Numerical Simulation of Reinforced Concrete Beam with Utilization of Elasto-plastic Material Model of Concrete

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Abstract: - Design of concrete structures in accordance with standards may suffer from less effectiveness of a final design. More realistic simulation of behaviour of structure can be achieved by incorporation of nonlinear material model within numerical simulation of given problem. However, this approach is connected with several difficulties. First of all, the utilization of nonlinear material model is confronted with problem of existence of material parameters that are not often known in advance. Second, the introducing reinforcement into finite element model is complicated by the choice of the correct element type and the correct bond between elements of both meshes. First of the above mentioned problems can be handled with by applying an inverse analysis based on utilization of optimization techniques. Such process of the inverse identification is based on the comparison of the experimental and numerical load displacement curves and minimization of the difference between them. The problem of the modelling of reinforcement can be resolved in more possible ways. The article shows the solution with simple truss elements that are modelled in according to finite element mesh of the concrete beam.

Key-Words: - ANSYS, reinforced concrete beam, optimization, identification, elasto-plastic material model

1 Introduction

Design of each structure is based on fulfilment of terms of the actual standards. The present European standards are based on methodology of the limit states and partial safety factors whose consistent application guarantees the safety, usability and durability of structure. The design of concrete structures is also based on assumptions of linear theory of plasticity. These assumptions may cause the loss of effectivity of the final design. It can be said that, thanks to advances in computer technology and software tools, and to the current level of development in the field of nonlinear material models, the application of such material models leads to more effective structural design [1].

The several evolutionary branches can be identified in the current field of nonlinear material models of concrete. One of this branch is based on assumptions of linear and nonlinear fracture mechanics [2]. The second evolutionary branch are based on utilization of pure plasticity theory. An overview of historical development of such material models is described by Cicekli [3] and also in work of Grassl [4]. However, as Grassl and Jirasek states, independent plasticity models are not sufficient when the decrease of stiffness due to cracking is required. This problem have led to development of the third branch of material models that are based on assumptions of the damage theory. However, the results of the research in this area show that independent use of this theory is not optimal. The reason is in inability of this group of material models to describe irreversible deformations and the inelastic volumetric expansions [4]. Other branches in the field of describing the nonlinear behaviour of concrete are represented by discrete element method (DEM) [5] and by extended finite element method (XFEM) [6]. The above mentioned insufficiency of the pure plasticity models and damage models are often removed by their mutual combination in one material model. Nowadays, we are able to use models that combine the plasticity theory with the damage theory or models that combine the plasticity theory with aspects of nonlinear fracture mechanics. The last mentioned approach is represented by database of elasto-plastic material models called multiPlas [7]. This database was developed for support of the nonlinear simulations in ANSYS [8] computational system.

The use of combined material models results in a problem in real life in the form of the large amount of mechanico-physical and fracture mechanical parameters which need to be known for the selected material model before the launch of the numerical simulation. This problem can be resolved by performance of the simple fracture test, followed by inverse identification of these material parameters from measured load-displacement curves. Inverse identification of the material parameters can be performed by methods based on the use of neural networks [9] or by optimization techniques. Basic principle of use of the optimization for identification lies in minimization of the difference between the experimental and numerical *L-d* curve.

The presented article contains study of nonlinear numerical simulation of reinforced concrete beam in ANSYS computational system with use of Menétrey-Willam material model from multiPlas database. The setup of correct material parameters was performed on the basis of results of optimization in ANSYS Workbench. The article deals with complex description of the whole numerical analysis and contains therefore description of input data, description of fracture experiment, description of computational and material model and process of inverse identification.

2 Problem Formulation

The basic objective of the study was numerical analysis of nonlinear behaviour of the reinforced concrete, beam where the beam was modelled by solid finite elements with nonlinear material model based on Menétrey-Willam plasticity surface. The goal of the numerical analysis was to obtain *L-d* curve of the reinforced beam for future comparison with experimental measurements on the real specimen. The formulated task was complicated by large amount of material parameters that were not known in advance. Due to this complication, the solution of the task was divided into two stages. The inverse identification of mechanico-physical and fracture-mechanical parameters of the given constitutive law was performed in the first stage. The second stage contained the numerical simulation of the concrete element with previously identified parameters.

2.1 Fracture experiment

The inverse identification of parameters for the material model from the multiPlas library was performed on the *L-d* curve obtained from the results of a three-point bending test performed on a notched concrete beam, which were published by Zimmermann *et al.* [10]. The dimensions of the beam were 360 x 120 x 58 mm and depth of the notch was 40 mm. The tested sample was made from concrete C20/25 with slump value F45 and with age of 28 days. The geometry of the fracture experiment is schematically depicted in Fig. 1.

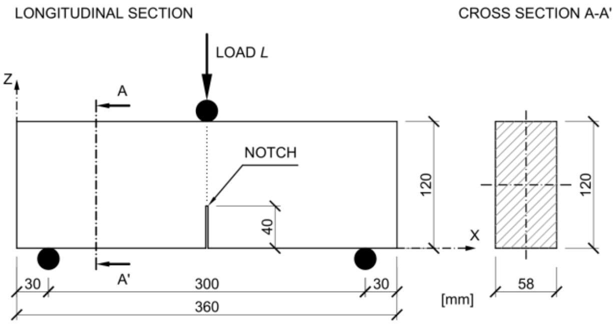

Fig. 1. Idealization of the testing configuration

2.2 Analyzed reinforced beam

The analysis of nonlinear behaviour of concrete beam was performed on reinforced concrete beam with dimensions 1190 x 140 x 100 mm. This type of beam was chosen with respect to the low production costs and because it is easy to get these specimens from window lintels. The properties of concrete for manufacturing of this beam was the same as in case of specimen for fracture experiment and therefore the data from inverse identification could be used for nonlinear analysis. The beam was reinforced with steel bars with diameter of 8 mm made from reinforcing steel B500B. The reinforced beam was also subjected to the three-point bending test whose configuration is depicted in Fig.2.

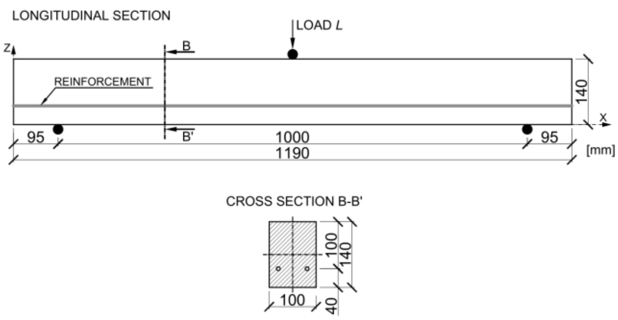

Fig. 2. Idealization of the tested reinforced beam

3 Inverse identification of material parameters

The inverse identification of parameters of material model from multiPlas library was performed with use of genetic algorithm so called MOGA in environment of ANSYS Workbench. Due to task automation requirements, the whole calculation was controlled by a programmed APDL macro which set up the geometry of the computational model and the parameters of the chosen constitutive law, input the solver settings, solved the tasks and finally exported the file containing the points for the *L-d* diagrams.

3.1 Geometry of computational model

The geometry of the computational model set up as part of the aforementioned macro was, with regard to the fact that the task was solved as 2D plane stress, formed by surfaces covered by PLANE182 planar four-node elements with three degrees of freedom at each of the nodes. In order to reduce the computation time requirement the geometry was covered by a regular mesh of 6.0 mm elements. The corresponding size of the computational model in the remaining third dimension was achieved at the finite element level by the prescription of an element width w of 58 mm.

The special modification of the computational model was also required by an area near the supports. With regard to the chosen boundary conditions this was an area where local stress peaks could potentially occur, and so the elements in a 60 mm-wide band above both supports was prescribed a linear-elastic constitutive relation between stress and deformation. This modification of the computational model was conducted outside the area of interest above the notch, and so should not have had a negative impact on the performed inverse analysis.

3.2 Menétrey-Willam material model

The used Menétrey-Willam material model ranks among the group of nonlinear material models incapable of expressing the influence of strain rate on the state of stress. Due to this characteristic it is thus suitable for the numerical simulation of the performance of a three-point bending test on a notched concrete beam. In this group of models, irreversible deformation occurs when predetermined criteria are fulfilled, during which it is expected that the whole strain vector $\boldsymbol{\varepsilon}_{tot}$ will decompose into an elastic $\boldsymbol{\varepsilon}_{el}$ and a plastic $\boldsymbol{\varepsilon}_{pl}$ part [11]. The criterion for the occurrence of plastic deformation is given by the prescription of a yield surface function. The Menétrey-Willam [12] material model selected from the multiPlas database is, as was previously mentioned, based on the Willam-Warnke [13] yield surface, which in contrast to the Drucker-Prager surface function is not only the first invariant of stress tensor and the second invariant of stress deviator but also the third invariant of stress deviator (the “lode angle”). This modification achieved the smoothing of the corners of deviatoric planes of the yield surface which, in addition, do not have a constant distance from the hydrostatic axis in Haigh-Westergaard space.

From the point of view of the use of FEM the chosen material model utilizes the smeared crack concept [14]. The given problem was solved with the aid of a softening function based on the dissipation of specific fracture energy G_{ft}, which thus acts as one of the sought parameters. With regard to the need to remove the negative dependence of the solution on the size of the network of finite elements, the nonlinear Menétrey-Willam model makes use of Bažant’s Crack Band concept. [15]. In order for the model to behave in the appropriate nonlinear manner it was necessary to predefine a total of 12 material parameters.

3.3 Inverse identification

The process of inverse identification of the given material model was based on repeated numerical solution of fracture experiment with parameters that had been generated via genetic optimization algorithm MOGA in ANSYS Workbench. Each numerical solution produced a set of points that formed resultant *L-d* curve. The comparison of the experimental and numerically obtained *L-d* curve was performed by calculation of RMSE error. The calculation of this error was executed using external application that had been programmed in MATLAB language. Use of this external utility was caused by a necessity of mapping the points of the numerical *L-d* curve with respect to positions of points on experimental *L-d* curve via linear interpolation.

3.3.1 Optimization

As follows from the above, the inverse identification was performed as optimization task, where the goal was defined as minimization of RMSE error. The prescription of this function was therefore a prescription of the objective function:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{1=1}^{n}\left(y_i^{*} - y_1\right)^2}{n}} \tag{1}$$

where the y_i^* was the value of the force at the i-th point of the curve and y_i denoted the value of the force calculated using the nonlinear material model at i-th point of the curve and n was the dimension of force vector. Beside the objective function, the inequality design conditions were also prescribed:

$$\arctan\frac{f_t}{\sqrt{2}f_c} < \psi < \arctan\frac{1}{\sqrt{2}} \quad (2)$$

$$\frac{f_c}{E} < \varepsilon_{ml} \quad (3)$$

Due to possible occurrence of an unconverged solution, the analysis required use of robust genetic optimization algorithm MOGA. The form of the design vector that entered the solution was as follows:

$$X = \{E, \nu, f_c, f_t, k, \varepsilon_{ml}, \psi, \Omega_{ci}, \Omega_{cr}, G_{fc}, \Omega_{tr}, G_{ft}\}^T \quad (4)$$

The optimization process was set to 40 generations of design vectors with 10 individuals in each of them. The resultant optimum with the lowest value of the RMSE was achieved in 346-th realization. The comparison of the *L-d* curve for identified parameters and experimental *L-d* curve is depicted in Fig. 3. It is clearly visible that curves match well and the identified parameters could be considered as sought mechanico-physical and fracture-mechanical parameters of the given concrete.

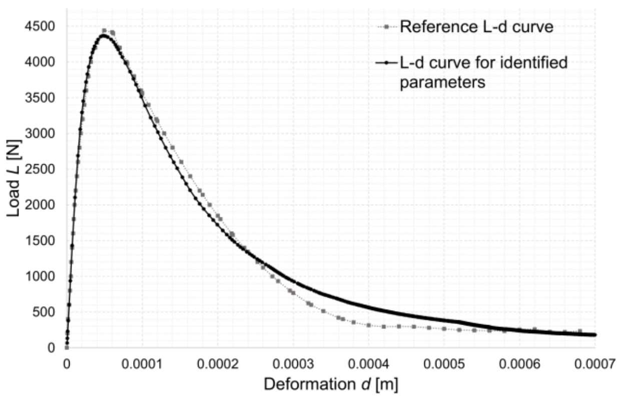

Fig. 3. Comparison of L-d curves

The values of the material parameters are shown in Table 1.

4 Nonlinear numerical simulation of reinforced concrete beam

The numerical simulation of reinforced beam, as in case of simulation of fracture experiment, was performed in classic environment of ANSYS 15.0. The nonlinear behaviour of the computational model was enabled by use of the Menétrey-Willam material model with previously identified parameters. Due to higher weight of the analyzed specimen, two variants of solutions were considered. The first solution was performed without consideration of the self-weight and the in the second solution, the acceleration with value of 9.81 ms^{-2} was added.

Table 1. Identified material parameters

Par.	Unit	Value
E	[Pa]	$41.843 \cdot 10^9$
N	[-]	0.216
f_c	[Pa]	$48.734 \cdot 10^6$
f_t	[Pa]	$2.323 \cdot 10^6$
K	[-]	1.245
Ψ	[°]	8.953
ε_{ml}	[-]	0.0019
G_{fc}	[Nm/m^2]	0.754
Ω_{ci}	[-]	0.138
Ω_{cr}	[-]	1044.837
G_{ft}	[Nm/m^2]	47.002
Ω_{tr}	[-]	$0.578 \cdot 10^{-2}$

4.2 Computational model of reinforced beam

The computational model of the reinforced concrete beam was more complex than previously mentioned model for simulation of fracture test. The geometry with dimensions, which were shown in Fig. 2, was covered with a spatial mesh of regular solid elements SOLID 185. These 8-nodes solid elements have three degrees of freedom at each of the nodes.

The reinforcement was idealized by truss elements LINK180. These elements are very simple and have only 2 nodes with three degrees of freedom at each of them. The use of these elements appeared to be more advantageous than utilization of more complex BEAM188 elements where the higher valence in nodes leads to necessity of rotational constraints. Due to nonlinear character of material model of concrete, the material model for steel reinforcement was also defined as elasto-plastic with constant hardening. The value of yield strength was set to 500 MPa.

The boundary conditions were defined with respect to the testing configuration that was shown in Fig. 3. In order to prevent the rigid body motion of the beam, the horizontal constraint was defined at the position of loading force. The vertical constraints were defined at position of supports.

4.2 Numerical solution

The numerical solution of both variants were, due to use of 3D solid finite elements, performed with PCG solver (pre-conditioned conjugate gradient). The resultant load-displacement curves are shown in Fig. 4.

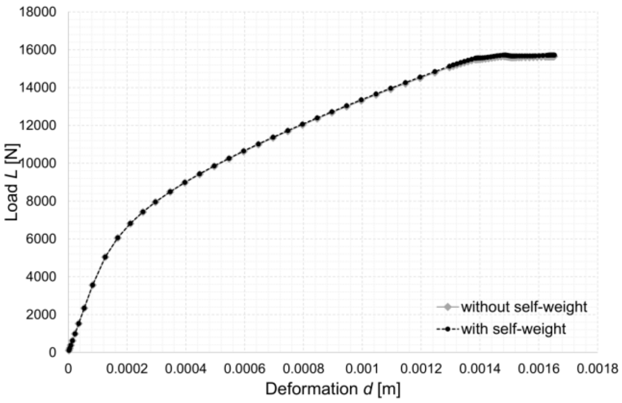

Fig. 4. Resultant L-d curves for reinforced beam

The Fig. 4 clearly shows a load-carrying capacity of the given reinforced concrete beam. The value of the load-carrying capacity respectively the maximum loading force was $L_{RC,max}$ = 15653.1 N in the variant without consideration of influence of self-weight. The maximum loading force in the second case was $L_{RC,max,g0}$ = 15721.0 N. The diagram in Fig. 4 also shows all 3 expected stages of nonlinear behaviour of loaded beam.

4 Conclusion

The main goal of the presented study was to find a load-carrying capacity of the given reinforced concrete beam with use of nonlinear material models. This goal was accomplished and it will be a part of our ongoing experimental research that will endeavour confirmation or refutation of acquired knowledge. The article also shows that the problem with unknown material parameters of the material models of concrete could be overcome with application of inverse analysis based on the optimization techniques. The shown process of the design of concrete structures with utilization of experimental research and advanced simulation techniques can be considered as bringing the design methods nearer to the real actions of structures.

Acknowledgement

This outcome has been achieved with the financial support of the project No. FAST-J-16-3562, “Implementation of Material Models of Concrete in ANSYS and their experimental verification” provided by the Brno University of Technology fund for specific university research.

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