The FEM analysis of coupled vibration of the elastics baffle with two sides' fluid in a rigid tank

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Abstract: - The coupled vibration of the elastics baffle with two sides' fluid, in a rigid tank, is carried out by FEM. Under the special boundary conditions, the equations of coupled vibration are derived in the form of pressure and displacement, which include the elementary matrix on mass, stiffness and coupled domain. It regards the structure and fluid element global degrees of freedoms as row index and column index, and the coupled matrix is assembled to the global matrix. Finally, the numerical results obtained with the proposed FEM are compared with the test example, which agree well.

Key-Words: - elastics baffles; two sides' fluid; coupled vibration; FEM

1 Introduction

The motion of liquid in flexible tanks is different from that in rigid tanks, especially under partially filling conditions. When the sloshing frequency is close to the frequency of structure, resonance would have a great effect on the coupling system.

To prevent resonating and restraining liquid sloshing, baffles are used widely as damping devices. Sloshing problems are investigated by the aerospace agency in early times, so there are a lot of researches on cylindrical tanks with baffles. K. C. Biswal[1] investigates dynamic response of a liquid-filled cylindrical tank with annular baffle, and the effects of the tank wall and baffle flexibility on the slosh response are also given. R. D. Firouz-Abadi[2] and M. A. Noorian[3] develop a boundary element method to determine the natural frequencies and mode shapes of liquid sloshing in 3D baffled tanks.

A zoning method is introduced to model arbitrary arrangements of baffles, and the effect of baffle on sloshing frequencies is investigated. Wang[4] studies the small amplitude sloshing of an ideal fluid in a partially fluid-filled cylindrical rigid tank with a rigid annular baffle. Separating the complicated fluid domain into several simple subdomains, the velocity potential functions corresponding to every fluid subdomain are studied. By introducing the modal expansion method, the general expressions of the modal shape functions of each subdomain are analytically deduced. According to the continuous conditions among the subdomain interfaces and the free fluid surface wave conditions, the solution and the natural sloshing frequencies are determined by using the Fourier series expansion and the Bessel series expansion.

M. A. Goudarzi[5] estimates the damping efficiencies of both vertical and horizontal baffles with various dimensions and locations using the velocity potential formulation and linear wave theory. According to the results of the investigations, the hydrodynamic damping is significantly affected by the size and location of baffles. Guan[6] simulates liquid sloshing in 3D tanks with baffles based on boundary element method(BEM). The numerical result is compared with the analytical solution and experimental data to verify the validity of the simulation, and the effects of the baffle parameters, such as dimension and position on sloshing are also provided.

For the simplification, those 3D problems are treated as two dimensions. K. C. Biswal[7] computes the non-linear sloshing response of liquid in a two dimensional rigid rectangular tank with rigid baffles. A re-gridding technique is applied to the free surface of the liquid, which effectively eliminates the numerical instabilities without the use of artificial smoothing. Guan [8] simulates the liquid sloshing in 2-D baffled tank. In the ALE method, the full Navier-Stokes equations are employed. The effects of the baffle parameters such as dimension and position on sloshing are provided. The dynamic force and moment acting on the tank are also computed.

Most of the published works are concerned with the baffled cylindrical tank, but little with the rectangular ones. The cubic tanks are generally used in ship buildings, such as double bottom tanks and oil tanks. To simply the problem in analyzing rectangular tanks, some reports treated the baffles as rigid ones, which affect the results significantly; others set the baffles under the free surface, seldom above the free surface. Therefore, the flexible baffle is put above the free surface in a rigid tank in this report, which cut the liquid into two parts. There is filled with liquid in each part, and the coupling vibration between the baffles and two sides' liquid are investigated. The vibration equations of flexible structure and liquid are constructed by FEM, and the coupling equations are deduced by interface connection. The progress of forming coupling matrices is given. The numerical results obtained by the proposed FEM are compared with the test example.

2 The vibration equations of structure coupling with two sides' liquid

A sketch of structure coupling with two sides' fluid is given as figure 1. The domain of structure is represented by Ω_s, and the height of structure is L_s. The two sides liquid domains are denoted by Ω_{fL} and Ω_{fR}, where B_1, B_2, B_7, B_8 boundaries are set to be rigid, B_3 and B_6 represent the interface between liquid and structure. B_4 and B_5 denote the two sides' liquid free surface. The width of two sides' liquid are shown as L_{fL} and L_{fR}.

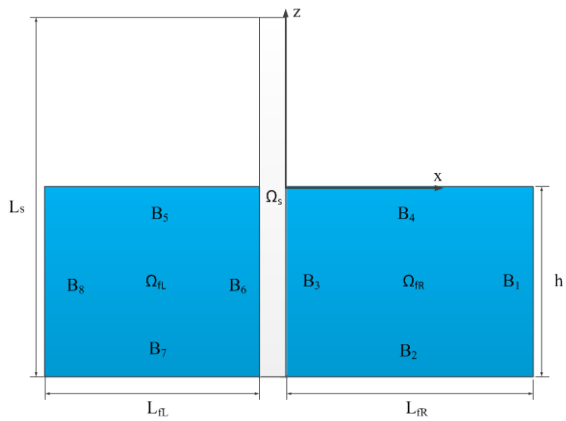


Fig.1 A sketch of structure coupling with two sides' fluid

2.1 liquid equations

The present formulations consider an inviscid compressible irrotational liquid. Two sides' liquid is satisfied with equation (1), where P is the liquid excess pressure. At the four boundaries, the pressure gradient along $\boldsymbol{n}$ is zero, where $\boldsymbol{n}$ stands for outward unit vector normal to the liquid boundary, as in equation (2). The outward unit vector connection between structure and two sides' liquid is given as (3). The structure vector direction is same with one side liquid, while is opposite with another side liquid. The relationship of liquid pressure and the acceleration of structure is given in (4) equation, where liquid density and beam displacement are represented by ρ_f and $\boldsymbol{u}_s$.For boundary B_4 and B_5, the linearized free surface wave is written in pressure form, where $\boldsymbol{g}$ denotes acceleration of gravity.

$\nabla^2 P = 0$ (in Ω_{fL} and Ω_{fR} domains), (1)

$\frac{\partial P}{\partial \boldsymbol{n}} = 0$ (at B_1, B_2, B_7, B_8 boundaries), (2)

$\boldsymbol{n} = \boldsymbol{n}_{fL} = \boldsymbol{n}_{fR}$ （at interface of B_3 and B_6） (3)

$\frac{\partial P_L}{\partial \boldsymbol{n}_{fL}} = \frac{\partial P_L}{\partial \boldsymbol{n}_{fL}} = -\rho_f \ddot{\boldsymbol{u}}_s$ (at interface of B_3 and B_6),(4)

$\frac{\partial^2 P}{\partial^2 t} = -\boldsymbol{g}\frac{\partial P}{\partial \boldsymbol{n}}$ (at B_4 and B_5 boundaries), (5)

Appling the functional variational principle on equation (1) and using Green-Gauss theorem on two sides' liquid leads to equations (6) (7).

$$\delta\Pi_{\Omega_{fR}} = \int_{B_{1+}B_2} \delta P \frac{\partial P_R}{\partial \boldsymbol{n}} ds + \int_{B_3} \delta P \frac{\partial P_R}{\partial \boldsymbol{n}} ds + \int_{B_4} \delta P \frac{\partial P_R}{\partial \boldsymbol{n}} ds - \int_{\Omega_{fR}} \nabla(\delta P)\nabla P_R \, dv \, , \quad (6)$$

$$\delta\Pi_{\Omega_{fL}} = \int_{B_7+B_8} \delta P \frac{\partial P_L}{\partial \boldsymbol{n}} ds + \int_{B_6} \delta P \frac{\partial P_L}{\partial \boldsymbol{n}} ds + \int_{B_5} \delta P \frac{\partial P_L}{\partial \boldsymbol{n}} ds - \int_{\Omega_{fL}} \nabla(\delta P)\nabla P_L \, dv \, , \quad (7)$$

And we set:

$P = \boldsymbol{N_f P^n}$ (8)

$u_s = \boldsymbol{N_s u_s^n}$ (9)

Where the shape functions for the structure and fluid domains are represented by $\boldsymbol{N_s}$ and $\boldsymbol{N_f}$, respectively. Time dependent nodal pressure is denoted by $\boldsymbol{P^n}$, and $\boldsymbol{u_s^n}$ is the nodal displacement.

Inserting equations (2) (3) (4) (8) (9) into (5) (6) leads to:

$$\delta\Pi_{\Omega_{fR}} = \int_{B_3} \delta P(-\rho_f \ddot{\boldsymbol{u}}_{s)} \, ds + \int_{B_4} \delta P(-\frac{1}{\boldsymbol{g}}\frac{\partial^2 P_R}{\partial^2 t}) \, ds - \int_{\Omega_{fR}} \nabla(\delta P)\nabla P_R \, dv, \quad (10)$$

$$\delta\Pi_{\Omega_{fL}} = \int_{B_6} \delta P(-\rho_f \ddot{\boldsymbol{u}}_{s)} \, ds + \int_{B_5} \delta P(-\frac{1}{\boldsymbol{g}}\frac{\partial^2 P_L}{\partial^2 t}) \, ds - \int_{\Omega_{fL}} \nabla(\delta P)\nabla P_L \, dv \, , \quad (11)$$

Substituting (8) (9) into (10)(11) gives the two sides' liquid vibration equations:

$[-\rho_f \boldsymbol{H}_L^T]\{\ddot{\boldsymbol{u}}_s^n\} + [\boldsymbol{M}_{fL}]\{\ddot{\boldsymbol{P}}_L^n\} + [\boldsymbol{K}_{fL}]\{\boldsymbol{P}_L^n\} = 0$ (12)

$[\rho_f \boldsymbol{H}_R^T]\{\ddot{\boldsymbol{u}}_s^n\} + [\boldsymbol{M}_{fR}]\{\ddot{\boldsymbol{P}}_R^n\} + [\boldsymbol{K}_{fR}]\{\boldsymbol{P}_R^n\} = 0$ (13)

Where $\boldsymbol{M_f}$ and $\boldsymbol{K_f}$ contain liquid mass and rigid matrix, $\boldsymbol{H}$ is liquid structure coupling matrix. The subscript denotes right or left liquid. The element matrices expressions are given in equation (14).

$\boldsymbol{M_f} = \frac{1}{\boldsymbol{g}}\int_{B_4 or B_2} \boldsymbol{N}_f^T \boldsymbol{N}_f ds$

$\boldsymbol{K_f} = \int_{\Omega_{fR} or \Omega_{fL}} (\nabla \boldsymbol{N}_f)^{\boldsymbol{T}} \nabla \boldsymbol{N}_f dv$ (14)

$\boldsymbol{H} = \int_{B_3 \, or \, B_6} \boldsymbol{N}_s^T \boldsymbol{N}_f ds$

2.2 structure equations

For the structure, the equation of motion can be written as (15).

$\boldsymbol{\nabla\sigma} + \boldsymbol{B_o} - \rho_s \ddot{\boldsymbol{u}}_s = \boldsymbol{0}$, (15)

$\boldsymbol{\sigma}$ denotes structure stresses, and $\boldsymbol{B_o}$ represents the body force, and ρ_s is the density of the structure. Appling Galerkin method of weighted residuals on equation (15) gives (16).

$\delta\Pi_s = \int_{\Omega_s} (\nabla\boldsymbol{\sigma} + \boldsymbol{B_o} - \rho_s \ddot{\boldsymbol{u}}_s) \, \delta\boldsymbol{u}_s dv$, (16)

Using partial integration on (16)'s left gives：

$$\int_{\Omega_s} \nabla\boldsymbol{\sigma} \, \delta\boldsymbol{u}_s dv = \int_{(\partial\Omega_{sd} + \partial\Omega_{sw})} \boldsymbol{\sigma}_{ij} \boldsymbol{n}_j \boldsymbol{\delta u}_j ds - \int_{\Omega_s} \boldsymbol{\sigma}_{ij} \, \delta\varepsilon_{ij} \, dv \, , \quad (17)$$

$\partial\Omega_{sd}$ and $\partial\Omega_{sw}$ represent structure dry and wet surfaces. The right fist term of equation (17) can be rewritten into structure dry surface stress and wet surface liquid load, which leads to equation (18).

$$\int_{(\partial\Omega_{sd} + \partial\Omega_{sw})} \boldsymbol{\sigma}_{ij} \boldsymbol{n}_j \boldsymbol{\delta u}_j ds = \int_{\partial\Omega_{sd}} f_s \delta\boldsymbol{u}_s ds + \int_{\partial\Omega_{sw}} (P_L - P_R)\boldsymbol{n}\delta\boldsymbol{u}_s ds \quad (18)$$

Substituting (18) into (16) gives：

$$\delta\Pi_s = \int_{\partial\Omega_{sd}} f_s \delta\boldsymbol{u}_s ds + \int_{\Omega_s} \boldsymbol{B_o} \, \delta\boldsymbol{u}_s dv + \int_{\partial\Omega_{sw}} (P_L - P_R) n\delta\boldsymbol{u}_s ds - \int_{\Omega_s} \rho_s \ddot{\boldsymbol{u}}_s \, \delta\boldsymbol{u}_s dv - \int_{\Omega_s} \boldsymbol{\sigma}_{ij} \, \delta\varepsilon_{ij} \, dv \, , \quad (19)$$

Inserting (8) (9) into (19) gives discretized structure vibration equation (20), where $\boldsymbol{F}^n$ is the nodal external load.

$$[\boldsymbol{M_s}]\{\ddot{\boldsymbol{u}}_s^n\} + [\boldsymbol{K}_s]\{\boldsymbol{u}_s^n\} - [\boldsymbol{H}_R]\{\boldsymbol{P}_R^n\} + [\boldsymbol{H}_L]\{\boldsymbol{P}_L^n\} = \boldsymbol{F}^n \quad (20)$$

Mass matrix $\boldsymbol{M_s}$ and rigid matrix $\boldsymbol{K_s}$ are given in equation (21), and $\boldsymbol{B}$ is displacement-strain matrix. $\boldsymbol{D}$ is the constitutive matrix.

$\boldsymbol{M_s} = \int_{\Omega_s} \boldsymbol{N}_s^T \boldsymbol{N}_s ds$

$\boldsymbol{K_s} = \int_{\Omega_s} \mathrm{B}^T \boldsymbol{DB} dv$ (21)

Coupling (12) (13) (20) gives the liquid structure coupling vibration equation (22).

$$\begin{bmatrix} \boldsymbol{M_s} & 0 & 0 \\ \rho_f \boldsymbol{H}_R^T & \boldsymbol{M}_{fR} & 0 \\ \rho_f \boldsymbol{H}_L^T & 0 & \boldsymbol{M}_{fL} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}}_s^n \\ \ddot{\boldsymbol{P}}_R^n \\ \ddot{\boldsymbol{P}}_L^n \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_s & -\boldsymbol{H}_R & \boldsymbol{H}_L \\ 0 & \boldsymbol{K}_{fR} & 0 \\ 0 & 0 & \boldsymbol{K}_{fL} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_s^n \\ \boldsymbol{P}_R^n \\ \boldsymbol{P}_L^n \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}^n \\ 0 \\ 0 \end{bmatrix}$$
(22)

3 The coupling process

According to coupling matrix relationship (14), the two nodes and six degrees of freedom beam element and four degrees of liquid element are used here. The coupling element matrix is 6×2, only two nodes of liquid element involves coupling, which contact with structure element, as in figure.2.

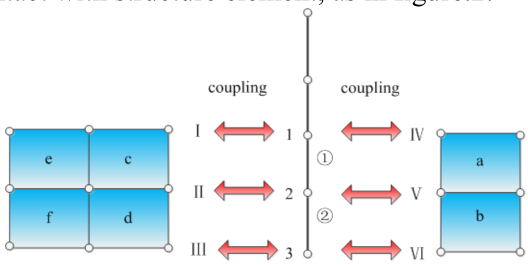


Fig.2 A sketch of liquid and structure elements interaction

After forming the coupling matrix $\boldsymbol{H}$, we need to add the coupling matrix to the global matrix, take the coupling beam elements (①、②) and liquid elements (c, d) for example. The assemble method is illustrated, where beam element ②'s degrees of freedom are set to be (4 5 6 7 8 9), and liquid's degrees are set to be (Ⅱ Ⅲ) , the coupling element matrix are (1 2 3 4 5 6 ;7 8 9 10 11 12), the coupling matrix is 9×6 . The global matrix row index is from beam element ②'s degrees of freedom, and the column index is from liquid element (d)'s degrees of freedom, the result is shown in figure 3. The beam element ① and liquid element (a) are assembled to global matrix by the same method. For the beam is coupling with two sides' liquid, two sides liquid element need to be meshed separately. At the interface, according to equation (22), two kinds of elements have same coordinates, but exist with different nodes. The coupling element matrices are need to be formed respectively for beam and liquid, which finally are added to global coupling matrix.

Fluid element

Structure element		II	III			
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
4	0	1	7	0	0	0
5	0	2	8	0	0	0
6	0	3	9	0	0	0
7	0	4	10	0	0	0
8	0	5	11	0	0	0
9	0	6	12	0	0	0

Fig.3 A sketch of the assembling coupled matrix of structure and fluid

4 Numerical experiments and results

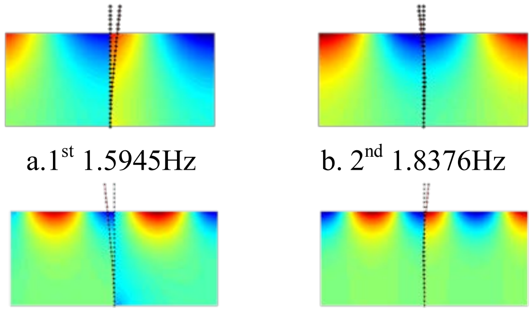
The numerical results are verified by the paper [9]. The experiment equipment is a rectangular container partially filled with water, and a cantilever baffle is clamped to the bottom of the rigid container, the top of the baffle is free, where an electromagnet is placed above the baffle. The baffle's first dry bending frequency is close to the first sloshing modes of the fluid, to enforce the coupling effect of liquid sloshing and baffle vibration.

The height of baffle is 0.266m, baffle thickness is 0.00057m, width is 0.099m, density is 7800kg/m^3, Young modulus is 2.1 × 10^{11}Pa. The width two sides' liquid is 0.210m, height is 0.231 m. The liquid is set to be water, whose density is 1000kg/m^3. The force on the baffle is vertical to the plate, and the coupling motion also occurs in plane, so the problem can be considered as two-dimensional. The baffle and liquid elements are used in calculations, the baffle is meshed by 27 elements and every node has three degrees of freedom. Two sides' liquid is meshed by 483 elements, every node has one DOF. Two kinds of elements couple at the interface.

Table.1 Comparisons of the proposed results with experiment

Mode	Paper [9] results(Hz)	Calculation results(Hz)	Error
1	1.56	1.5945	2.2%
2		1.8376	
3	2.39	2.4323	1.8%
4		2.6134	
5	2.95	2.9931	1.5%

The results are shown in table 1. For the exciting way in the experiment, the antisymmetric coupling vibration can be test. The first three orders are corresponding to the one, three and five orders of calculations. The maximum error of calculations is 2.2%, which valid this FEM model.



a. 1st 1.5945Hz b. 2nd 1.8376Hz

c. 3rd 2.4323Hz d. 4th 2.6134Hz

Fig.4 Structure displacement and liquid contour map

Structure displacement and liquid pressure contour map is shown in figure 4. The existence of plate decreases the coupling vibration frequencies, where one and three orders are corresponding to the test, they are antisymmetric modes.

5 Conclusion

The coupled vibration of elastics baffle with two sides' liquid, in a rigid tank, is carried out by FEM, and the equation of coupled vibration are derived, then the form of coupling matrices of liquid and structure is given. The numerical results obtained by the proposed method have good agreement with the test one, which valid the model. Coupling vibration shape is different from the liquid sloshing. This method can be used in baffled tanks with different two sides' liquid height.

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References:

[1] K.C. Biswal, S.K. Bhattacharyya, P.K. Sinha. Dynamic response analysis of a liquid-filled cylindrical tank with annular baffle [J]. *Journal of sound and vibration*, 2004, 274(1): 13-37.

[2] R. D. Firouz-Abadi, H. Haddadpour, M. A. Noorian, et al. A 3D BEM model for liquid sloshing in baffled tanks [J]. *International journal for numerical methods in engineering*, 2008, 76(9): 1419-1433.

[3] M. A. Noorian, R D. Firouz-Abadi, H. Haddadpour. A reduced order model for liquid sloshing in tanks with flexible baffles using boundary element method[J]. *International Journal for Numerical Methods in Engineering*, 2012, 89:1652-1664.

[4] Wang Jia-dong;Zhou Ding;Liu Wei-qing. Analytical Study of Small Amplitude Sloshing of Liquid in a Cylindrical Tank with an Annual Baffle. *Journal of Vibration and shock* , 2010, 29(2): 54-59.

[5] M. A. Goudarzi, S. R. Sabbagh-Yazdi,W. Marx. Investigation of sloshing damping in baffled rectangular tanks subjected to the dynamic excitation [J]. *Bulletin of Earthquake Engineering*, 2010, 8(4): 1055-1072.

[6] Guan Yanmin ,Ye Hengkui ,Chen Qi ng ren ,Feng Dakui. Numerical simulating liquid sloshing in 3D tanks with baffles based boundary element method[J]. *Journal of Huazhong University of Science and Technology (Natural Science Edition)*, 2010 (4): 102-104.

[7] K.C. Biswal, S.K. Bhattacharyya, P.K. Sinha. Non-linear sloshing in partially liquid filled containers with baffles [J]. *International journal for numerical methods in engineering*, 2006, 68 (3): 317-337.

[8] Guan Yanmin ,Ye Hengkui ,Chen Qi ng ren ,Feng Dakui.Comparison of ALE Finite Element Method and Boundary Element Method for Simulation of Liquid Sloshing in 2-D Baffled Tank[J]. *Journal of Ship Mechanics*,2010,14(9),958-966.

[9] X.-J. Chai. Influence de la gravité sur les interactions fluide-structure pour un fluide dans undomaine borné à surface libre [D]. PhD thesis, Institut National Polytechnique de Lorraine, 1996.