t)} x(s)ds \\
&\quad -e^{2\alpha h_2}\Big(\frac{1}{h_2-h_1}\int_{t-h(t)}^{t-h_1} x(s)ds\Big)^T(h_2-h_1)^2 \\
&\quad \times P_8\Big(\frac{1}{h_2-h_1}\int_{t-h(t)}^{t-h_1} x(s)ds\Big) \\
&\quad -e^{2\alpha h_2}\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h(t)} x(s)ds\Big)^T(h_2-h_1)^2 \\
&\quad \times P_8\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h(t)} x(s)ds\Big) \\
&\quad +d^2x^T(t)W_2x(t) \\
&\quad -e^{-2\alpha d}\Big(\int_{t-d}^{t} x(s)ds\Big)^T W_2\Big(\int_{t-d}^{t} x(s)ds\Big) \\
&\quad -2\alpha V_3(t).
\end{aligned}$$

Using Lemma 2, Lemma 4, Lemma 6, Lemma 7 and Lemma 8 , we get that

$$\begin{aligned}
\dot{V}_4(t) &\le z^T(t)\Big[h_1^2P_9 + h_1^2P_{10}\Big]z(t) \\
&\quad +e^{-2\alpha h_1}\begin{bmatrix} x(t) \\ x(t-h_1) \end{bmatrix}^T \begin{bmatrix} -P_9 & P_9 \\ * & -P_9 \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t) \\ x(t-h_1) \end{bmatrix} \\
&\quad -e^{-2\alpha h_1}\Big(\int_{t-h_1}^{t} z(s)ds\Big)^T P_{10}\Big(\int_{t-h_1}^{t} z(s)ds\Big) \\
&\quad +z^T(t)\Big[h_2^2P_{11} + h_2^2P_{12} + h_2^2P_{13} + h_2P_{14}\Big]z(t) \\
&\quad +e^{-2\alpha h_2}\begin{bmatrix} x(t) \\ x(t-h_2) \end{bmatrix}^T \begin{bmatrix} -P_{11} & P_{11} \\ * & -P_{11} \end{bmatrix} \\
&\quad \times \begin{bmatrix} x(t) \\ x(t-h_2) \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
&-e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} z(s)ds\Big)^T h_2^2 P_{12}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} z(s)ds\Big)\\
&-e^{-2\alpha h_2}\int_{t-h(t)}^{t} z^T(s)dsP_{13}\int_{t-h(t)}^{t} z(s)ds\\
&-e^{-2\alpha h_2}\int_{t-h_2}^{t-h(t)} z^T(s)dsP_{13}\int_{t-h_2}^{t-h(t)} z(s)ds\\
&+e^{-2\alpha h_2}\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T\\
&\times\begin{bmatrix} N_1+N_1^T & -N_1^T+N_2 & 0\\ * & N_1+N_1^T-N_2-N_2^T & -N_1^T+N_2\\ * & * & -N_2-N_2^T\end{bmatrix}\\
&\times\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}\\
&+h_2e^{-2\alpha h_2}\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T\begin{bmatrix} N_3 & N_4 & 0\\ * & N_3+N_5 & N_4\\ * & * & N_5\end{bmatrix}\\
&\times\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}+(h_2-h_1)^2z^T(t)P_{15}z(t)\\
&+z^T(t)[(h_2-h_1)^2P_{16}+(h_2-h_1)P_{17}]z(t)\\
&+e^{-2\alpha h_2}\begin{bmatrix} x(t-h_1)\\ x(t-h_2)\\ \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}\end{bmatrix}^T\begin{bmatrix} -4P_{15} & -2P_{15} & 6P_{15}\\ * & -4P_{15} & 6P_{15}\\ * & * & -12P_{15}\end{bmatrix}\\
&\times\begin{bmatrix} x(t-h_1)\\ x(t-h_2)\\ \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}\end{bmatrix}+e^{-2\alpha h_2}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T\\
&\times\begin{bmatrix} -P_{16} & P_{16}-S & S\\ * & -2P_{16}+S+S^T & P_{16}-S\\ * & * & -P_{16}\end{bmatrix}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}\\
&+e^{-2\alpha h_2}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T\\
&\times\begin{bmatrix} K_1+K_1^T & -K_1^T+K_2 & 0\\ * & K_1+K_1^T-K_2-K_2^T & -K_1^T+K_2\\ * & * & -K_2-K_2^T\end{bmatrix}\\
&\times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}\\
&+(h_2-h_1)e^{-2\alpha h_2}\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}^T\\
&\times\begin{bmatrix} K_3 & K_4 & 0\\ * & K_3+K_5 & K_4\\ * & * & K_5\end{bmatrix}\\
&\times\begin{bmatrix} x(t-h_1)\\ x(t-h(t))\\ x(t-h_2)\end{bmatrix}+z^T(t)\Big[d^2W_3+d^2W_4\Big]z(t)\\
&+e^{-2\alpha d}\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}^T\begin{bmatrix} -W_3 & W_3\\ * & -W_3\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\end{bmatrix}\\
&-e^{-2\alpha d}\Big(\int_{t-d}^{t} z(s)ds\Big)^T W_4\Big(\int_{t-d}^{t} z(s)ds\Big)-2\alpha V_4(t).
\end{aligned}
$$

From Lemma2 and Lemma5, we have the following inequalities

$$
\begin{aligned}
\dot V_5(t) \quad\leq\quad & h_1^2\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}^T\begin{bmatrix} R_1 & R_2\\ * & R_3\end{bmatrix}\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}\\
&+h_2^2\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}^T\begin{bmatrix} R_4 & R_5\\ * & R_6\end{bmatrix}\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}\\
&+(h_2-h_1)^2\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}^T\begin{bmatrix} R_7 & R_8\\ * & R_9\end{bmatrix}\begin{bmatrix} x(t)\\ z(t)\end{bmatrix}\\
&-h_1e^{-2\alpha h_1}\begin{bmatrix} \int_{t-h_1}^{t} x(s)ds\\ \int_{t-h_1}^{t} z(s)ds\end{bmatrix}^T\begin{bmatrix} R_1 & R_2\\ * & R_3\end{bmatrix}\\
&\times\begin{bmatrix} \int_{t-h_1}^{t} x(s)ds\\ \int_{t-h_1}^{t} z(s)ds\end{bmatrix}\\
&+e^{-2\alpha h_2}\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t} x(s)ds\\ \int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}^T\\
&\times\begin{bmatrix} -R_6 & R_6 & 0 & -R_5^T & 0\\ * & -R_6-R_6 & R_6 & R_5^T & -R_5^T\\ * & * & -R_6 & 0 & R_5^T\\ * & * & * & -R_4 & 0\\ * & * & * & * & -R_4\end{bmatrix}\\
&\times\begin{bmatrix} x(t)\\ x(t-h(t))\\ x(t-h_2)\\ \int_{t-h(t)}^{t} x(s)ds\\ \int_{t-h_2}^{t-h(t)} x(s)ds\end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
&+e^{-2\alpha h_2}\begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix}^T \\
&\times \begin{bmatrix} -R_9 & R_9 & 0 & -R_8^T & 0 \\ * & -R_9-R_9 & R_9 & R_8^T & -R_8^T \\ * & * & -R_9 & 0 & R_8^T \\ * & * & * & -R_7 & 0 \\ * & * & * & * & -R_7 \end{bmatrix} \\
&\times \begin{bmatrix} x(t-h_1) \\ x(t-h(t)) \\ x(t-h_2) \\ \int_{t-h(t)}^{t-h_1} x(s)ds \\ \int_{t-h_2}^{t-h(t)} x(s)ds \end{bmatrix} + d^2 \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}^T \begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \\
&-de^{-2\alpha d}\begin{bmatrix} \int_{t-d}^{t} x(s)ds \\ \int_{t-d}^{t} z(s)ds \end{bmatrix}^T \begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} \begin{bmatrix} \int_{t-d}^{t} x(s)ds \\ \int_{t-d}^{t} z(s)ds \end{bmatrix} \\
&-2\alpha V_5(t).
\end{aligned}
$$

Calculating the derivative of $\dot{V}_6(t)$, we get

$$
\begin{aligned}
\dot{V}_6(t) \leq\ & h_2^2 z^T(t)P_{18}z(t) \\
&-e^{-4\alpha h_2}\int_{-h_2}^{0}\int_{t+s}^{t} z^T(t)P_{18}z(t)d\theta ds \\
&+(h_2-h_1)h_2 z^T(t)P_{19}z(t) \\
&-e^{-4\alpha h_2}\int_{-h_2}^{-h_1}\int_{t+s}^{t} z^T(t)P_{19}z(t)d\theta ds \\
&-2\alpha V_6(t).
\end{aligned}
$$

By the use of Lemma9 and Lemma10, two integral terms in $\dot{V}_6(t)$ can be estimated as follows

$$
\begin{aligned}
\dot{V}_6(t) \leq\ & h_2^2 z^T(t)P_{18}z(t) + (h_2-h_1)h_2 z^T(t)P_{19}z(t) \\
&+e^{-4\alpha h_2}\begin{bmatrix} x(t) \\ \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -2P_{18} & 2P_{18} \\ * & -2P_{18} \end{bmatrix} \\
&\times \begin{bmatrix} x(t) \\ \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \end{bmatrix} - \frac{2e^{-4\alpha h_2}}{h_2^2-h_1^2} \\
&\times\Big(\int_{-h_2}^{-h_1}\int_{t+s}^{t} z(\theta)d\theta ds\Big)^T P_{19} \\
&\times\Big(\int_{-h_2}^{-h_1}\int_{t+s}^{t} z(\theta)d\theta ds\Big) - 2\alpha V_6(t),
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & h_2^2 z^T(t)P_{18}z(t) + (h_2-h_1)h_2 z^T(t)P_{19}z(t) \\
&+e^{-4\alpha h_2}\begin{bmatrix} x(t) \\ \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} -2P_{18} & 2P_{18} \\ * & -2P_{18} \end{bmatrix} \\
&\times \begin{bmatrix} x(t) \\ \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \end{bmatrix} - \frac{2e^{-4\alpha h_2}}{h_2^2-h_1^2}\Big((h_2-h_1)x(t) \\
&-\int_{t-h(t)}^{t-h_1} x(s)ds - \int_{t-h_2}^{t-h(t)} x(s)ds\Big)^T P_{19} \\
&\times\Big((h_2-h_1)x(t) - \int_{t-h(t)}^{t-h_1} x(s)ds \\
&-\int_{t-h_2}^{t-h(t)} x(s)ds\Big) - 2\alpha V_6(t).
\end{aligned}
$$

From the utilization of zero equation, the following equation is true for any real matrices, $L_k, k = 1, 2, ..., 9$ with appropriate dimensions

$$
\begin{aligned}
&2\Big[x^T(t)L_1^T + x^T(t-h(t))L_2^T \\
&+\int_{t-h(t)}^{t} z^T(s)dsL_3^T\Big] \\
&\times\Big[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} z^T(s)ds\Big] = 0, \quad (24)
\end{aligned}
$$

$$
\begin{aligned}
&2\Big[x^T(t)L_4^T + x^T(t-h_1)L_5^T + \int_{t-h_1}^{t} z^T(s)dsL_6^T\Big] \\
&\times\Big[x(t) - x(t-h_1) - \int_{t-h_1}^{t} z^T(s)ds\Big] = 0, \quad (25)
\end{aligned}
$$

$$
\begin{aligned}
&2\Big[x^T(t)L_7^T + x^T(t-h(t))L_8^T + \int_{t-h(t)}^{t} z^T(s)dsL_9^T\Big] \\
&\times\Big[-z(t) + (A+B)x(t) - B\int_{t-h(t)}^{t} z^T(s)ds \\
&+Cx(t-d) + f(t,x(t)) + g(t,x(t-h(t))) \\
&+w(t,x(t-d))\Big] = 0. \quad (26)
\end{aligned}
$$

From (7)-(9), we obtain for any scalars $\epsilon_1, \epsilon_2, \epsilon_3 > 0$

$$
\epsilon_1 f^T(t,x(t))f(t,x(t)) \leq \epsilon_1\eta^2 x^T(t)x(t), \quad (27)
$$

$$
\begin{aligned}
&\epsilon_2 g^T(t,x(t-h(t)))g(t,x(t-h(t))) \\
&\leq \epsilon_2\rho^2 x^T(t-h(t))x(t-h(t)), \quad (28)
\end{aligned}
$$

$$
\begin{aligned}
&\epsilon_3 w^T(t,x(t-d))w(t,x(t-d)) \\
&\leq \epsilon_3\gamma^2 x^T(t-d)x(t-d). \quad (29)
\end{aligned}
$$

Let us define $U = P_1G$. From (23), (24)-(29), we obtain

$$
\dot{V}(t) + 2\alpha V(t) \leq \xi^T(t)\sum\xi(t) \quad (30)
$$

where $\xi^T(t) = \Big[x^T(t) \;\; x^T(t-h(t)) \;\; \int_{t-h(t)}^{t} z^T(s)ds$
$z^T(t) \;\; x^T(t-h_1) \;\; x^T(t-h_2) \;\; \int_{t-h_1}^{t} x^T(s)ds$
$\frac{1}{h_2}\int_{t-h_2}^{t} x^T(s)ds \;\; \int_{t-h(t)}^{t-h_1} x^T(s)ds \;\; \int_{t-h_2}^{t-h(t)} x^T(s)ds$
$\frac{1}{h_2-h_1}\int_{t-h(t)}^{t-h_1} x^T(s)ds \;\; \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h(t)} x^T(s)ds$
$\int_{t-h_1}^{t} z^T(s)ds \;\; \frac{1}{h_2}\int_{t-h_2}^{t} z^T(s)ds \;\; \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x^T(s)ds$
$\int_{t-h(t)}^{t} x^T(s)ds \;\; f^T(t,x(t)) \;\; g^T(t,x(t-h(t)))$
$x^T(t-d) \;\; \int_{t-d}^{d} x^T(s)ds \;\; \int_{t-d}^{d} z^T(s)ds \;\; w(t,x(t-d))$
$\int_{t-h_2}^{t-h(t)} z^T(s)ds\Big]$

and $\sum$ is defined in (10). It is a fact that, if $\sum < 0$, then

$$\dot{V}(t) + 2\alpha V(t) \leq 0, \qquad \forall t \in R^+, \qquad (31)$$

which gives

$$V(t) \leq V(0)e^{-2\alpha t}, \qquad \forall t \in R^+. \qquad (32)$$

From (32), it is easy to see that

$$\begin{aligned} \lambda_{\min}(P_1)\|x(t)\|^2 &\leq V(t) \leq V(0)e^{-2\alpha t}, \quad (33) \\ V(0) &= \sum_{i=1}^{6} V_i(0), \end{aligned}$$

where

$$\begin{aligned} V_1(0) &= x^T(0)P_1x(0), \\ V_2(0) &= \int_{-h_1}^{0} e^{2\alpha s}x^T(s)P_2x(s)ds \\ &+ \int_{-h_2}^{0} e^{2\alpha s}x^T(s)P_3x(s)ds \\ &+ \int_{-h_2}^{-h_1} e^{2\alpha s}x^T(s)P_4x(s)ds \\ &+ \int_{-d}^{0} e^{2\alpha s}x^T(s)W_1x(s)ds, \\ V_3(0) &= h_1\int_{-h_1}^{0}\int_{s}^{0} e^{2\alpha\theta}x^T(\theta)P_5x(\theta)d\theta ds \\ &+h_2\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}x^T(\theta)P_6x(\theta)d\theta ds \\ &+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}x^T(\theta)P_7x(\theta)d\theta ds \\ &+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}x^T(\theta)P_8x(\theta)d\theta ds \\ &+d\int_{-d}^{0}\int_{s}^{0} e^{2\alpha\theta}x^T(\theta)W_2x(\theta)d\theta ds, \\ V_4(0) &= h_1\int_{-h_1}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)[P_9+P_{10}]z(\theta)d\theta ds \\ &+h_2\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{11}z(\theta)d\theta ds \\ &+h_2\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{12}z(\theta)d\theta ds \\ &+h_2\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{13}z(\theta)d\theta ds \\ &+\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{14}z(\theta)d\theta ds \\ &+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{15}z(\theta)d\theta ds \\ &+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{16}z(\theta)d\theta ds \\ &+\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)P_{17}z(\theta)d\theta ds \\ &+d\int_{-d}^{0}\int_{s}^{0} e^{2\alpha\theta}z^T(\theta)[W_3+W_4]z(\theta)d\theta ds, \\ V_5(0) &= h_1\int_{-h_1}^{0}\int_{s}^{0} e^{2\alpha\theta}\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}^T\begin{bmatrix}R_1 & R_2\\ * & R_3\end{bmatrix} \\ &\times\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}d\theta ds \\ &+h_2\int_{-h_2}^{0}\int_{s}^{0} e^{2\alpha\theta}\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}^T\begin{bmatrix}R_4 & R_5\\ * & R_6\end{bmatrix} \\ &\times\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}d\theta ds \\ &+(h_2-h_1)\int_{-h_2}^{-h_1}\int_{s}^{0} e^{2\alpha\theta}\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}^T\begin{bmatrix}R_7 & R_8\\ * & R_9\end{bmatrix} \\ &\times\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}d\theta ds \\ &+d\int_{-d}^{0}\int_{s}^{0} e^{2\alpha\theta}\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}^T\begin{bmatrix}R_{10} & R_{11}\\ * & R_{12}\end{bmatrix} \\ &\times\begin{bmatrix}x(\theta)\\z(\theta)\end{bmatrix}d\theta ds, \\ V_6(0) &= \int_{-h_2}^{0}\int_{\theta}^{0}\int_{s}^{0} e^{2\alpha(\tau+s)}z^T(\tau)P_{18}z(\tau)d\tau d\theta ds \\ &+\int_{-h_2}^{-h_1}\int_{\theta}^{0}\int_{s}^{0} e^{2\alpha(\tau+s)}z^T(\tau)P_{19}z(\tau)d\tau d\theta ds. \end{aligned}$$

Therefore, we get

$$\lambda_{\min}(P_1)\|x(t)\|^2 \leq V(0)e^{-2\alpha t} \leq N\|\phi\|^2e^{-2\alpha t}, \quad (34)$$

where

$$\begin{aligned} N &= \lambda_{\max}(P_1) + h_2\lambda_{\max}(P_2+P_3+P_4) \\ &\quad +d\lambda_{\max}W_1 \\ &\quad h_2^3\lambda_{\max}(P_5+P_6+P_7+P_8) \\ &\quad +d^3\lambda_{\max}(W_2+W_3+W_4) \\ &\quad +h_2^3\lambda_{\max}(P_9+P_{10}+h_2P_{11}+h_2P_{12} \\ &\quad +h_2^3\lambda_{\max}(h_2P_{13}+P_{14}) \\ &\quad +h_2^2\lambda_{\max}(h_2P_{15}+h_2P_{16}+P_{17}) \\ &\quad +h_2^3\lambda_{\max}(P_{18}+P_{19}) \\ &\quad +h_2^3\lambda_{\max}\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} + h_2^3\lambda_{\max}\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \\ &\quad +h_2^3\lambda_{\max}\begin{bmatrix} R_7 & R_8 \\ R_8^T & R_9 \end{bmatrix} + d^3\lambda_{\max}\begin{bmatrix} R_{10} & R_{11} \\ R_{11}^T & R_{12} \end{bmatrix}. \end{aligned}$$

From (34), we get

$$\|x(t,\phi)\| \le \sqrt{\frac{N}{\lambda_{\min}(P_1)}}\|\phi\|e^{-\alpha t}, \quad t \in R^+. \quad (35)$$

This means that the system (11) is exponentially stable. The proof of the theorem is completed. □

4 Robust exponential stability analysis

In this section, we study the robust stability criteria for the uncertain linear systems with multiple delays and nonlinear perturbations of the system (1).

Theorem 2 The system (1) is robust exponentially stable, if there exist positive definite symmetric matrices P_i, $i = 1,2,\ldots,19$, any appropriate dimensional matrices $G,S,N,J,Q_j,N_k,K_k,R_m,L_n$, $j=1,2,...,3$, $k=1,2,...,5$, $m=1,2,\ldots,12$, $n=1,2,\ldots,9$, and positive real constants δ satisfying the following LMIs :

$$\begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0, \quad (36)$$

$$\begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} > 0, \quad (37)$$

$$\begin{bmatrix} R_6 & R_7 \\ * & R_9 \end{bmatrix} > 0, \quad (38)$$

$$\begin{bmatrix} R_{10} & R_{11} \\ * & R_{12} \end{bmatrix} > 0, \quad (39)$$

$$\begin{bmatrix} P_{14} & N_1 & N_2 \\ * & N_3 & N_4 \\ * & * & N_5 \end{bmatrix} \ge 0, \quad (40)$$

$$\begin{bmatrix} P_{17} & K_1 & K_2 \\ * & K_3 & K_4 \\ * & * & K_5 \end{bmatrix} \ge 0, \quad (41)$$

$$\begin{bmatrix} \sum & F & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I \end{bmatrix} < 0. \quad (42)$$

Moreover, the solution $x(t,\phi)$ satisfies the inequality

$$\|x(t,\phi\| \le \sqrt{\frac{M}{\lambda_{\min}(P_1)}}\|\phi\|e^{-\alpha t}, \quad t \in R^+, \quad (43)$$

where

$$\begin{aligned} M &= \lambda_{\max}(P_1) + h_2\lambda_{max}(P_2+P_3+P_4) \\ &\quad +d\lambda_{\max}W_1h_2^3\lambda_{\max}(P_5+P_6+P_7+P_8) \\ &\quad +d^3\lambda_{\max}(W_2+W_3+W_4) \\ &\quad +h_2^3\lambda_{\max}(P_9+P_{10}+h_2P_{11}+h_2P_{12}) \\ &\quad +h_2^3\lambda_{\max}(h_2P_{13}+P_{14}) \\ &\quad +h_2^2\lambda_{\max}(h_2P_{15}+h_2P_{16}+P_{17}) \\ &\quad +h_2^3\lambda_{max}(P_{18}+P_{19}) \\ &\quad +h_2^3\lambda_{\max}\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} + h_2^3\lambda_{\max}\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \\ &\quad +h_2^3\lambda_{\max}\begin{bmatrix} R_7 & R_8 \\ R_8^T & R_9 \end{bmatrix} + d^3\lambda_{max}\begin{bmatrix} R_{10} & R_{11} \\ R_{11}^T & R_{12} \end{bmatrix}. \end{aligned}$$

Proof. Replacing A, B and C in (10) with $A = A + E\Delta(t)G_1$, $B = B + E\Delta(t)G_2$ and $C = C + E\Delta(t)G_3$, respectively, we find that condition (10) is equivalent to the following condition

$$\sum + F\Delta(t)N + N^T\Delta(t)^TF^T < 0,$$

where $\sum$ is define in (10), $F^T = \Big[E^T(Q_3 + L_7)\ E^TL_8\ E^TL_9\ E^TQ_2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\Big]$, $N = \Big[G_1 + G_2\ 0\ -G_2\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ G_3\ 0\ 0\ 0\ 0\ 0\ 0\Big]$. By using lemma3, we can find that (44) is equivalent to the LMIs as follows,

$$\begin{bmatrix} \sum & F & \delta N^T \\ * & -\delta I & \delta J^T \\ * & * & -\delta I \end{bmatrix} < 0, \quad (44)$$

where δ is positive real constant. From **Theorem1** and condition (36)-(44), system (1) is robust exponentially stable. The proof of theorem is complete. □

5 Numerical Examples

In this section, we give numerical example in order to compare several existing criteria and those obtained in this paper. All the numerical results are calculated via the LMIs toolbox of Matlab.

Example1 Consider the linear system (1) with the following coefficient matrices:

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, C = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, G_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, G_3 = J = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By using the LMIs Toolbox of Matlab and Conditions (2.7)-(2.9) of Theorem 2 for Example 1 with $d = 0.1$, $\alpha = 0.1$, $\eta = 0.1$, $\rho = 0.1$, $\gamma = 0.1$, $h_1 = 0.3$ and upper bound $h_2 = 0.693$. The solutions of LMIs verify as follows,

$$P_1 = \begin{bmatrix} 0.2108 & -0.0186 \\ -0.0186 & 0.0458 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.0466 & 0.0010 \\ 0.0010 & 0.0290 \end{bmatrix},$$
$$P_3 = \begin{bmatrix} 0.0476 & -0.0033 \\ -0.0033 & 0.0194 \end{bmatrix},$$
$$P_4 = \begin{bmatrix} 0.1522 & -0.0264 \\ -0.0264 & 0.0445 \end{bmatrix},$$
$$P_5 = \begin{bmatrix} 0.0044 & -0.0027 \\ -0.0027 & 0.0019 \end{bmatrix},$$
$$P_6 = \begin{bmatrix} 0.0011 & -0.0007 \\ -0.0007 & 0.0005 \end{bmatrix},$$
$$P_7 = \begin{bmatrix} 0.0032 & -0.0020 \\ -0.0020 & 0.0014 \end{bmatrix},$$
$$P_8 = \begin{bmatrix} 0.0054 & -0.0033 \\ -0.0033 & 0.0022 \end{bmatrix},$$
$$P_9 = \begin{bmatrix} 0.0867 & -0.0029 \\ -0.0029 & 0.0203 \end{bmatrix},$$
$$P_{10} = \begin{bmatrix} 0.7730 & -0.4664 \\ -0.4664 & 0.3266 \end{bmatrix},$$
$$P_{11} = 10^{-4} \times \begin{bmatrix} 0.7730 & -0.4664 \\ -0.4664 & 0.3266 \end{bmatrix},$$
$$P_{12} = 10^{-4} \times \begin{bmatrix} 0.9776 & -0.5876 \\ -0.5876 & 0.4169 \end{bmatrix},$$
$$P_{13} = 10^{-3} \times \begin{bmatrix} 0.4351 & -0.2650 \\ -0.2650 & 0.1811 \end{bmatrix},$$
$$P_{14} = 10^{-3} \begin{bmatrix} 0.5502 & -0.3350 \\ -0.3350 & 0.2288 \end{bmatrix},$$
$$P_{15} = 10^{-3} \begin{bmatrix} 0.3569 & -0.2175 \\ -0.2175 & 0.1487 \end{bmatrix},$$
$$P_{16} = 10^{-3} \begin{bmatrix} 0.1560 & -0.0948 \\ -0.0948 & 0.0655 \end{bmatrix},$$
$$P_{17} = \begin{bmatrix} 0.0188 & -0.0013 \\ -0.0013 & 0.0131 \end{bmatrix},$$
$$P_{18} = 10^{-4} \begin{bmatrix} 0.6962 & -0.4196 \\ -0.4196 & 0.2929 \end{bmatrix},$$
$$P_{19} = 10^{-3} \times \begin{bmatrix} 0.1109 & -0.0672 \\ -0.0672 & 0.0466 \end{bmatrix},$$
$$G = 10^5 \times \begin{bmatrix} 1.3211 & 1.2306 \\ 6.4709 & -0.6849 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 0.0044 & -0.0027 \\ -0.0027 & 0.0019 \end{bmatrix},$$
$$R_2 = 10^{-6} \times \begin{bmatrix} 0.9103 & -0.6462 \\ -0.6462 & 0.4561 \end{bmatrix},$$
$$R_3 = \begin{bmatrix} 0.0867 & -0.0029 \\ -0.0029 & 0.0203 \end{bmatrix},$$
$$R_4 = \begin{bmatrix} 0.0012 & -0.0008 \\ -0.0008 & 0.0005 \end{bmatrix},$$
$$R_5 = 10^{-5} \times \begin{bmatrix} -0.1343 & 0.0739 \\ 0.0739 & -0.0491 \end{bmatrix},$$
$$R_6 = 10^{-3} \times \begin{bmatrix} 0.4026 & -0.2451 \\ -0.2451 & 0.1675 \end{bmatrix},$$
$$R_7 = \begin{bmatrix} 0.0032 & -0.0020 \\ -0.0020 & 0.0014 \end{bmatrix},$$
$$R_8 = 10^{-4} \begin{bmatrix} 0.5306 & -0.3293 \\ -0.3293 & 0.2323 \end{bmatrix},$$
$$N_1 = 10^{-3} \begin{bmatrix} -0.7913 & 0.4826 \\ 0.4826 & -0.3284 \end{bmatrix},$$
$$N_2 = 10^{-3} \begin{bmatrix} 0.7913 & -0.4826 \\ -0.4826 & 0.3284 \end{bmatrix},$$
$$N_3 = \begin{bmatrix} 0.0013 & -0.0008 \\ -0.0008 & 0.0005 \end{bmatrix},$$
$$N_4 = \begin{bmatrix} -0.0012 & 0.0007 \\ 0.0007 & -0.0005 \end{bmatrix},$$
$$N_5 = \begin{bmatrix} 0.0013 & -0.0008 \\ -0.0008 & 0.0005 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -0.0475 & 0.0031 \\ 0.0031 & -0.0333 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.0475 & -0.0031 \\ -0.0031 & 0.0333 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.1207 & -0.0078 \\ -0.0078 & 0.0845 \end{bmatrix},$$
$$K_4 = \begin{bmatrix} -0.1206 & 0.0077 \\ 0.0077 & -0.0845 \end{bmatrix},$$
$$K_5 = \begin{bmatrix} 0.1207 & -0.0078 \\ -0.0078 & 0.0845 \end{bmatrix},$$
$$Q_1 = 10^4 \times \begin{bmatrix} -3.0543 & -5.3984 \\ -5.3984 & 1.0255 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.0567 & -0.0085 \\ -0.0085 & 0.0224 \end{bmatrix},$$
$$Q_3 = 10^7 \times \begin{bmatrix} -3.1607 & -0.0516 \\ -0.0516 & -0.2641 \end{bmatrix},$$
$$L_1 = 10^4 \times \begin{bmatrix} 1.4710 & 2.6769 \\ 2.6769 & -0.4830 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} -0.0246 & 0.0116 \\ 0.0116 & 0.0117 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 0.1367 & 0.0051 \\ 0.0051 & 0.0573 \end{bmatrix},$$
$$L_4 = \begin{bmatrix} -0.0897 & -0.0099 \\ -0.0099 & -0.0331 \end{bmatrix},$$
$$L_5 = \begin{bmatrix} 0.0871 & 0.0056 \\ 0.0056 & 0.0393 \end{bmatrix},$$
$$L_6 = \begin{bmatrix} -0.0526 & -0.0011 \\ -0.0011 & 0.0061 \end{bmatrix},$$
$$L_7 = 10^7 \times \begin{bmatrix} 3.1607 & 0.0516 \\ 0.0516 & 0.2641 \end{bmatrix},$$
$$L_8 = \begin{bmatrix} 0.0519 & -0.0005 \\ -0.0005 & 0.0236 \end{bmatrix},$$
$$L_9 = \begin{bmatrix} 0.0726 & -0.0028 \\ -0.0028 & 0.0200 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.0228 & -0.0130 \\ -0.0130 & 0.0169 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 0.0269 & -0.0162 \\ -0.0162 & 0.0115 \end{bmatrix},$$
$$W_3 = \begin{bmatrix} 0.0824 & -0.0231 \\ -0.0231 & 0.0590 \end{bmatrix},$$
$$W_4 = \begin{bmatrix} 0.0033 & -0.0020 \\ -0.0020 & 0.0014 \end{bmatrix},$$

$\epsilon_1 = 1.5251, \quad \epsilon_2 = 0.8023, \quad \epsilon_3 = 1.5111$, and $\delta = 0.1570$.
Assume that the nonlinear perturbations satisfy (7) - (9) respectively and the delay $h(t)$ satisfies (2.3). Now we calculate the allow able upper bound of h_2 that guarantees the robust stability of system (2.1) under different and listed in Table 1, 2.

Table 1: Upper bounds of time delays h_2 for Example 1 with
$$\alpha = \gamma = \eta = \rho = d = 0.1.$$

h_1	h_2
0.0	0.6489
0.3	0.6930
0.5	0.7164
0.7	0.7444

Table 2: Upper bounds of time delays h_2 for Example 1 with
$$\gamma = \eta = \rho = d = 0.1 \text{ and } h_1 = 0.$$

α	h_2
0.0	0.7000
0.1	0.6489
0.2	0.6077
0.3	0.5733

Example 2 Consider the follwing system [17] :

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t-h(t)) + f(t, x(t)) \\ &\quad + g(t, x(t-h(t))) \end{aligned} \tag{45}$$

with

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix},$$

where $\eta \geq 0$ and $\rho \geq 0$.
Assume that the nonlinear perturbations $f(t, x(t))$ and $g(t, x(t-h(t)))$ satisfy (7) and (9) respectively and the delay $h(t)$ satisfies (2). Now we calculate the allowable upper bound of h_2 that guarantees the exponential stability of system (45) under different and listed in Table 3, 4.

Table 3: Upper bounds of time delays h_2 for Example 2 for $h_1 = 0, \eta = 0$ and $\rho = 0.1$

	$\dot{h}(t) \geq 1$	no restriction on $\dot{h}(t)$
Chen [15]	0.7355	-
Qiu [16]	0.9284	-
Botmart [17]	-	1.1045
Theorem1	-	1.3598

Table 4: Upper bounds of time delays h_2 for Example 2 for $h_1 = 0, \eta = 0.1$ and $\rho = 0.1$

	$\dot{h}(t) \geq 1$	no restriction on $\dot{h}(t)$
Chen [15]	0.7147	-
Qiu [16]	0.8865	-
Botmart [17]	-	1.0600
Theorem1	-	1.2152

6 Conclusion

The research proposed the robust exponential stability criteria for delay-dependent robust exponential stability for uncertain linear systems with multiple non-differentiable time-varying delays and nonlinear perturbations. The method combining Lyapunov-Krasovskii functional, model transformation, Jensent's inequality Lemma, Wirtinger-base integral inequality, Peng-Paik's integral inequality, Leibinz-Newton formula and utilization of zero equations has been adopted to study the research. Exponential stability criteria have formulated in terms of LMIs for the systems. Finally, numerical example showed that the proposed criteria are less conservative than some existing stability criteria.

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