Optimization of a Drive Shaft using PSO Algorithm

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Abstract: - Mechanical design involves several continuous variables associated with the calculation of elements that compose the parts implemented in different processes. However, when the values associated with several design variables are selected, the range of each such variable may result in infinite solutions or oversized solution spaces. Thus, the choice and fit of different variables related to the mechanical parts under analysis pose a challenge to designers. This is the case of drive shaft design: the variables that represent the diameters of several transversal sections of each of its elements directly affect its weight and resistance to mechanical stresses. Therefore, the selection of variables should not be at random. This article presents the optimization of the design of a drive shaft composed of three transversal sections using the metaheuristic technique particle swarm optimization (PSO). Such problem is solved to obtain an optimal and reliable part. For that purpose, a nonlinear mathematical model was developed to represent this problem as a function of the physical features of the mechanical system. The objective function is the reduction of the weight of the shaft and the variables are the diameters of each section. The set of constraints in this problem considers the general equation to design a fatigue-safe shaft as well as a constructive constraint to establish the minimum step distance for coupling the mechanical elements. Due to the nonlinearity of the mathematical model, this work proposes PSO as optimization technique. This algorithm has proven to be an efficient tool to solve continuous nonlinear problems. Finally, the solution provided by the optimization technique is validated in ANSYS® software, thus demonstrating that the answer meets all the design criteria previously selected.

Key-Words: - machinery design, drive shaft, particle swarm optimization, ANSYS® simulation.

1 Introduction

Designing mechanical parts for industrial machinery is a complex and delicate task that should consider several factors for construction. They include the stresses the part will undergo, the type of material to be used and the correct geometry for coupling the elements. Finding an adequate solution to different mechanical design problems requires time to simulate, build and validate several prototypes of the part under analysis. This entails a hard and long process that requires an investment of time and economic resources. The latter are the most significant limitation because, in many cases, companies lack the funding to conduct mechanical resistance tests of the elements they design. In the industrial field, mechanical design ensures quality and safety in the manufacture of components for different machinery and equipment. This design process can be divided into several stages: First, a preliminary or conceptual design of the part is produced. During this stage a vision of what is wanted, along with the required measurements, is provided. During the second stage, simulation and analysis are conducted to certify that the element will be able to withstand the loads and stresses it will be subjected to. The next step is prototyping to experimentally validate the element before building it. This process guarantees that the element will not suffer premature failures [1]. Since drive shafts experience variable loads, their correct operation in conditions of fatigue should be guaranteed because the failure of this element may cause a total shutdown of a piece of machinery [2]. Different tools have been implemented in the field of mechanical design of drive shafts to obtain an adequate and reliable part. This is the case of the work by Cerón et al. in 2006 [3]. They presented a study to redesign a drive shaft that had suffered a fatigue failure in one of the keyseats. In their analysis, these authors proposed a mathematical model to calculate the safety factors and diameters where gears are located. Subsequently, a simulation was conducted using Finite Element Analysis (FEA) software to find the root cause of the problem and propose an improvement for future manufacturing. In 2011, Momcilovic et al. [4] analyzed the failure of a drive shaft in a hydraulic turbine. To solve this problem, they started with an experimental case and found the effects that caused the failure in the turbine. Afterward, they developed a mathematical model to identify the stresses the turbine and the shaft experience. Additionally, the loads were modeled and simulated in ANSYS® to locate the critical points of the element under analysis, thus finding the problems the shaft presented and possible solutions. This analysis demonstrates the importance of employing mathematical models to analyze failures in shafts and that results can be validated using software or specialized design tools.

In recent years, mathematical tools known as optimization algorithms have been widely implemented to solve mathematical models associated with different mechanical design problems [5]. They enable to obtain different design parameters that meet all the mechanical and construction constraints for the part under analysis [6] based on a specific criterion or set of criteria selected by the designer (e.g. weight, length, diameter, or thickness). This type of algorithms explore the solution space associated with a given problem by implementing the mathematical model that represents said problem [7]. The models are composed of the objective function and the set of constraints that represent the problem, thus limiting the solution space. The mono- or multi-objective function evaluates each individual generated by the optimization technique and the impact of each proposed solution on the main objective(s) set for the problem. It should be highlighted that the objective function may be maximized or minimized depending on the type of problem. The set of constraints fixes ranges for different variables considered in the problem as well as technical and construction limitations so that the solution space is restricted to the technical aspects each scenario presents.

Evolutionary algorithms, especially metaheuristic techniques, stand out from the group of optimization techniques implemented in mechanical design problems [8]. They are part of the group of inexact or numerical convergence methods that, despite not having mechanisms that guarantee a global optimal solution, enable to obtain good quality solutions to optimization problems with acceptable computational effort. Said effort involves two important aspects: calculation time and amount of memory required by the process. Metaheuristic techniques guide and modify the operations of subordinate heuristics to provide high quality solutions in an efficient fashion by implementing successful search strategies and bioinspired algorithms [9]. Some of the most commonly used metaheuristic techniques are genetic algorithms, particle swarm, simulated annealing, tabu search, immune algorithms, ant colony, and bee swarm [10]. Most of them have been implemented in mechanical design, as in the work by Lampinen [11]. The latter focused on designing cams using a genetic algorithm as solution technique, and it demonstrated that computational tools enable to find an excellent solution faster than traditional design methods. In 2017, Abdessamed et al. [12] conducted a study aimed at maximizing the nominal efficiency of turbomachinery. They solved the proposed mathematical model by implementing bio- and socio-inspired algorithms and found similar answers by different methods. At the end, the PSO algorithm provided the best answer among the algorithms under analysis. In 2015, Hanafi et al. [13] researched the cutting speed and depth of cut values that produced CNC machining with good surface finishing. They developed said model by means of PSO and found that this tool allowed to determine the optimal values that produced minimal roughness in the finishing of the most economical tools; besides, excellent machining times were achieved. Other metaheuristic optimization algorithms have been adopted to solve mechanical design problems, such as League Championship Algorithm (LCA) and Multi View Differential Evolution (MVDE); Particle Swarm Optimization (PSO) is one of the most widely used [14]–[16].

Generally, the objective function presented in works in the specialized literature on mechanical design is the minimization of production costs, weight, the load as a function of the stress of bearings, and production cost as a function of volume [14]–[16]. In all these cases the limitations are the mechanical stresses that each scenario should withstand without compromising its function, which enable to establish basic constraints for a mechanical design problem. However, it should be clarified that they vary from one problem to the next.

Optimization algorithms should be implemented in the field of mechanical drive (e.g. couplings and shafts) to quickly and accurately find feasible solutions that improve manufacture and assembly. This enables to reduce production costs as well as design and manufacturing times. For that reason, this article proposes to delve into the minimization of the weight of a drive shaft with an abrupt change in the transversal section. The objective function in this work is the reduction of the weight of the shaft and the variables in the problem are the diameters of the transversal sections. Nevertheless, there are two constraints: calculating diameters with the general equation for designing a fatigue-safe shaft [17] and a constructive limitation to establish the minimum distance of the step for coupling of the mechanical elements. The PSO algorithm is proposed as a solution technique for the mathematical model described above. It was selected because it has been widely implemented to solve mechanical design problems, it offers good quality, low computational cost, and excellent results for optimizing problems with continuous variables, such as in this study. Finally, the answer provided by the PSO was analyzed with specialized software (ANSYS®) to validate the final solution found by the simulation and hence guarantee that the shaft will withstand the loads described in the problem while experiencing adequate deformation.

This document is divided as follows. Section 2 analyzes and explains the PSO optimization algorithm. Section 3 presents the mathematical formulation of the problem and lists the data that enable to study the problem and define its variables. Section 4 describes the coding proposed for the problem. Section 5 formulates the mathematical model for the optimal design of a drive shaft; it also explains the objective function and the set of constraints associated with the problem. Section 6 presents the method proposed to solve the mathematical model based on the parameters necessary to implement the algorithm; besides, it details the way the load analysis and the validation in ANSYS® were carried out. The results found by PSO and their corresponding analyses are included in Section 7. Finally, Section 8 discusses the conclusions of this study.

2 PSO Algorithm

Developed by Eberhart and Kennedy in 1995, this type of metaheuristic and bio-inspired algorithm is based on the flocking behavior of birds and fish schools [18]. It works based on the way these groups of animals explore the ocean or a given region looking for a common source of food for all the group. Each animal is modeled as a particle, which turns the scout group into a swarm of particles scattered in a solution space limited by the set of constraints of each problem. The main characteristic of PSO is the way each particle moves in the solution space, because every step considers the information of each particle as well as that of the particle that represents the best answer in the swarm at each iteration or movement. It offers the possibility of controlling the pace and consider a random component that prevents the algorithm from being trapped in local optima.

It should be mentioned that the position of the particle in the solution space is given by the possible values of each of the variables that represent the solution to the problem. There are two versions of this technique: continuous and binary. In the first case, the particles can take real values in each dimension. In the second, each dimension of the particles can take a value of 0 or 1. This work implements the continuous alternative to identify the optimal dimensions of a drive shaft, which are represented by real numbers. The equations that define the PSO and its iterative process are presented below.

Vector X_i contains the variables of the optimization problem so that, when the iterative process is complete, it provides the set of values that result in the optimal solution to the problem; see equation (1), where i denotes the i-th particle. Likewise, velocity vector V_i contains the velocities of each of the variables in the problem as presented in equation (2). Both vectors are updated at every iteration so that the particles advance towards the solution to the problem. It is worth mentioning that, at the start of the problem, a set of particles of size P is created. The starting point of each variable is randomly set, as well as the velocities of the first iteration. Additionally, these values should be assigned maximum and minimum allowed limits which are directly related to the problem under analysis (in the case of X_i) and the convergence speed (in the case of velocities).

$$X_i = [x_1, x_2, x_3, \dots, x_N] \quad \forall i \in P \tag{1}$$

$$V_i = [v_1, v_2, v_3, \dots, v_N] \quad \forall i \in P \tag{2}$$

In this algorithm, the movement of a particle at each iteration is composed of three vectors, as shown in equation (3). Vector X_i^t corresponds to the values assigned to the movement of the particle at iteration t, and it is composed of the sum of vector $X_i^{(t-1)}$, which corresponds to the previous position of the particle at iteration t-1 and its movement speed at iteration t.

$$X_i^t = X_i^{t-1} + V_i^t \tag{3}$$

It should be mentioned that in equation (3) the movement of the particle in the exploration space is given by V_i^t, which is obtained from the values assigned to each variable at the previous iteration and the implementation of two adaptation functions. By analyzing the objective function of each particle (*aptitude_x*$_i$), said functions enable to identify the best position of the i-th particle (*Betterpos*$_i$), its objective function (*aptitude_Betterpos*$_i$), the position of the best solution in the swarm of particles (*Betterpos*$_g$), and its objective function (*aptitude_Betterpos*$_g$) until the iteration under analysis. Furthermore, these positions are updated at each iteration as long as the answer provided by the particle and the swarm of particles of the current iteration outperforms the one obtained at the previous iteration. Equation (4) presents the formula to calculate the velocity at each iteration.

$$V_i^t = \Omega^{t-1}.V_i^t + \varphi_1.random_1.\left(Betterpos_i - X_i^{t-1}\right) + \varphi_2.random_2.\left(Betterpos_g - X_i^{t-1}\right) \tag{4}$$

where $X_i^{(t-1)}$ represents the position vector of particle i at the previous iteration and Ω^{t-1}, the inertia factor at the previous iteration, which is updated at each iteration so that it grows as the algorithm moves forward. $\phi 1$ and $\phi 2$ represent the cognitive and social components. *Random*$_1$ and *Random*$_2$ are random values between 0 and 1, which prevent the technique from being trapped in local optima [19]. Figure 1 presents the flowchart that describes the iterative process of the PSO algorithm.

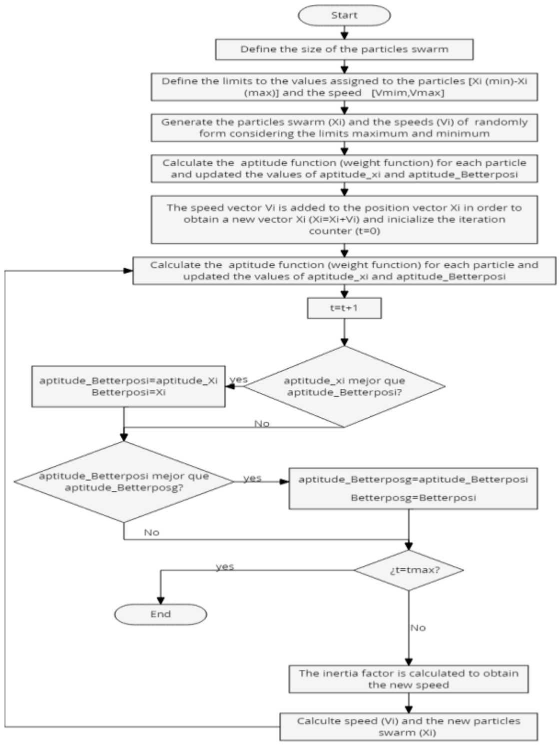

Fig.1. Flowchart of the PSO algorithm [19].

3 Problem Formulation

Drive shafts are essential elements in industrial machinery and the correct operation of the system depends on them because they transmit power to the different elements that compose a piece of machinery. In most cases, they are subjected to loads that fluctuate over time, which means the shaft should be manufactured to withstand different loads efficiently [1]. This article describes the study of a drive shaft with an abrupt chance in the transversal section operating at variable loads. The main objectives of the proposed method are to find the optimal diameters of each section that can withstand the loads described in the problem and reduce weight. The description and characteristics of the problem were taken from another study [20] that presents a stepped drive shaft divided into 3 transversal sections, whose lengths are L1=5.9 in, L=10 in, and L3=10 in. Points A, B, C and D are identified for the mathematical analysis. A pulley of 20 in pitch diameter and a weight of 60 lbs. is coupled to point A. Besides, a spur gear of 10 in in pitch diameter and 25 lbs. is coupled to point C. The transmitted power is 20 hp and the engine reaches a speed of 1200 rpm. The drive shaft is manufactured with AISI 1040 CD steel, and the assigned factor of safety is 2.2. The shaft rotates at 360 rpm. The support points are B and D, where rigid ball bearings are coupled.

To solve the problem, a mono-objective genetic algorithm of the PSO type iterates in the solution space varying the values of the 3 diameters between maximum and minimum limits. At each iterative cycle, the weight function and the set of constraints described in the model are evaluated.

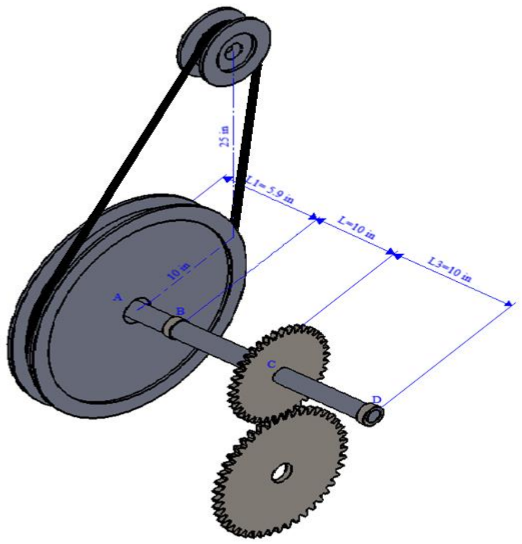

Fig.2. Illustration of the problem. Own source, Autodesk Inventor®.

4 Problem Codification

A 1x3 vector (1 row and 3 columns) was implemented to codify this problem and represent different solutions provided by the optimization technique within the solution space (see fig.3.). The first column of said vector contains the value assigned to the diameter of the segment whose length is L1; the second column, the value of the diameter assigned to the segment of length L. In turn, the third column stores the diameter assigned to section L3 of the drive shaft being analyzed and designed. It should be highlighted that the values assigned in this codification are limited by the set of constraints posed by the problem. They are listed in Section 5.

section	L_1	L_2	L_3
diameter	d_1	d_2	d_3

Fig.3. Problem Codification. Own source.

5 Mathematical model

The mathematical model described in equations (5) to (21) was utilized to formulate the problem introduced above.

$$F = f_1 + Pen \tag{5}$$

$$f_1 = \frac{\gamma\pi}{4}(L_1 d_1^2 + L d_2^2 + L_3 d_3^2) \tag{6}$$

$$Pen = (p_1 + p_2 + p_3 + p_4 + p_5)Fp \tag{7}$$

Equation (5) presents the objective function selected for the problem. It is composed of function f_1 and equation (6), which represents the weight of the shaft as a function of different diameters and lengths, where γ represents the specific weight of AISI 1040 CD steel (0.2834 Lb/in^3). Diameters d_1, d_2 and d_3 are variables in the problem that correspond to the diameters of the transversal sections assigned to each segment of the shaft: L_1, L y L_3, respectively. Penalty function *Pen* in expression (7) improves the exploration of the solution space, which enables the technique to move within the infeasible region. The latter is defined as the sum of penalties associated with each constraint multiplied by a penalty factor (Fp=1.5e6), which allows to add *Pen* to f_1 and consider even the slightest violations of the constraints that represent the problem. Each penalty takes a value in equation (7) when any of the constraints mentioned above is violated. Equation (8) outlines different penalties presented in this work, where the maximum yields zero if the constraint is met and a value other than zero if the constraint is violated. The set of penalties associated with the global penalty is presented in equations (9) to (13), where it will take the maximum value from p_1 to p_5 at each iteration when constraints (g_1 a g_5) are evaluated.

$$p_i = \max\{0, restriccioni\} \tag{8}$$

$$\forall\, i = 1,2,3,4\ y\ 5$$

$$p_1 = \max\{0, k(d_1) - d_1\} \tag{9}$$

$$p_2 = \max\{0, k(d_2) - d_2\} \tag{10}$$

$$p_3 = \max\{0, k(d_3) - d_3\} \tag{11}$$

$$p_4 = \max\{0, d_1 - d_2 + 0.0787\} \tag{12}$$

$$p_5 = \max\{0, d_3 - d_2 + 0.0787\} \tag{13}$$

The following are the constraints that guarantee that the variables selected for the diameters of the drive shaft withstand the loads and mechanical stresses it undergoes. Expression (14) is the general equation to design a fatigue-safe drive shaft [17]. It establishes the minimum diameter that meets the factor of safety considering the loads it should withstand.

$$k(di) = \left\{ \frac{32N_f}{\pi} \left[\frac{\sqrt{(K_f M_a)^2 + \frac{3}{4}(K_{fs} T_a)^2}}{S_f} + \frac{\sqrt{(K_{fm} M_m)^2 + \frac{3}{4}(K_{fsm} T_m)^2}}{S_{ut}} \right] \right\}^{\frac{1}{3}} \quad \forall\, i = 1{,}2\; y\; 3 \tag{14}$$

where N_f is the desired factor of security, 2.2; S_{ut}, the maximum strength to mechanical stress, 75000 psi; and S_f , fatigue stress corrected in a selected lifecycle. The latter is calculated applying Norton's theory [17], which considers corrections to ultimate strength experimentally reported for this material in websites like MatWeb [21]. This value is multiplied by the factors that depend on operating temperature (1), type of material of the drive shaft, surface finishing (0.8318), size or diameter of the shaft (0.869), and type of load it experiences (1). Moreover, the fatigue strength (37500 psi) and safety factor (0.897) set for this design should be included. The multiplication of these two parameters was found to equal 24314.3354 $d_i^{(-0.097)}$ psi (d_i represents the diameter of each segment). K_f is the stress concentration factor for the alternating component of the normal bending stress (2.05 and 2.4 for d_1 and d_3, respectively); K_{fs}, the stress concentration factor for the alternating component of the torsional shear stress (2.05 for d_1 and d_3); K_{fm}, the stress concentration factor of the normal mean component (2.05 and 2.4 for d_1 and d_3, respectively; and K_{fsm}, the stress concentration factor for the shear mean component (2.05 for d_1 and d_3). The concentration factors were taken from the theories by Norton [17] and Mott [2]. To find these values it was necessary to define a 1-mm fillet radius, which is the value recommended by a bearings manufacturer [22] in order to determine the sensitivity of the groove given said radius and the relationship of the diameters. This step enabled to find the values of normal and shear stress concentration. A stress concentration factor of 1.0 was considered in the case of d_2 because this area does not experience stress concentration due to its larger diameter. M_a and T_a refer to alternating moment and torque and M_m and T_m, to average moment and torque. They are calculated by analyzing bending loads and applying the static theory of physics [23]. For this purpose, it was necessary to calculate the forces involved in the problem to find the moments. This calculation required the decomposition of two principal forces: the force the pulley exerts on its horizontal and vertical components, and the force between the teeth of the pinion and the gear in the radial and tangential components of the rigid transmission. They are applied on points A and C, respectively. Based on said forces, the forces on supports B and D are deduced. Subsequently, a free body diagram of the shaft was drawn in each orthogonal plane to find maximum and minimum moments. Furthermore, a 2D vector analysis was conducted on the "horizontal" and "vertical" planes considering maximum and minimum forces. The tool MDSolids 4.0® was used to draw diagrams of internal shear and bending stress that served as a basis to calculate, by vector addition, the critical loads the drive shaft experiences. Afterward, the state of the load was characterized by calculating average and alternating moments. Since every segment of the drive shaft undergoes different forces, the alternating moments that occur in B and C were calculated: 1154.7 and 1097.7 [lb in], respectively. Besides, their average moments were 1619.8 and 1828.8 [lb in], respectively. The values of average and alternating torque reached 2187.5 and 1312.5 [lb in], respectively. The constraint of minimum diameters in (15) establishes the difference between the diameter provided by the algorithm and the minimum diameter analyzed in equation (14). Such difference should be equal or greater than 0.

$$g_i = d_i - k(d_i) \geq 0 \quad \forall\, i = 1{,}2\; y\; 3 \tag{15}$$

where d_i denotes the diameter evaluated at each iteration. Since the drive shaft under analysis is composed of 3 transversal sections, a constraint should be defined for each of them; they are represented in equations (16), (17) and (18). The moments that occur in point B were used for constraint g1 and those in critical point C, for constraints g2 and g3. This is because greater loads take place in point C.

$$g_1 = d_1 - k(d_1) \geq 0 \tag{16}$$

$$g_2 = d_2 - k(d_2) \geq 0 \tag{17}$$

$$g_3 = d_3 - k(d_3) \geq 0 \tag{18}$$

$$g_4 = (d_2 - d_1) \geq 0{,}0787\ in \tag{19}$$

$$g_5 = (d_2 - d_3) \geq 0{,}0787\ in \tag{20}$$

Due to construction criteria, an additional set of constraints needed to be defined for the correct coupling of the mechanical elements. Thus, a minimal distance of 0.0787 in was established between d_1 and d_2 and between d_2 and d_3, as expressed in equations (19) and (20). This value is provided by bearing manufacturers [22]. Finally, equation (21) presents the maximum and maximum limits of the diameters for segments L_1, L, and L_2.

$$1\ in \leq\ d_i\ \leq 4\ in \qquad \forall\ i = 1{,}2\ y\ 3 \tag{21}$$

6 Proposed Method

In order to solve the mathematical model, an PSO is used. In this section the methodology of the load analysis and the verification in ANSYS® is shown in order to explain how the parameters, named before, was calculated.

6.1 Parameters for the optimal sizing of the drive shaft

The parameters implemented to apply the PSO algorithm are listed in Table 2.

Table 2. Parameters to calculate weight and constraints. Own source.

Name	Value
Maximum value of the diameters	4 in
Minimum value of the diameters	1 in
Stopping criteria	convergence
Maximum number of iterations	300
Number of particles	30
Dimensions of the problem	3
Maximum inertia	0.7
Minimum inertia	0.001
Cognitive component	1.494
Social component	1.494
Maximum speed value	0.01
Penalty criterion	1.5e6

6.2 Load analysis

Analyzing the loads is also necessary to identify different parameters implemented in the mathematical model of the problem. The analysis of the loads on the drive shaft considered a torque calculated based on the velocity and power at which the system operates in each scenario: maximum and minimum loads. Besides, the average and alternating torque that characterize each situation were later established: 3500 lb in (maximum torque) and 875 lb in (minimum torque). MDSOLIDS® was used to find the moments. This piece of software enables to detail the internal bending moments the bar undergoes due to external forces on the drive elements and bearings. Fig.4. shows the free-body diagram for the analysis of an overhanging beam and the forces the pulley and the spur gear exert on it.

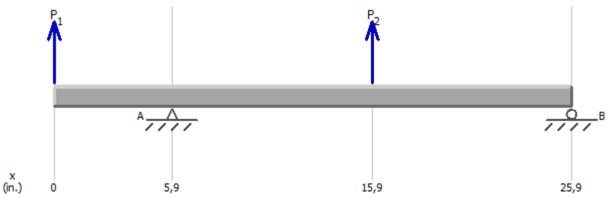

Fig.4. Free-body diagram, MDSOLIDS®. Own source.

This tool produces a diagram (Fig.5.) of internal bending moments at each point along the beam based on the forces that were configured, separating vertical and horizontal planes and maximum and minimum loads.

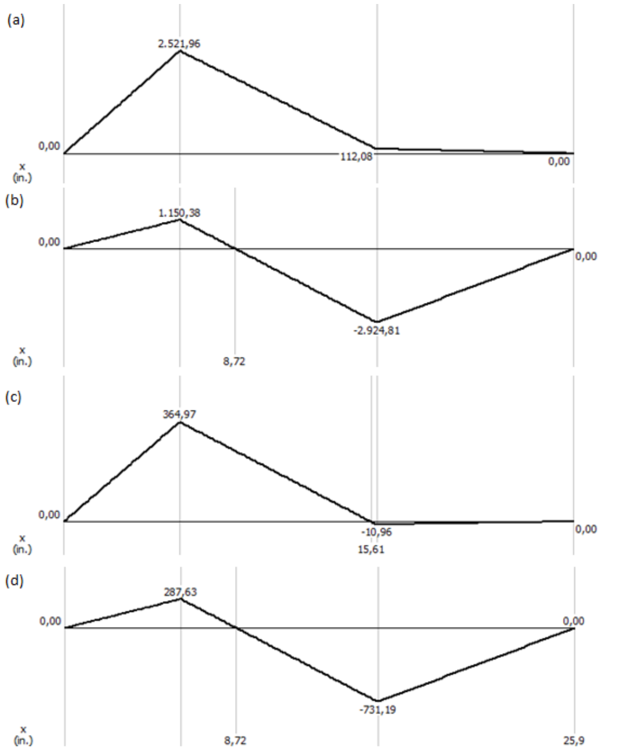

Fig.5. (a) Internal moments in the vertical plane with maximum load. (b) Internal moments in the horizontal plane with maximum load. (c) Internal moments in the vertical plane with minimum load. (d) Internal moments in the horizontal plane with minimum load. Own source.

Maximum and minimum moments were calculated based on these results. Maximum moments: 2927 N in point C and 2772 N in point B. Minimum moments: 731 N in point C and 465 N in point B. This indicates that in both scenarios the critical point is C, where the greatest deformation would occur

6.3 Validation of the solution using ANSYS®

At this stage, the deformation of the drive shaft with optimal diameters provided by the PSO algorithm was compared with the results. Such analysis was conducted using ANSYS® and the tool StaticStructural®, and the simulation considered maximum loads and torques. To start the process, ANSYS® requires a geometry that was modeled with the tool DesingModeler®. Besides, the diameter and fillet radius were parameterized to change values easily and conduct different analyses more quickly. Afterward, the system was discretized by creating the mesh with the tool Mechanical®. The latter was used to carry out a refinement that revealed changes in the transversal section and the phenomenon of stress concentration that can be observed in Figure 6. Finally, the forces and twisting moment are presented as vectors in Figure 7.

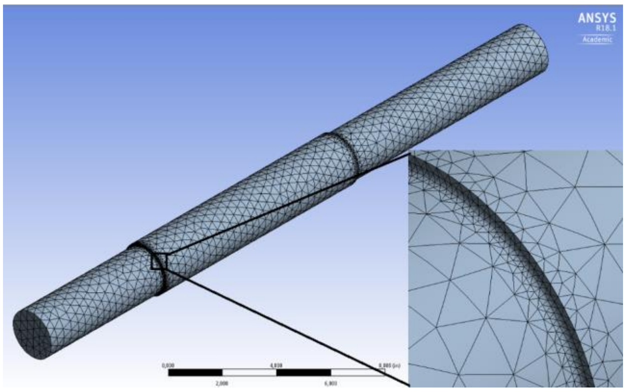

Fig.6. Mesh in the module Mechanical® with zoom on the details of the curvature. Own source.

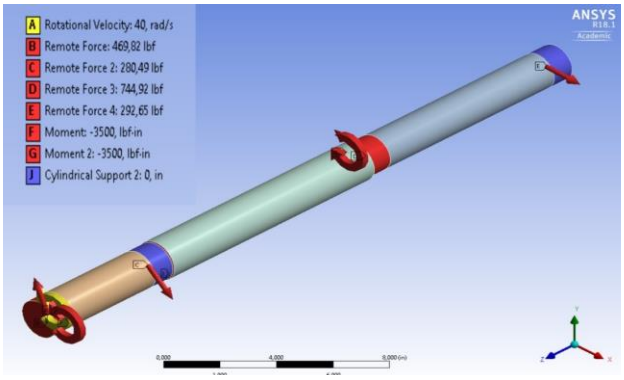

Fig.7. Location of forces and moment. Mechanical®. Own source.

4 Results and discussion

This article presents a PSO algorithm as a technique to solve a mechanical design problem. It proved to be a fast and effective tool to find an excellent solution while meeting a set of constraints that represent physical conditions the material should have to ensure a correct operation. Moreover, ANSYS® simulation provided an additional factor of safety when it verified the results in a digital scenario with the elements that the drive shaft would have coupled in the problem. The following section is an analysis of the results.

7.1 PSO results

The PSO algorithm managed to converge in 170 iterations in a computation time under 1s, and it found the diameters in Table 3 as the optimal values. This solution is considered valid because it does not violate the constraints described in the Method section. This proves that, with the diameters obtained in this study, the drive shaft will withstand the loads it will experience. Besides, a 0.0787 in difference can be observed between D1 and D2, which represents an adequate step for the coupling of the bearing at point B and the satisfaction of this technical criterion.

Table 3. Optimal diameters. Own Source.

d1	d2	d3
1.6805 in	1.7592 in	1.5669 in

With these diameters, the drive shaft would weigh 16.062 lb, which corresponds to 71.26 N and equals (7.27 kg). This value is understood as the minimum weight that satisfies the constraints imposed on the optimization algorithm. When these results were compared to the work by Guzmán and Delgado [20], a big difference was found between the weight they established and the one in this document. This is because the weight obtained from their geometries ranges from 7.5066 MN (766.511 Ton) to 23.169 MN (2362.737 Ton). An in-depth analysis of the results in that work revealed that the specific weight [20] does not correspond to the weight of the material in the description of the problem. As a result, the solutions grow and produce inviable geometries at physical and constructive levels. For that reason, compared to the reference, the solution found in this work represents a better option that defines a single solution satisfying all the design specifications under analysis. Nevertheless, the algorithm used by Guzmán and Delgado [20] is a useful tool because it presents several solutions and enables to study two objective functions for the same problem during the same iterative cycles. Although more computational time is necessary for development, it may be used for problems that require more than one optimal solution, as may be the case of the design of more complex mechanisms affected by more than one variable.

7.2 ANSYS® Verification

Subsequently, ANSYS® was used to verify deformation. This is an important design criterion for drive shafts that use bearings on the supports and on which rigid transmission elements are installed. The result was acceptable since the maximum value was 0.00466 in. Fig.8. shows the location of maximum and minimum deformation values. Points B and D experience the effect the least because they are the locations of the supporting bearings. On the other hand, it can be noticed that point C is where the effect of the loads results in the maximum deformation, which demonstrates the criticality of said spot. This indicates that the optimization and simulation method proposed in this article provides an excellent solution to the problem of optimal sizing of drive shafts.

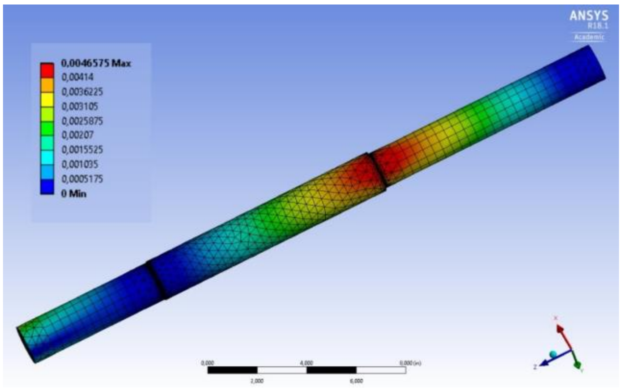

Fig.8. Image of the deformation of the beam and critical point C. Own source.

4 Conclusions

The PSO algorithm was used to solve the problem of the mechanical design of a drive shaft. An appropriate solution to the objective function was quickly found with a low computational cost, which makes this strategy an efficient way to produce a single solution to the problem being addressed.

Conventional machine design is based on iterative manual work that means long hours to solve a problem and, although the constraints may be met, an optimal solution is not guaranteed. For that reason, the introduction of optimization algorithms in this field provides great help to design or improve the design of the mechanical parts in a machine, thus reducing solution times and guaranteeing adequate solutions based on the described objective function.

The combination of an optimization algorithm and validation by ANSYS® simulation results in the satisfaction of all the mechanical design criteria because the error margin in manufacturing is reduced, which constitutes an adequate tool for mechanical design.

As future work, several optimization algorithms could be studied to solve this or other mechanical design problems that are common in the industrial field. Furthermore, said problems could include the analysis of the stresses and deformations that drive shafts undergo to thus determine an optimal geometry that satisfies the requirements.

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