Effect of an Acoustic Field on Disturbances Development and a Laminar-turbulent Transition in a Supersonic Boundary Layer

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Abstract: - This paper describes a method for estimating of disturbances development and the laminar-turbulent transition location on a base of the linear stability theory in presence of the acoustic field. In a stable region amplitudes of perturbation velocities near the location of a stability loss are determined by disturbances which are excited by external sound waves. Because of proximity of a sound wave and eigen fluctuations parameters, the received value of amplitude is accepted equal to amplitude of the growing boundary layer wave. At some point down the Reynolds stress becomes equal to several percents of the laminar boundary layer stress. On the basis of experimental data the criterion is accepted according to which the Reynolds stress equal to 12% of the laminar stress in the beginning of the laminar-turbulent transition.
On the basis of a spectrum of external disturbances and the accepted criterion the satisfactory consent of the calculated and experimental transition Reynolds numbers of is received. Up to transition the disturbance increase in a boundary layer is described by linear stability equations well enough.

Key-Words: - **S**upersonic boundary layer, laminar-turbulent transition, Reynolds stresses, hydrodynamic stability, external perturbations, spectrum

1 Introduction

One of the most difficult problems in the theoretical aerodynamics is the prediction of the laminar-turbulent transition location. Existing examples of real flight and wind tunnels tests demonstrate the dramatic effect of the transition on the flow characteristics. One of the possible methods to determine the position of the transition can be based on experimental studies. These methods are limited areas of experimental researches, and need to be clarified on the basis of more fundamental approaches, in particular, the application of the linear stability theory. Although linear theory cannot fully describe the transition process (due to obvious non-linear processes) it may be useful in cases of small initial perturbation, such as in flight, because the length of the linear area in these cases considerably bigger than the plots of the nonlinear interaction.

So far the e^N-method is widely applied to definition laminar and turbulent transitions [1, 2]. The essence of a method consists in the assumption that the disturbance amplitude in unstable area increases in e^N of times at constant value N. The detailed review of works on application of e^N-method is available in [3].

It should be noticed that the more reasonable should recognize amplitude method [4]. The essence of this method consists that in the field of the beginning transition the disturbance amplitude reaches the size A_t. The complexity of its application consists in need of calculation of initial amplitude (near a stability loss point) of disturbances. It can only be solved within the framework of the receptivity theory, which turns out to be more complex than the stability theory of parallel flows. Therefore, in the first phase the amplitude method was not used and most researchers used e^N-method.

In the case of the parameters closeness of external disturbances and neutral eigen waves it is possible to be guided by the theory of a quasiresonant excitation of oscillations in a boundary layer [5]. The gist of this theory is the distribution of disturbances in the boundary layer for both types of perturbations are close between themselves. Therefore near a neutral point the amplitude of neutral perturbations can be compared with the forced vibration amplitude caused by external waves. Without references to quasiresonant interaction this method was used in [4].

2 Transition criterion

Apparently the amplitude method was first proposed in [6]. There the assumption has been made that transition of a laminar boundary layer into a turbulent layer happens in a location where the Reynolds stress generated by perturbations are equals the mean viscous stress. In practice the position of the transition start is determined by a visible deviation of the measured Reynolds stresses from laminar stresses. For example if the start of the transition is determined by a minimum local friction on a wall or a dynamic pressure (Pitot's tube) it can be noticed that their minimum value exceeds the corresponding quantity at a laminar flow only by several percent. As an example the dependence of the measured friction on the heat-insulated plate on longitudinal coordinate for Mach number M=1.99 taken from [7] are shown in Fig. 1. If to extend the line of the measured laminar friction at x = 13" up to the position corresponding to a minimum, it will be received $\tau_w/q=0.5\ 10^{-3}$. Comparison with minimum value indicates that the last value approximately 12% higher than the value calculated for laminar law. In particular in [4] it has been shown that a satisfactory predictions of the position of the transition can be achieved on the assumption if to take that the value of Reynolds stresses equal to 14% of the laminar value.

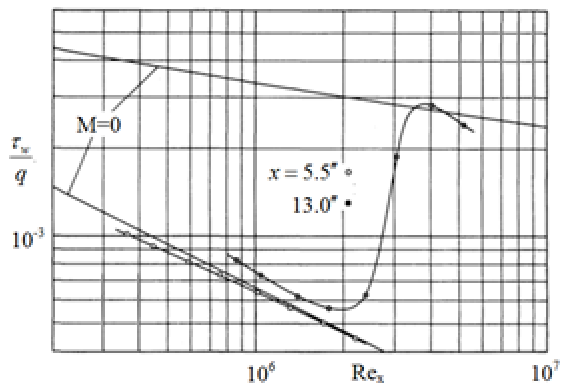

Fig.1. Local drag coefficient in conditions of laminar-turbulent transition, M = 1.97

Based on the above as a transition criterion was adopted the condition:

$$\langle uv\rangle_{\max} = 0.12\bar{\mu}/(\mathrm{Re}_{\delta t}(\bar{\rho}(d\bar{u}/d\bar{y}))_{\max}) \quad (1)$$

Here $\tilde{\mu},\tilde{u},\tilde{\rho},u,v$ — dynamic viscosity, velocity and density, as well as perturbations of velocities, normalized on corresponding values on a boundary layer edge; normal coordinate $\tilde{y}=y/\delta$, where δ — boundary layer thickness; $\mathrm{Re}_{\delta t}$ — transition Reynolds number, built according to the thickness of the boundary layer. In the case of single-mode perturbations the right-hand side is determined by the eigen function of the stability theory and its amplification downstream. In formulating the problem of a hydrodynamic stability of quasi-parallel flows [8–10] velocities u and v are represented in zeroth approximation as follows:

$$u,v = A_0\,\mathrm{Re}\,al\{[f(\tilde{y},\varepsilon x),\varphi(\tilde{y},\varepsilon x)]\cdot\exp(i\vartheta)\}$$

For perturbations growing in the longitudinal direction can be taken

$$\vartheta_i = -\int_{x_0}^{x}\alpha_1^*(x)dx;\ \ \vartheta_r = \int_{x_0}^{x}\alpha_r^*(x)dx + \beta^* z - \omega^* t\,,$$

where $\alpha^* = \alpha_r^* + i\alpha_i^*$ — eigen wave number, f and φ are the corresponding eigenfunctions of the stability problem of a locally parallel flow for fixed values of β^* and ω^*; x_0 is the coordinate of the neutral oscillations on the lower branch of the neutral curve. An asterisk indicates dimensional values. In the future it will be accepted that the maximum value of an amplitude f inside the layer is equal to unity, $|f|_{\max}=1$. It can be shown that $\langle uv\rangle = A_0^2\exp(-2\vartheta_i)\mathrm{Re}\,al(f\varphi^*)/2$, where A_0 — maximum amplitude of eigen oscillationы of longitudinal speed inside the layer at $x= x_0$, an asterisk denotes complex conjugation.

The amplitude of neutral oscillations A_0 is generated as a result of non-stationary impact on the boundary layer, in particular, by external disturbances with amplitude A_1. According to [4] it is assumed that $A_0= A_z A_1$, , where A_z — receptivity factor which dependces on the wave number β^* and frequency ω^*. With this in mind relation (1) can be rewritten in the form:

$$\langle uv\rangle_{\max} = A_z^{\,2}A_1^{\,2}\left|\mathrm{Re}\,al(f\varphi^*)\right|_{\max}\frac{I^2}{2} = \frac{0.12}{\mathrm{Re}_{\delta t}}\frac{\tilde{\mu}}{\tilde{\rho}}\frac{d\tilde{u}}{d\tilde{y}}\bigg|_{\max},$$

where $I^2 = \exp(-2\vartheta_i)$; or in another form:

$$A_t = A_z A_1 I = \frac{0.35}{\mathrm{Re}_\delta^{1/2}}\frac{\left(2\tilde{\mu}\tilde{u}_{\tilde{y}}/\bar{\rho}\right)_{\max}^{1/2}}{((\mathrm{Re}\,al(f\varphi^*))_{\max})^{1/2}} \quad (2)$$

Taking into account that $(f\varphi^*)_{\max}$ is the slowly varying function of the longitudinal coordinate, and the amplification coefficient depends exponentially on x, the perturbations amplitude near the transition for very small external perturbations will practically does not depend on the transition Reynolds number. Therefore, it is assumed that the amplitude of the longitudinal velocity perturbations near the transition is constant, as is customary in [4].

Let us estimate the amplitude A_t for the boundary layer on the thermally insulated plate. In this case

$$\left(\frac{2\tilde{\mu}}{\tilde{\rho}}\left|\frac{d\tilde{u}}{d\tilde{y}}\right|_{\max}\right)^{1/2}=\left(\frac{2\tilde{\mu}}{\tilde{\rho}}\frac{d\tilde{u}}{d\tilde{y}}\right)_w^{1/2}=0.8\tilde{\mu}_w^{1/2}$$

at $\delta=\sqrt{x(\mu/\rho U)_e}$ and $(\tilde{u}_{\tilde{y}})/\tilde{\rho})_w=0.332$.

According to [11] $\varphi=\alpha_r\varepsilon f$, $\varepsilon=(\alpha_r \mathrm{Re}_{\delta t})^{-1/3}$. It follows from (2) that for *f=1* $A_t=A_zA_1I\simeq 0.28\alpha_r^{-1/3}/\mathrm{Re}_{\delta t}^{1/3}$.

At a fixed frequency the wave number α_r is associated with a Reynolds number by relationship: $\alpha_r\approx(\omega^*\nu_e/u_e^2)\mathrm{Re}_\delta)/c_r=F\,\mathrm{Re}_\delta/c_r$. Due to the fact that the phase velocity c_r varies only slightly with the Reynolds number, it's possible to take $\alpha_r=B\,\mathrm{Re}_\delta$ or $\alpha_r=\alpha_0(\mathrm{Re}_\delta/\mathrm{Re}_0)$. Because at low frequencies on the lower branch of the neutral curve $\alpha_0\mathrm{Re}_0=D\approx const$ [12] it's possible to obtain

$\alpha_r=D(\mathrm{Re}_\delta/\mathrm{Re}_0^2)$. So $A_t\approx 0.28D^{-1/3}\left(\frac{\mathrm{Re}_0}{\mathrm{Re}_{\delta t}}\right)^{2/3}$.

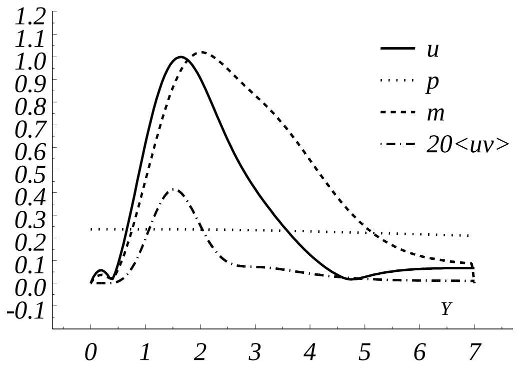

Fig. 2. The amplitudes dependence of pressure (p), velocity (u), mass flow rate (m) and <uv> on the normal coordinate, $dY=\tilde{\rho}dy/\delta$.
Re=1400, χ=arctg(β/α)=45°, M=2.

In [12] it was shown that for the boundary layer on a heat-insulated plate with M = 2 D≈10, therefore $A_t=A_zA_1I\approx 0.13\left(\mathrm{Re}_0/\mathrm{Re}_{\delta t}\right)^{2/3}$. In addition, in wind tunnels $\mathrm{Re}_0/\mathrm{Re}_{\delta t}=0.3\div 0.5$, therefore $A_t\approx 0.06$, that is 6% of the main velocity at the boundary layer edge . For subsonic flows in [4] the value of the longitudinal velocity pulsations near the transition position was assumed to be four percent (4%). As to the supersonic boundary layer, in [4] as a transition criterion was taken the pressure perturbation amplitude equal to 1%. But pressure perturbations inside the layer are about in 4 times lower than the longitudinal velocity perturbations the according to the theory of stability, Fig.2. Therefore it's possible to consider that in [4] the value of the amplitude of velocity perturbations at the beginning of the transition was taken to be about 4% as in the case of a subsonic boundary layer.

In Fig.2 along with the longitudinal velocity and pressure amplitudes the distribution of the value *<uv>* (which is proportional to the Reynolds stresses) and the mass flow are shown at M = 2, Re_δ=1400, χ=arctg(β/α)=45°, $F=0.1\cdot 10^{-4}$. From the above results it can be seen that the positions of the amplitudes maxima of the disturbances and Reynolds stresses (which are proportional to *<uv>*) are in the same position approximately. Additional calculations showed that their location is practically independent of frequency at a fixed angle *χ*, which justifies the use of (2).

In the case of multi-frequency oscillations at fixed values of *x, y, z* the longitudinal velocity oscillations are written in the form: $u(t)=\mathrm{Re}\,al(\sum_{i=1}^{8}A_i\exp(i\omega_i t))$. In this case $\langle uv\rangle=\sum_{i=1}^{n}A_{zi}{}^2A_{1i}{}^2\,\mathrm{Re}\,al(f_i\varphi_i^*)I^2/2$. If the positions of the maximum amplitudes of the longitudinal velocity A_{imax} do not depend on ω_i inside the boundary layer then, omitting the index $_{\max}$, instead of (2) we will have:

$$\sum_{i=1}^{n}A_{zi}{}^2A_{1i}{}^2\,\mathrm{Re}_{\delta t}(\mathrm{Re}\,al(f_i\varphi_i^*)I_i^2=0.24\frac{\bar{\mu}}{\bar{\rho}}\frac{d\bar{u}}{d\bar{y}}\bigg|_{\max} \quad (3)$$

At the same time at the beginning of the laminar-turbulent transition the mean-square perturbations of the longitudinal velocity $\langle u\rangle=(\sum_{i=1}^{n}A_{ti}^2/2)^{1/2}$, where $A_{ti}=A_{1i}A_{zi}I_i$.

3 Disturbances in the supersonic boundary layer and its laminar-turbulent transition in conditions of the wind tunnel T-325

This section demonstrates the efficiency of the stability theory of supersonic boundary layers by comparing its data with experimental results obtained in a wind tunnel T-325 of Khristianovich Institute of Theoretical and Applied Mechanics SB RAS.

3.1 Disturbances amplification in the transition region of the boundary layer

As an example we consider experiments [13] which were carried out at the unit Reynolds number $Re_1 = 12.51 \cdot 10^6$/m and M = 2 in the boundary layer of a flat plate. Fig. 3 shows the mass flow spectrum approximation in the boundary layer. Dependence of root-mean-square perturbations of the mass flow in the boundary layer on the Reynolds number on the Reynolds number is shown in Fig. 4 (round icons). The mass flow spectrum was approximated by the dependence:

$m(t) = \sum_{i=1}^{8} m_i \sin(\omega_i t + \psi_i)$. The amplitude spectrum

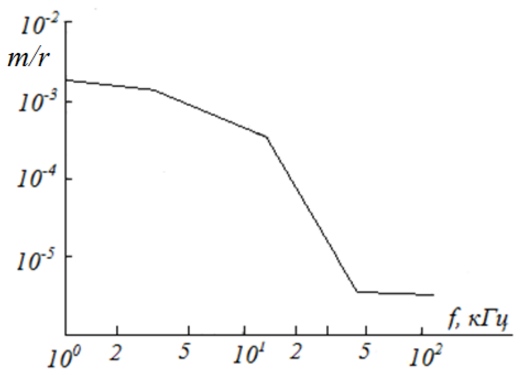

Fig.3 Amplitude spectrum of the mass flow in the boundary layer, $Re_\delta = 700$

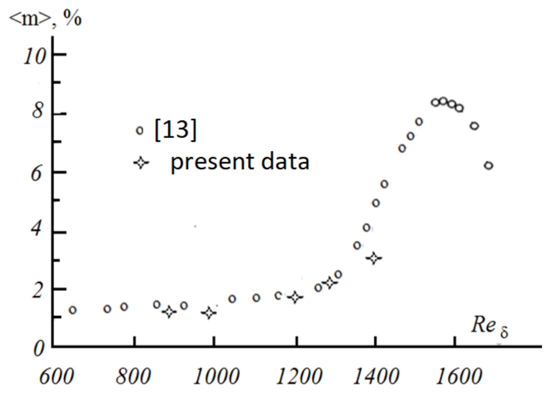

Fig.4. Dependence of root-mean-square perturbations of the mass flow in the boundary layer on the Reynolds number

in [20] was obtained with an accuracy of the normalization factor therefore it is designated as m/r in Fig. 3.

Table 1 shows the values of m_i/r as a function of a frequency parameter (second column), were obtained from Fig. 3, as well as theirs squares (third column). Based on experimental values of the mass flow perturbation amplitude, Fig. 4, it was assumed that at Re = 700. <m> = 0.014, that is 1.4% of the main mass flow of the external flow. Using the relation $r^2 \sum_{i=1}^{8} (m_i / r)^2 / 2 = (0.014)^2 = 0.196 \cdot 10^{-3}$

Table 1. Calculations of root-mean-square pulsations of a mass flow in a dependence on the Reynolds number

$F \cdot 10^4$	experiment, Re=700			Calculation $m_i^2 \cdot 10^6$ based on experimental data				
	m_i/r	m_i^2/r^2	$m_i^2 \cdot 10^6$	Re =840	1000	1200	1300	1400
0.05	10	100	300	318	318	288	280	317
0.10	4.8	23.4	70.2	67	56.4	181.8	230.4	418.1
0.15	2.5	6.25	18.8	24.7	51.1	199.5	433.5	945.6
0.20	0.85	0.72	2.2	4.5	13.0	67.0	138.4	251.7
0.25	0.32	0.10	0.3	0.8	2.9	12.0	18.7	24
0.30	0.13	0.02	0.06	0.2	0.7	1.6	2.0	1.7
0.35	0.07	0.01	0.03	0.1	0.3	0.3	0	0
0.40	0.04	0.00	0.01	0.0	0	0	0	0
sum		131	392	413	441	747	1096	1950
<m>%			1.4	1.37	1.31	1.75	2.3	3.1

can be obtained $r^2 = 3 \cdot 10^{-6}$. The m_i values for Re = 840, 1000.1200, 1300 and 1400 were obtained by multiplying the experimental value of the perturbation at Re = 700 by the amplification coefficient at the interval of the Reynolds number, in accordance with the stability equations [14]. For frequency parameters $F=(0.05, 0.1) \cdot 10^{-4}$ at which the critical Reynolds numbers are more than700, the coefficient of an increase (decrease) of disturbances were determined by the ratio $A_{zi}(\mathrm{Re})/A_{zi}(\mathrm{Rec})$ according to Fig. 5 when Re<Rec, and when Re>Rec by multiplying $A_{zi}(\mathrm{Rec})/A_{zi}(\mathrm{Re}=700)$ by *I* (see (2). corresponding experimental values were multiplied by

$$I_1 = \exp(-\int_{x(\mathrm{Re}=700)}^{x(\mathrm{Re})} \alpha_i^* dx) \ .$$

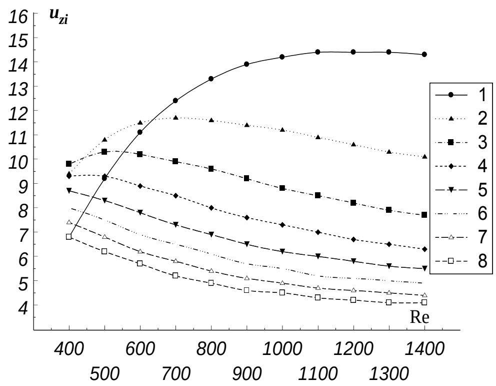

Fig.5. Dependence of u_{zi} on Re_δ. Numbers 1,2, 3,4, 5,6,7,8 represent to frequency parameters *F=(0.05,0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40)·10*$^{-4}$

Calculations show that if $F=0.05\cdot10^{-4}$ then A_{zi}(Re)/A_{zi}(Re=700)=1.03,1.03,0.98,0.96for Re=840, 1000, 1200, 1260, and for $F=0.10\cdot10^{-4}$ A_{zi}(Re)/A_{zi}(Re=700)=0.96 at Re= Re_c =840.

Calculations results of mass flow perturbations, <m>%, are shown in Fig. 4 and in Table 3. The deviation of the experimental data from the calculation is observed in the interval of the Reynolds numbers $1300<\mathrm{Re}_\delta<1400$, where nonlinear processes are observed, [13].

3.2 Location of laminar-turbulent transition

Because $\left(\bar{\mu}\bar{U}'/\bar{\rho}\right)_{\max} = 0.33\bar{\mu}_w$, $\bar{\mu}_w \approx 1.4$ and (3) is rewrited as

$$K_{tr} = \sum_{i=1}^{n} Ki = 0.11; \quad Ki = A_{1i}^2 A_{zi}^2 Di\,, \quad (4)$$

where $Di = \mathrm{Re}_{\delta t} \left|\mathrm{Re}\,al(f_i\varphi_i^*)\right|_{\max} I_i^2$

The performance of the criterion (4) is verified by the example of experiments on the laminar-turbulent transition of a supersonic boundary layer using the spectral composition of the perturbations in the wind tunnel T-325 of ITAM of SB RAS [15]. Experiments were carried out at Mach number M = 2 and a unit Reynolds number $\mathrm{Re}_1=25\cdot10^6$/m [16]. The T-325 was upgraded periodically after which the composition of the perturbations in the wind tunnel changed. Therefore it is important to emphasize that spectrum measurements [15] and experiments [16] were carried out at about the same time, and it can be assumed that the background of external disturbances was approximately the same in both cases. Studies [15, 17] found that the pressure perturbation level in the working part at M = 2 and $Re_1=25\cdot10^6/m$ was approximately 0.4%, the angle of inclination of the acoustic waves front relatively to the direction of the main flow was approximately 45° in the frequency range $4\div40kH$ what corresponds to the phase velocity $c=\omega/\alpha_r$= 0.3. But the pressure amplitude and the amplitude of the longitudinal velocity outside the boundary layer are related by the equality $p=-\gamma M^2(1-c)u$.

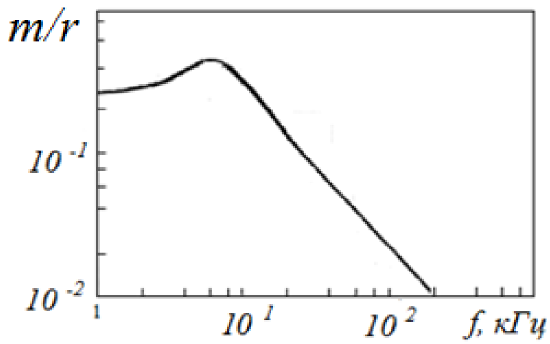

Fig.6. Amplitude mass flow spectrum in T-325, $\mathrm{Re}_1=30\cdot10^6$/m.

To calculate the spectral composition of longitudinal velocity perturbations on the basis of the mass flow spectrum, Fig. 6, we use the relation: $u=m-\rho=m-p/\gamma=m+M^2(1-c)u$, from which it follows that

$$|\mathrm{u}| = |\mathrm{m}/(M^2(1-c)-1)| \approx 0.56|\mathrm{m}|\,. \qquad (5)$$

Table 2. Calculation results of the velocity disyurbances in the T-325 on a spectrum (6)

	m_i/r	u_{1i}/r	u_{1i}^2/r^2	$u_{1i}^2\cdot10^8$
0.05	22.76	12.75	162.4	140.1
0.10	10.62	5.95	35.4	30.55
0.15	6.8	3.81	14.5	12.5
0.20	4.95	2.77	7.68	6.6
0.25	3.87	2.17	4.7	4.1
0.30	3.17	1.78	3.15	2.7
0.35	2.68	1.50	2.25	1.9
0.40	2.31	1.29	1.67	1.4
sum			232	200

In the frequencies interval $f=10\div200kHz$ the spectral curve, Fig. 6, is practically independent on a unit Reynolds number and is approximated by the relation

$$m/m_0=(f/f_0)^{-1.1}. \qquad (6)$$

The calculations were carried out at f_0 = *40kHz*, which corresponds to the frequency parameter $F_0 = 0.352\cdot 10^{-4}$, and $m_0/r = 6\cdot10^{-2}$ at $\mathrm{Re}_1 = 30\cdot10^6/m$ and the velocity in the wind tunnel $u_e \approx 500 m/s$

Unfortunately the normalization factor for the presented mass flow spectra was not given in [15], therefore the following technique was used. In the accordance with (6) the values of m_i/r were calculated for frequency parameters $F=(0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40)\cdot10^{-4}$ and corresponding values of u_i were obtained on (5). Perturbations of the longitudinal velocity were represented as $u(t) = (\sum_{i=1}^{8} u_i \sin(\Delta_F t + \phi_i))$, where $\Delta_F=0.05\cdot10^{-4}$. The values of m_i/r, u_{1i}/r and their squares are given in 2, 3 and 4 columns of Table 2. From the relation $\langle u^2 \rangle = r^2 \sum_{i=1}^{8} (u_i^2/r^2)/2 = 10^{-6}$ and the data of Table 2 it follows that $r^2=0.863\cdot10^{-8}$. The squares of the amplitudes of the velocity perturbations are shown in the last column of the table.

To determine the u_{zi} the calculations of the interaction of external oblique sound waves (χ=45°) with the boundary layer were performed in the low frequency approximation (the pressure disturbance in the boundary layer is constantly) in accordance with [14].

Table 3. Values of u_{zi}, Re_c, K_i and D_i in the dependence on the frequency parameter

$F\cdot10^4$	Re_c	u_{zi}/r	Re=1400				Re=1500			
			Di	*Ki*	I^2	$10^3 u_{ti}^2$	*Di*	*Ki*	I^2	$10^3\cdot u_{ti}^2$
0.05	1260	14.4	12.5	0.0038	1	0.291	17.2	0.0050	1	0.291
0.10	840	1.5	198	0.0080	7	0.283	446	0.0180	14	0.566
0.15	680	0.0	2120	0.0265	51	0.637	4922	0.0602	106	1.321
0.20	580	9.0	6988	0.0379	142	0.759	11760	0.0638	221	2.38
0.25	500	8.3	7553	0.0213	142	0.401	8917	0.0250	157	0.89
0.30	460	7.7	4215	0.0065	75	0.120	3741	0.0058	63	0.101
0.35	420	6.8	1563	0.0014	27	0.023	1049	0.0001	17	0.014
0.40	400	6.7	454	0.0003	8	0	215	0.0002	3	0
sum				0.106		2.514		0.178		5.56

The relations of the maximum amplitude of perturbations of the longitudinal velocity in the boundary layer to the amplitude of the external sound wave are shown in Fig.5. Table 3 shows the values of the coefficients u_{zi} at critical Reynolds numbers, Re_c, as well as *Di* and *Ki*, in a dependence on the frequency parameter for the numbers Re = 1400 and 1500. It is easy to see that condition (4) is practically satisfied when the Reynolds number is $\mathrm{Re}_x = x\mathrm{Re}_1 \approx 2\cdot10^6$. In this case the amplitude of perturbations of the velocity u_t reaches 3% of u_e at Re = 1400, and approximately 5% at Re = 1500.

It should be noted that the sound field of three wind tunnel walls affects on the model boundary layer Therefore in calculations of the transition location should take the amplitude of external disturbances equal to 75% of measured experimentally in free stream and a criterion value of K_{tr} should be increased approximately by 1.8 times and it is taken approximately equal to 0.2. In this case, the transition Reynolds number will slightly exceed the experimental value $\mathrm{Re}_{xtr}=2.2\cdot10^6$ of the paper [16].

4 Conclusion

Thus, for the first time, on the basis of the spectrum of external acoustic disturbances, the position of the laminar-turbulent transition of a supersonic boundary layer is calculated, which is in agreement with the experimental ones. In the pre-transition region the natural disturbances are described well by the linear theory.

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