Fuzzy Analysis of Serviceability Limit State of Slender Beams

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Abstract: - The maximum permissible deflection values in design standards are determined subjectively and can thus be considered as fuzzy numbers. Using the finite element method, it has been found that the lateral deflection of a slender beam under major axis bending can be relatively high. Nonlinear load-deflection analysis was applied for the compilation of the presented numerical studies. The serviceability limit state of beams with initial imperfections was studied. The limit moment, which yields the deflection that is equal to the maximum permissible value, was defined. Fuzzy analysis of the limit moment was evaluated using the general extension principle. Slendernesses of members for which the attainment of the maximum permissible value of deflection is the deciding limit state for design were determined.

Key-Words: - Fuzzy logic, resistance, imperfections, stability, serviceability, limit state, beam, bending

1 Introduction

Building structures should be designed for strength and serviceability during their service life, namely without any significant functionality loss or necessity of excessive, and/or unforeseen maintenance.

The basis of the design of steel structures is the safeguarding of their safety and reliability. The steel structures usually are slender, and require paying increased attention to stability problems. The analysis of ultimate limit states of slender steel members is based on calculation models in closed form [1, 2] or on finite element method [3, 4]. The influence of material and geometrical characteristics on the model output can be assessed by means of the advanced sensitivity analysis method, see, e.g., [1, 5]. Advanced approaches analyse the reliability by applying numerical simulation methods of the Monte Carlo type [6].

The safeguarding of serviceability represents the second equally important part of the design. The evaluation of the serviceability limit state is important in particular in recent time when advanced nonlinear calculation methods enable to analyse the deformations of light steel structures [7]. Serviceability limit state design of structures includes factors such as deflection, cracking and excessive vibration, durability, overall stability, and fire resistance. Basic design criteria of applicability of statically stressed structures are connected with larger deflections, above all the occurrence of which in current service should not exceed the maximum permissible value of deflection (limits) specified by design standards [8, 9].

The limit values of deflection are determined in a very subjective manner because they are connected with appearance of structure, contentment of users and serviceability but also with damage of surfaces or non-supporting members. The limit values of deflections are thus of subjective character, and they can be perceived as fuzzy numbers rather than as random numbers or fields.

Typical exemplars are buckling and lateral-torsional buckling of slender beams with initial imperfections. The design of slender beam according to ultimate limit states [8, 9] is based on buckling curves. In design standards, the assessment according to serviceability limit state has not the background as strong as ultimate limit state.

The maximum permissible values of deflection associated with buckling are not specified at all by design standards for stability problems of buckling and lateral torsional buckling. Experimental research [10] have shown that the lateral deflection of slender imperfect beams under major axis bending may, in some cases, reach relatively high values. This problem will be studied, in the present article, using the software Ansys [11] and the nonlinear load-deflection analysis of the FEM [12].

The paper is divided into five sections including the introduction and conclusion. In the second section the model, loading, boundary conditions and initial geometric imperfections are described. The third section discusses the calculation of limit deflection. In the fourth section the fuzzy analysis is presented.

2 Problem Formulation

The computational model is presented by a simply supported beam of a hot-rolled profile IPE220 and steel grade S235. The non-dimensional slenderness in lateral-torsional buckling $\bar{\lambda}_{LT}$ [8] was chosen within the interval $\bar{\lambda}_{LT} \in \langle 0.2, 4 \rangle$. Since the beam length L is a function of this value, it was changed depending on the non-dimensional slenderness. The beam is loaded on both ends by equal bending moments M of the same magnitude and of opposite direction. This represents the case of pure bending. The load is applied by means of compression (tension) on the web p_1 and flange p_2 to determinate the sizes of loading effects p_1 and p_2. Equations (1) to (4) describe how the sizes of loading effects p_1 and p_2 are calculated. Normal stress σ_x has a linear distribution along the cross section due to bending; see Fig.1. The highest absolute value $\sigma_{x,\max}$ is reached at the outer edges.

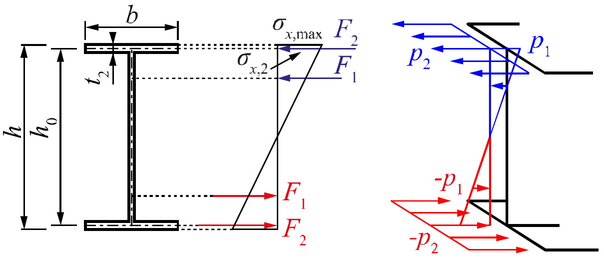

Fig.1: Stress distribution and application of compression and tension at the end sections of the beam

$$\sigma_x = \frac{M}{I_y} z , \quad \sigma_{x,\max} = \frac{M}{I_y} \frac{h}{2} \tag{1}$$

The resultant moment M is given as the sum of moments M_1 and M_2 from the force couples F_1 and F_2.

$$M = M_1 + M_2 = \frac{2}{3} F_1 h_0 + F_2 h_0 = \frac{2}{3} F_1 (h - t_2) + \\ + F_2 (h - t_2) \tag{2}$$

The force F_2, the point of action of which is on the central line of the flange, is given by multiplying the respective stress and flange area A_2.

$$F_2 = \sigma_{x,2} A_2 = \sigma_{x,\max} \left(1 - \frac{t_2}{h}\right) b t_2 = \\ = \frac{1}{2} \frac{M (h - t_2) b t_2}{I_y} \tag{3}$$

Substituting Eq. (3) into Eq. (2), the force F_1 and pressures p_1 and p_2 [Nm^{-1}] can then be subsequently determined as

$$F_1 = \frac{3M}{2(h-t_2)} \left[1 - \frac{1}{2} \frac{(h-t_2)^2 b t_2}{I_y} \right], \\ p_1 = \frac{6M}{(h-t_2)^2} \left[1 - \frac{1}{2} \frac{(h-t_2)^2 b t_2}{I_y} \right], \\ p_2 = \frac{M}{2} \frac{(h-t_2) t_2}{I_y} . \tag{4}$$

Since the beam is hinged at both ends, the boundary conditions are considered according to [2, 13] as follows

$$(y)_0 = (y)_L = 0, \\ (y)_0 = (y)_L = 0, \\ (\varphi)_0 = (\varphi)_L = 0 \\ \left(\frac{\partial^2 y}{\partial x^2}\right)_0 = \left(\frac{\partial^2 y}{\partial x^2}\right)_L = 0, \\ \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_0 = \left(\frac{\partial^2 \varphi}{\partial x^2}\right)_L = 0 . \tag{5}$$

The beam was modelled in finite element software Ansys [11]. The model was created using shell element SHELL181 which is a 4-node structural element suitable for analyzing thin to moderately-thick shell structures. The pressure loads were applied using element SURF156. The geometric nonlinear solution was applied using the incremental Newton-Raphson iterative method. The linear elastic stress-strain relationship was used for steel. Residual stress was not considered.

2.1 Initial Imperfections

In calculation models, the influence of imperfections on the behavior of structures is considered in various ways, see, e.g., [14-15]. In general, imperfections can be considered as deterministic or random quantities [16]. In this work, the initial imperfections are considered as deterministic quantities.

The initial out-of-plain imperfection of the beam axis (in the *xy* plane, see Fig.2), is described by function

$$y = e_0 \sin\left(\frac{\pi x}{L}\right), \tag{6}$$

where e_0 is the value of amplitude of initial curvature of the beam axis and given as $e_0 = L/1000$ [1]. In the case of an ideal straight beam, deformation in the *xy* plane does not occur as long as the bending moment *M* does not reach the critical value M_{cr}. Then the beam starts to deflect even in this plane and to rotate. When the axis of the beam has an initial curvature, it starts to deflect in the *xy* plane and rotate immediately after the application of loading *M*. An example of a finite element model created in Ansys software is depicted in Fig.3.

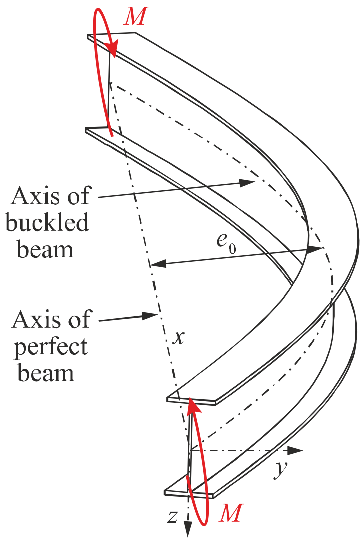

Fig.2: Initial imperfection

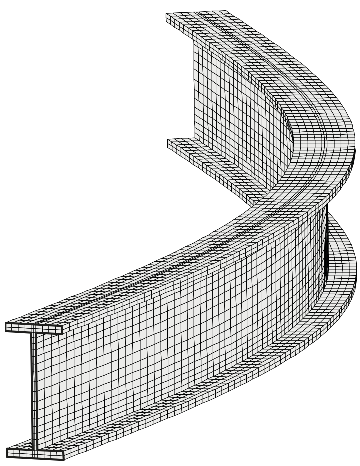

Fig.3: Finite element model from Ansys

3 Beam deflections

During loading the maximal deflection occurs in the middle of the span length $x = L/2$. The deflection value (or the vector deflection) *u* is calculated as the maximum of lengths |AA‘|, |BB‘|, |CC‘|, |DD‘|, see Fig.4. It is a function of the length *L* and the dimensions of cross section *b*, *h* and it is sought during calculation.

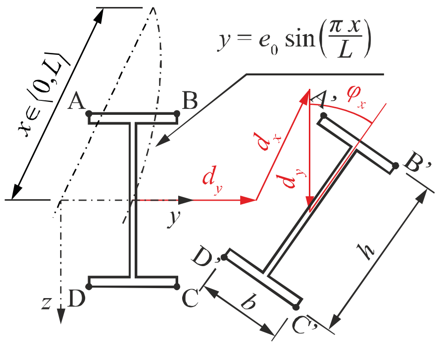

Fig.4: Deflections of the cross section

The deformation of any cross section in $x \in \langle 0, L \rangle$ can be described by the components of displacement on each axes u_x, u_y a u_z, and by rotation of cross sections φ_x, φ_y a φ_z around these axes. For the cross section in the middle of span length the rotations φ_y a φ_z are negligible

$$\left(\varphi_y\right)_{x\in\langle 0,L\rangle} \approx 0,\ \left(\varphi_z\right)_{x\in\langle 0,L\rangle} \approx 0. \tag{7}$$

But the deflection in the x-direction u_x is not negligible because one end of the beam is free to deflect in the direction of axis *x*. Therefore, the cross sections are shifted on this axis as well. For the vector deflection *u* it can be written

$$u = \sqrt{\begin{array}{l} u_x^2 + \left[u_y + u_{\phi_{x,y}} + e_0 \sin\left(\frac{\pi x}{L}\right)\right]^2 + \\ + \left(u_z + u_{\phi_{x,z}}\right)^2 \end{array}}. \tag{8}$$

Deflections $u_{\varphi_{x,y}}$ and $u_{\varphi_{x,z}}$ are calculated as

$$\begin{array}{l} u_{\varphi_{x,y}} = -\mathrm{sgn}(z)\sqrt{b^2 + h^2}\sin\left(\frac{\varphi_x}{2}\right) \\ \cos\left(\frac{\varphi_x}{2} - \mathrm{sgn}(y)\,\mathrm{sgn}(z)\arctan\left(\frac{b}{h}\right)\right) \\ \text{for } \forall\, y : y \in \left\{-\frac{b}{2}, \frac{b}{2}\right\} \text{ and } \forall\, z : z \in \left\{-\frac{h}{2}, \frac{h}{2}\right\}, \end{array} \tag{9}$$

$$u_{\varphi_{x,z}} = -\operatorname{sgn}(z)\sqrt{b^2+h^2}\sin\left(\frac{\varphi_x}{2}\right)$$
$$\sin\left(\frac{\varphi_x}{2} - \operatorname{sgn}(y)\operatorname{sgn}(z)\arctan\left(\frac{b}{h}\right)\right) \tag{10}$$
$$for\ \forall\ y : y \in \left\{-\frac{b}{2}, \frac{b}{2}\right\}\ and\ \forall\ z : z \in \left\{-\frac{h}{2}, \frac{h}{2}\right\}.$$

The non-dimensional slenderness $\bar{\lambda}_{LT}$ of the beams was considered within the interval $\bar{\lambda}_{LT} \in \langle 0.2, 4 \rangle$. Due to the relation between slenderness and length, the lengths are within the interval $L \in \langle 0.5, 42 \rangle$. Together, 416 beams with the length increase of 0.5 m were created. For each value L, there is a corresponding value $\bar{\lambda}_{LT}$, see Eurocode 3.

4 Fuzzy Analysis

The application of fuzzy sets in the construction industry/civil engineering is undergoing significant development, particularly in areas involving multiple criteria decision making [17]. The application of fuzzy analysis to the verification of the reliability of the design of structures is most frequently focused on the serviceability [18] or the safety of structures [19].

As the moment M increases from zero to M_{max}, the deflection u increases from the initial value of $e_0 = L/1000$ to u_{max}. The limit moment M_{lim} will occur when maximal deflection u_{max} is equal to the maximum permissible value of deflection u_{lim}. As u_{lim} is a fuzzy number, the limit moment M_{lim} is also a fuzzy number. u_{lim} has a membership function $\mu_{u_{lim}}(u) = \Lambda\,(u, L/450, L/250, L/173)$, see Fig.5.

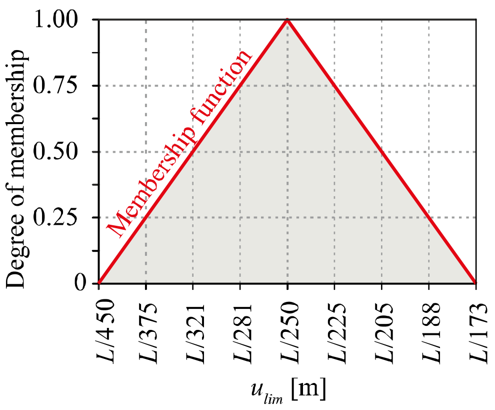

Fig.5: Membership function of u_{lim}

The relationship (11) is based on the general extension principle and it was applied for the fuzzy analysis of membership function of fuzzy number M_{lim} [20, 21].

$$\mu_{M_{lim}}(M) = \sup_{f(u)=M} \mu_{u_{lim}}(u) \tag{11}$$

The fuzzy analysis of two selected slendernesses is presented in Fig.6 and Fig.7. The support of M_{lim} is bounded by deflection values $L/450$ and $L/173$. These are considered to be the maximum vector deflections. The value of the centre of gravity (COG) of the area below the membership function is the defuzzified value.

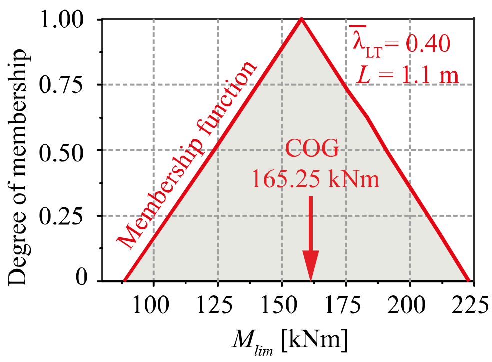

Fig.6: Fuzzy analysis M_{lim} of slenderness 0.4

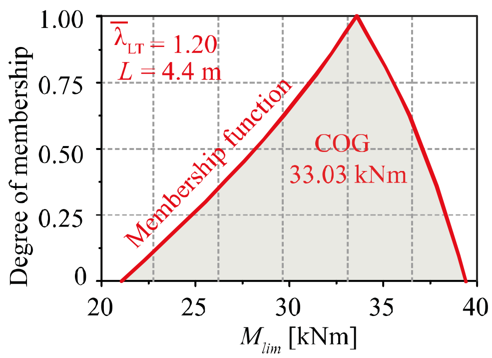

Fig.7: Fuzzy analysis M_{lim} of slenderness 1.2

The fuzzy analysis M_{lim} for other non-dimensional slendernesses is presented in Fig. 8.

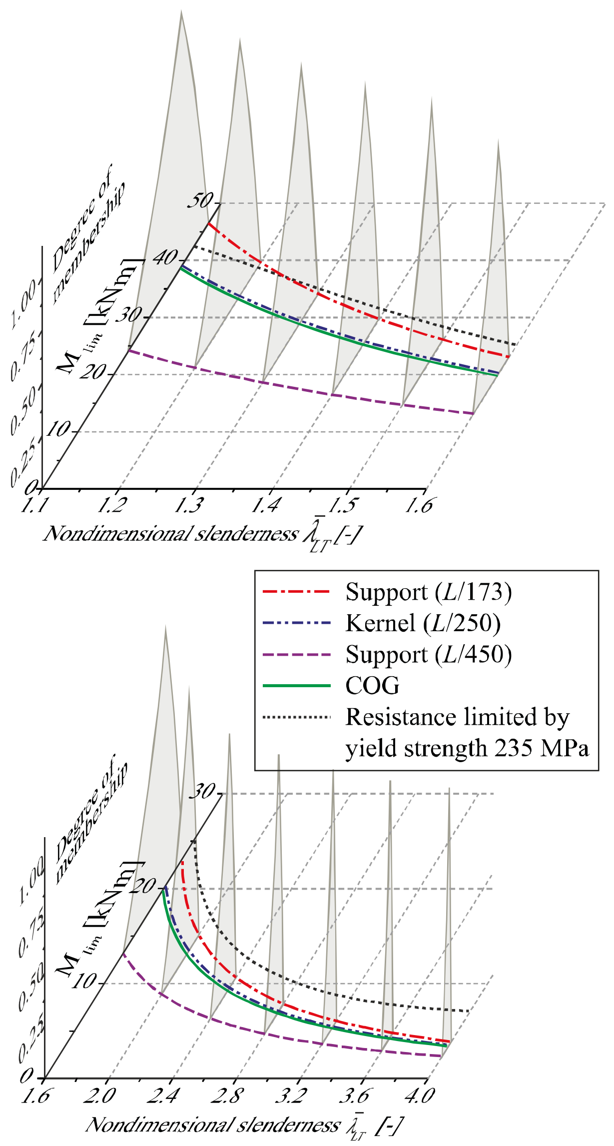

Fig.8: Fuzzy analysis M_{lim} vs slendernesses

The elastic resistance R_{fy}, see, e.g., [2], is plotted in Fig.8 as a black dash line. Its values correspond to the moment at which the yield strength is reached in the most stressed point.

5 Conclusion

The article was focused on a fuzzy analysis of the maximum moment M_{lim} at which the maximum permissible deflection u_{lim} has been reached. A nonlinear dependence between the deformation and the loading moment was discovered. u_{lim} is a subjectively chosen value. Thus it can be considered as a fuzzy number. Although u_{lim} was considered as a linear symmetrical fuzzy number Λ, M_{lim} is not linear and symmetric anymore, see Fig.7 and Fig.8.

The defuzzified values (COG) are greater than the kernel for $\bar{\lambda}_{LT} \leq 0.865$ ($L \leq$ 2.7m). The biggest difference between COG and the kernel is reached for the lowest values of slenderness. From the slenderness $\bar{\lambda}_{LT} > 0.865$ the COG values are lower than the kernel, and the difference becomes more significant with increasing value of $\bar{\lambda}_{LT}$.

It has been found out from the numerical studies that the lateral deflections (y axis) are substantially smaller than those in the vertical plane (z axis) for lower slenderness $\bar{\lambda}_{LT} \leq 0.6$. With increasing value of slenderness, the lateral deformations increase as well. Lateral and vertical deformations are approximately the same for $\bar{\lambda}_{LT} > 2.0$.

The yield strength is a random variable that is evaluated experimentally [22]. The elastic resistance R_{fy} is the physical limit of the moment M above which there is an objective reduction in safety due to initialization of permanent deformations. The curve of R_{fy} intersects the curve of the COG at the point $\bar{\lambda}_{LT} \approx 1.00$ (L = 3.3 m). The difference between COG and R_{fy} decreases with increasing slenderness, and the difference is negligible for slenderness $\bar{\lambda}_{LT} > 0.976$.

It is not possible to make a general conclusion of comparison of COG and R_{fy} because R_{fy} was calculated from the characteristic value of yield strength f_y = 235 MPa, and the initial curvature of the beam e_0 was considered as a non-random value L/1000. However, R_{fy} can be more accurately considered as a fuzzy random variable because the yield strength f_y and initial curvature of the beam e_0 are real random variables. The article provides topics intended for elaboration in further numerical studies of reliability analysis focused on the design of slender steel beam structures using stochastic methods and fuzzy logic. It also contributes to the discussions on the appropriateness of using fuzzy or stochastic approaches depending on the type of task. Studying serviceability limit state using stochastic methods cannot be considered sufficient, because the limit deformation in design standards are determined subjectively by humans. Fuzzy analysis is suitable in these cases.

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