Analytical Solution for MHD flow Due to a Permeable Stretching Surface Embedded in a Porous Medium

M.THIAGARAJAN[1,*], K. SENTHILKUMAR[2]
Department of Mathematics,
PSG College of Arts and Science,
Coimbatore-641014, INDIA.
[1]thiyagu2665@gmail.com, [2]ksenthilkumar1911@gmail.com
[*] Corresponding author: Mobile: +91-9842289873

Abstract: - This paper is devoted to introduce analytic solutions by Differential Transform Method-Pade approximants with theoretical study for the problem of the hydromagnetic flow due to a permeable stretching surface embedded in a porous medium in the presence of transverse magnetic field. The governing momentum equation which is a nonlinear partial differential equation is reduced into a nonlinear ordinary differential equation by using similarity transformation and then solved numerically by DTM-Pade. The accuracy of present method is tested by numerical shooting method and the results are found to be in an excellent agreement. Numerical results are displayed by means of graphs. The effect of porous parameter, magnetic parameter and suction/injection parameter on skin friction and velocity are thoroughly studied.

Keywords: - MHD, Porous medium, DTM, Pade approximant, Magnetic parameter, Skin friction.

1 Introduction

The hydrodynamic flow over a continuously stretching surface is an important problem in many engineering processes with applications in industries such as polymer extrusion, wire and fiber coating, melt-spinning, hot rolling, glass-fiber production, petroleum production, exotic lubricants and suspension solutions. Much work on the boundary layer Newtonian fluids has been carried out both experimentally and theoretically. An analytical solution to the boundary layer equations for the steady two dimensional flow due to a stretching surface in a quiescent incompressible fluid taking into account the case of a linear stretching surface was analyzed by crane [1]. Thereafter, many authors [2-8] has been extended the problem of Newtonian flow past a stretching surface in various ways.

In recent years, the study of fluid flow through porous media has received considerable attention because of numerous applications in various engineering disciplines, such as transfer ground water pollution, oil recovery processes, cooling of electronic components, food processing, etc., Hooper et al. [9] has analyzed the effects of surface injection or suction on mixed convection from a vertical plate in porous media. The effect of surface mass transfer on mixed convection in non-Newtonian fluids in porous media was examined by Taker [10].

The study of hydromagnetic viscous incompressible flow has many important engineering and industrial applications in devices such as MHD accelerators, the design of heat exchangers, the cooling of reactors, power generation. In MHD, the problem of stagnation point flow of electrically conducting fluids in the presence of large transverse magnetic field strengths was studied by Ariel [11].

In general no analytical solution is available to solve nonlinear differential equation problems and usually these are solved numerically subject to boundary conditions, are of which is prescribed at infinity. The differential transform method played an important role in recent researchers and applied for solving many of nonlinear problems in science and engineering [12-17]. This method builds for differential equations an analytical solution in the form of a power series. In addition, power series are not very much useful for large values of η, say $\eta \to \infty$. It is now well known that Pade approximation [18-19] have the advantage of manipulating the polynomial approximation into rational functions of polynomials. It is therefore essential to combination of the series solution, obtained by the DTM with the Pade approximation to provide an effective tool to handle boundary value problems at infinity domains. The first successful application of the DTM to boundary layer equations was presented by Rashidi [20].

Motivated to the above work, the present paper considered the nonlinear steady two dimensional laminar MHD flow of an electrically conductivity viscous fluid against a permeable stretching surface through a porous media. A transverse magnetic field is applied and the fluid is assumed to have a constant properties. The main aim of the present study is to find the approximate analytic solutions by the combinations of DTM and Pade approximants. Numerical results are displayed graphically by means of graphs. The effects of porous parameter, magnetic parameter and suction/injection parameter on velocity and skin friction are thoroughly discussed.

2 Formulation of the Problem

Let us consider a steady laminar boundary layer flow of an incompressible electrically conducting fluid over a permeable stretching surface embedded in a porous medium under a influence of a constant transverse magnetic field B_0. The origin is located at a slit, through which the sheet is drawn through the fluid medium. The x-axis is chosen along the sheet and y-axis is taken normal to it. Assume that the velocity of the continuous stretching surface $U = ax$ where x is the coordinate measured along the stretching surface and $a > 0)$ is a constant for a stretching. It is also assumed that the surface to be porous and the suction $(V_w > 0)$/injection $(V_w < 0)$ is taken into consideration. The fluid properties are assumed to be constant.

Under the usual boundary layer approximations for the Newtonian fluid, the steady two dimensional laminar MHD boundary layer equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\mu_e}{\rho k}u - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

where u and v are the velocity components in the x and y directions, respectively, k is the permeability of the porous medium, μ_e is the dynamic viscosity and ρ is the density of the fluid.

The corresponding boundary conditions are

$$u = U, \quad v = -v_w \quad at \quad y = 0 \tag{3}$$
$$u \to 0, \quad as \quad y \to \infty$$

Introduce the following nondimensional variables

$$\eta = y\sqrt{\frac{a}{\nu}},\ u = axf'(\eta),\ v = -\sqrt{a\nu}\,f(\eta) \tag{4}$$

Where η is the similarity variable, $f(\eta)$ is dimensional stream function and ν is the kinematic viscosity.

Using Eq. (4), Eq. (1) is identically satisfied and Eq. (2) becomes

$$f''' - f'^2 + ff'' - \beta f' - M f' = 0 \tag{5}$$

with the boundary conditions

$$f = S,\ f' = 1,\ at\ \eta = 0 \tag{6}$$
$$f' \to 0, \quad as \quad \eta \to \infty$$

Where $\beta = \mu_e / \rho a k$ is the porous parameter, $S = \left(v_w / \sqrt{a\nu}\right) > 0$ is suction or $S = \left(v_w / \sqrt{a\nu}\right) < 0$ is injection velocity parameter and $M = \sigma B_0^2 / \rho a$ is the magnetic parameter.

3 Differential Transformation Method

Consider a function $u(x)$ which is analytic in a domain T and let $x = x_0$ represent any point in T. The function $u(x)$ is then represented by a power series whose center is located at x_0. The differential transform of the function $u(x)$ is given by

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \tag{7}$$

where $u(x)$ is the original function and $U(k)$ is the transformed function. The inverse transformation is defined as follows

$$u(x) = \sum_{k=0}^{\infty}(x - x_0)^k U(k) \tag{8}$$

Combining Eqs. (7) and (8), we get

$$u(x) = \sum_{k=0}^{\infty}\frac{(x - x_0)^k}{k!}\left[\frac{d^k u(x)}{dx^k}\right]_{x=x_0} \tag{9}$$

Inspection of Eq. (9) indicates that the concept of differential transform is derived from Taylor series expansion. However, this method does not evaluate the derivatives symbolically. In actual applications, the function $u(x)$ is expressed by a finite series and Eq. (8) can be rewritten as follows:

$$u(x) \cong \sum_{k=0}^{m} (x - x_0)^k U(k) \quad (10)$$

which means that $u(x) \cong \sum_{k=m+1}^{\infty} (x - x_0)^k U(k)$ is negligibly small. Usually, the value of m is decided by convergence of the series coefficients. The operations for the one-dimensional differential transform method are provided in Table 1.

4 Analytical Approximations by Means of the DTM-Pade

The fundamental mathematical operations performed by DTM are listed in Table 1. Taking the differential transform of Eq. (5), we obtain

$$\begin{aligned}&(k+1)(k+2)(k+3)F(k+3)\\&-\sum_{r=0}^{k}(r+1)(k-r+1)F(r+1)F(k-r+1)\\&+\sum_{r=0}^{k}(k-r+2)(k-r+1)F(r)F(k-r+2)\\&-\beta(k+1)F(k+1)-M(k+1)F(k+1)=0\end{aligned} \quad (11)$$

where F (k) is the differential transform of $f(\eta)$. The transformed boundary conditions are

$$F(0)=S,\ F(1)=1,\ F(2)=\alpha \quad (12)$$

Moreover, substituting Eq. (12) into Eq. (11) and by a recursive method we can calculate the values of $F(k)$, where α is a constant that is computed from the boundary condition. For computing their values, the problem is solved with initial condition Eq. (12) and boundary conditions Eq. (6) are applied. The ideal method for enlarging the convergence radius of the truncated series solution is the Pade approximant i.e. converting the polynomial approximation into a ratio of two polynomials. The analytical solution obtained by the DTM, cannot satisfy boundary conditions at infinity. It is therefore essential to combine the series solution, obtained by DTM with the Pade approximant to provide an effective tool for accommodating boundary value problems in infinite domains.

5 Results and Discussion

In order to have a clear insight of the physical problem, numerical results are displayed with the help of Table 2 and graphical illustrations by fixing several values for the magnetic parameter M, porous parameter β and suction/injection parameter S on velocity and skin friction. The accuracy of present method is tested by numerical shooting method and the results are found to be in an excellent agreement and comparison between the DTM-Pade method and numerical shooting method are presented in Table 2

Table1: The operators for the one-dimensional differential transform method

Original function	Transformed function
$h(x)=u(x)\pm v(x)$	$H(k)=U(k)\pm V(k)$
$h(x)=\lambda u(x)$	$H(k)=\lambda U(k)$, λ is a constant
$h(x)=\dfrac{du(x)}{dx}$	$H(k)=(k+1)U(k+1)$
$h(x)=\dfrac{d^r u(x)}{dx^r}$	$H(k)=(k+1)(k+2)(k+3)\ldots(k+r)U(k+r)$
$h(x)=u(x)v(x)$	$H(k)=\sum_{r=0}^{k}U(r)V(k-r)$
$h(x)=\dfrac{du(x)}{dx}\dfrac{dv(x)}{dx}$	$H(k)=\sum_{r=0}^{k}(r+1)(k-r+1)U(r+1)V(k-r+1)$
$h(x)=u(x)\dfrac{dv(x)}{dx}$	$H(k)=\sum_{r=0}^{k}(k-r+1)U(r)V(k-r+1)$

Fig. 1 presents the dimensionless velocity profile $f'(\eta)$ for different values of porous parameter β when $S=0$ and $M=0$. It is observed that the porous parameter β increases, velocity $f'(\eta)$ decreases. Illustrating the fact that the effect of porous parameter is to decelerate the velocity.

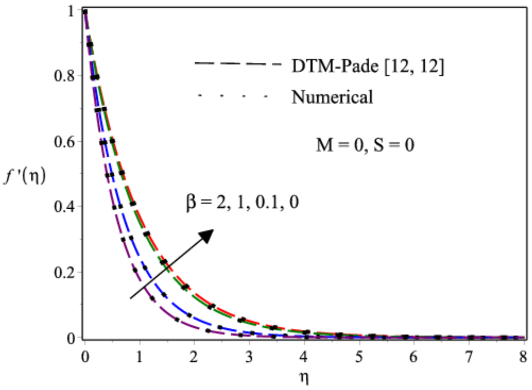

Fig. 1. Velocity profiles $f'(\eta)$ for various values of porous parameter β .

Fig. 2 demonstrates the plot of dimensionless velocity field for magnetic parameter M when $S=0$ and $\beta=1$. The effect of magnetic parameter M is decreasing the velocity.

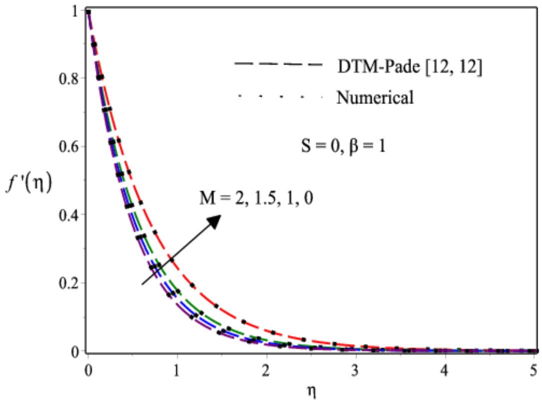

Fig. 2. Velocity profiles $f'(\eta)$ for various values of magnetic parameter M .

Dimensionless velocity field $f'(\eta)$ for suction/blowing parameter $S=-1,0,1$ when $M=2, \beta=1$ and $M=2, \beta=0$ are presented graphically through Fig. 3 and Fig. 4 respectively. It is noticed that for increasing values of suction/injection parameter S , the velocity $f'(\eta)$ decreases which physically conveys the fact that the effect of suction/blowing parameter S is to reduce the velocity.

Table 2. Values of $f''(0)$ for various values of S, β and M

S	β	M	DTM-Pade [12,12]	Numerical
0	0	0	-1.000000000	-1.000000000
-1	1	2	-1.561552813	-1.561552812
0	0.1	0	-1.048808848	-1.048808900
0	1	0	-1.414213562	-1.414213530
-1	0	2	-1.302775638	-1.302775638
0	1	1.5	-1.870828693	-1.870828690
0	1	2	-2.000000000	-2.000000000
-1	0	0	-0.618033989	-0.618034060
0	1	1	-1.732050808	-1.732050807
1	0	2	-2.302775637	-2.302775600
-1	0	1	-1.000000000	-1.000000000
0	0	1	-1.414213562	-1.414213530
1	0	1	-2.000000000	-2.000000000
1	0	0	-1.618033989	-1.618031000
1	1	2	-2.561552813	-2.561552810

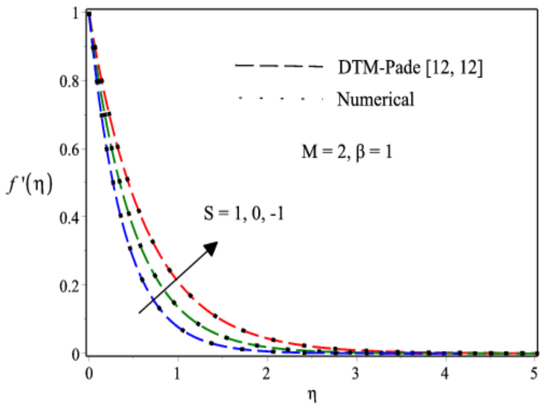

Fig. 3. Velocity profiles $f'(\eta)$ for various values of suction/blowing parameter S .

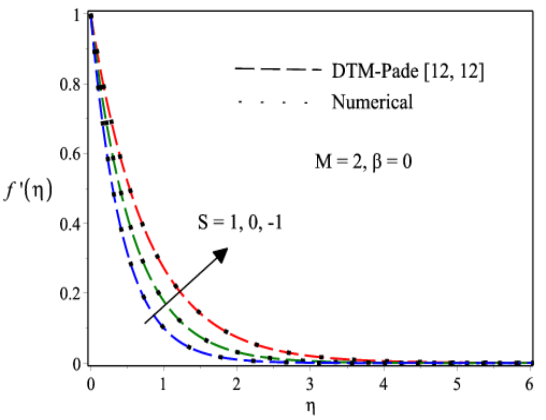

Fig. 4. Velocity profiles $f'(\eta)$ for various values of suction/blowing parameter S .

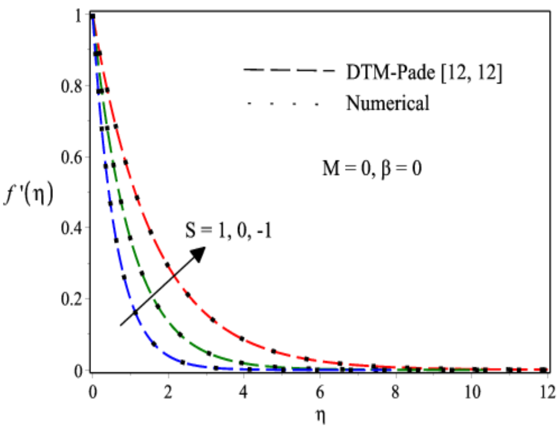

Fig. 5. Velocity profiles $f'(\eta)$ for various values of suction/injection parameter S .

Fig. 5 demonstrates the dimensionless velocity profile $f'(\eta)$ for suction/injection parameter in the absence of magnetic parameter M and absence of porous parameter β . It is seen that the effect of suction/blowing parameter is to reduce the velocity.

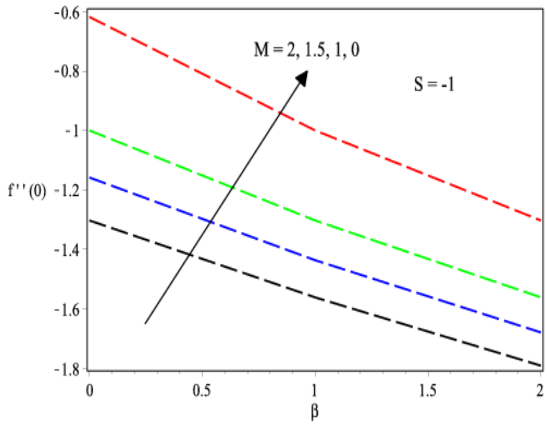

Fig. 6. Variation of $f''(0)$ with β for various values of magnetic parameter M .

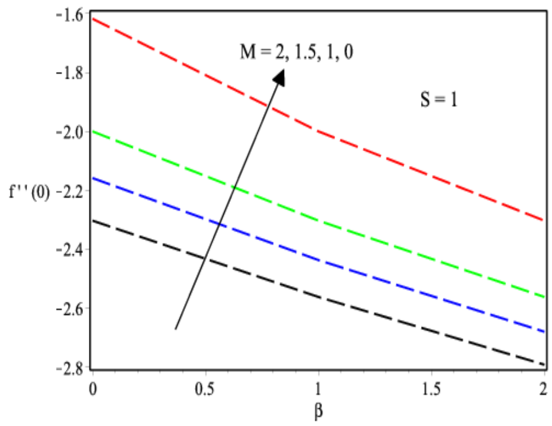

Fig. 7. Variation of skin friction with β for various values of magnetic parameter M .

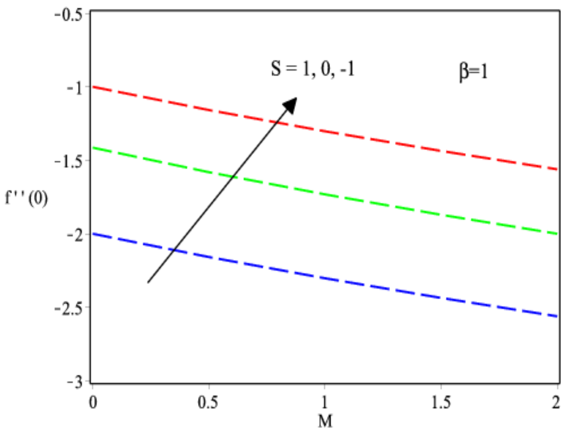

Fig. 8. Variation of skin friction with M for various values of suction/injection parameter S .

The effect of porous parameter β on skin friction $f''(0)$ for different values of magnetic parameter M with $S=-1$ and $S=1$ are shown in Fig. 6 and Fig. 7 respectively. It is inferred that the effect of magnetic field M and the effect of porous parameter β have the similar effect over skin friction so as to reduce it.

Fig. 8 displays the skin friction $f''(0)$ against magnetic parameter M for different suction/blowing parameter S with $\beta = 1$. It is observed that the effect of magnetic field is to decrease the skin friction. It is also seen that the skin friction decreases for $S > 0$ whereas increases for $S < 0$.

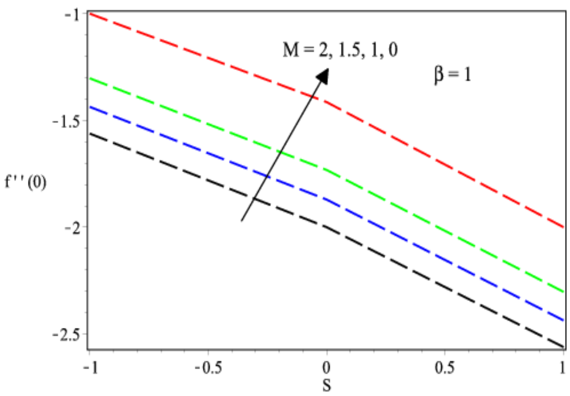

Fig. 9. Variation of skin friction with S for various values of magnetic parameter M.

Fig. 9 portrays the skin friction against suction/blowing parameter S for different values of magnetic parameter M with porous parameter $\beta = 1$. Illustrating the fact the effects of magnetic parameter and suction/injection parameter have the similar effect over skin friction so as to reduce it.

6 Conclusions

In this study, the DTM-Pade approximant for the problem of steady MHD flow of an electrically conductivity viscous fluid against a permeable stretching surface through a porous media is studied. Based on these studies, we made the following conclusions:

- The effect of magnetic field is to decreases the velocity and skin friction.
- Dimensionless velocity is reduced due to the influence of porous parameter.
- The effect of magnetic field due to porous parameter is to decrease the skin friction for both the case of suction/injection.

References:

[1] J. Crane, Flow past a stretching surface, *Z. Angew. Math. Phys*. Vol. 21,1970, pp. 645-647.

[2] N. Afzal, A. Badaruddin and A. A. Elgarvi, Momentum and heat transport on a continuous flat surface moving in a parallel stream, *International Journal Heat and Mass Transfer*, Vol. 36, No. 13, 1993, pp. 3399-3403.

[3] M. Yurusoy, Unsteady boundary layer flow of power-law fluid on stretching sheet surface, *International Journal of Engineering Sciences*, Vol. 44, 2006, pp. 325-332.

[4] S. Mukhopadhyay, G.C. Layek, Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface, *International Journal of Heat and Mass Transfer*, Vol. 51, 2008, pp. 2167-2178.

[5] R. Tsai, J. S. Huang, Heat and mass transfer for soret and Dufour's effects on Hiemenz flow through porous medium onto a stretching surface, *International Journal of Heat and Mass Transfer*, Vol. 52, 2009, pp. 2399-2406.

[6] P. R. Chappidi, F. S. Gunnerson, Analysis of heat and momentum transport along a moving surface, *International Journal of Heat and Mass Transfer*, Vol. 32, 1989, pp. 1383-1386.

[7] N. Afzal, A. Baderuddin, A.A. Elgarvi, Momentum transport on a continuous flat surface moving in a parallel stream, *International Journal of Heat and Mass Transfer*, Vol. 36, 1993, pp. 3399-3403.

[8] M. Thiagarajan, A.S. Sangeetha, Magnetohydrodynamic flow over a power-law stretching plate with free stream pressure gradient, *Journal of Energy Heat and Mass Transfer*, Vol. 32, 2010, pp. 375-384.

[9] W.B. Hooper, T.S. Chen, B.F. Armaly, Mixed convection from a vertical plate in porous media with surface injection or suction. Numerical Heat Transfer Part A-applications, Vol. 25, No. 3, 1994, pp. 317-329.

[10] H.S. Takhar, R.C. Ram, Magnetohydrodynamic free convection flow of water at $4^{\circ}C$ through a porous medium. *International Communications in Heat and Mass Transfer*, Vol. 21, 1994, pp. 371–376.

[11] R.D. Ariel, Hiemenz flow in hydromagnetics. *Acta Mechanica*, Vol. 103, 1994, pp. 31–43.

[12] N. Bildik, and A. Konuralp, The Use of Variational Iteration Method, Differential Transform Method and Adomian Decomposition Method for Solving Different Type of Nonlinear Partial Differential Equations, *International Journal of Nonlinear Science and Numerical Simulations*, Vol. 7, No. 1, 2006, pp. 65–70.

[13] O. Ozdemir, and M. O. Kaya, Flapwise Bending Vibration Analysis of a Rotating Tapered Cantilever Bernoulli-Euler Beam by Differential Transform Method, *J. Sound Vib.*, Vol. 289, No. 2006, pp. 413–420.

[14] M.M. Khader, On the Numerical Solutions for the Fractional Diffusion Equation, *International Journal of Communications in Nonlinear Science and Numerical Simulation*, Vol. 16, 2011, pp. 2535–2542.

[15] M. M. Khader, and A. M. Megahed, 2013, Numerical Simulation Using the Finite Difference Method for the Flow and Heat Transfer in a Thin Liquid Film Over an Unsteady Stretching Sheet in a Saturated Porous Medium in the Presence of Thermal Radiation, *Journal of King Saud Univ., Eng. Sci.*, Vol. 25, 2013, pp. 29–34.

[16] N. H. Sweilam, M. M. Khader, and A. M. Nagy, Numerical Solution of Two-Sided Space Fractional Wave Equation Using Finite Difference Method, *Comput. Appl. Math*, Vol. 235, 2011, pp. 2832–2841.

[17] M. M. Khader, N. H. Sweilam, N. H., and A. M. S. Mahdy, Numerical Study for the Fractional Differential Equations Generated by Optimization Problem Using Chebyshev Collocation Method and FDM, *Appl. Math. Inf. Sci.*, Vol. 7, No. 5, 2013, pp. 2011–2018.

[18] G.A. Baker, *Essential of Pade approximants*, London, Academic Press. 1975.

[19] G. A. Baker, and P. Graves-Morris, *Encyclopedia of mathematics and its application, Parts I and II: Pade approximants*, Addison-wesley publishing company, New York, 1981.

[20] M.M. Rashidi, G. Domairry, New analytical solution of the three-dimensional Navier–Stokes equations, *Modern Physics Letters B*. Vol. 23, No. 26, 2009, pp. 3147–3155.