Measuring and Evaluation of the Shape and Size of Initial Geometrical Imperfections of Structural Glass Members

ONDŘEJ PEŠEK, JINDŘICH MELCHER
Institute of Metal and Timber Structures
Brno University of Technology
Veveří 331/95, 602 00 Brno
CZECH REPUBLIC
pesek.o@fce.vutbr.cz; melcher.j@fce.vutbr.cz http://www.kdk.fce.vutbr.cz

Abstract: - Buckling resistance (including flexural buckling and lateral-torsional buckling) is major design parameter for many structural glass components. Buckling resistance is influenced by several parameters: boundary conditions, slenderness and initial imperfections. Initial imperfections, in particular initial geometrical out-of plane curvatures are very poorly presented in literature. In this paper measuring and determining of initial geometrical imperfections of fifteen specimens are presented. Measured specimens had the same dimensions but they had different composition – there are made from monolithic glass or laminated glass using PVB or EVA foil bonding together annealed glass panes or fully tempered glass panes. The shape of initial overall bow was measured using CARL ZEISS testing equipment. Information about initial curvature shapes and amplitudes are major conclusions resulting from statistical evaluation of tests.

Key-Words: - Structural glass, initial geometrical imperfections, flexural buckling, normal distribution, log-normal distribution

1 Introduction

Glass has been established as a material of load carrying members and structures in the end of twentieth century and its importance still grows today [9].

Due to slender of glass members it is necessary to design them on stability problems – flexural buckling or lateral torsional buckling. Design methods of steel and timber structures are not completely usable for glass structures because of several differences (initial imperfections, brittle behaviour and laminated glass behaviour [10]. The shape and size of initial geometrical imperfections are poorly published in recent publications.

$$f\left(w_0\right)_x = w_0 \frac{N}{N_{cr} - N} \cdot \cos \frac{\pi \cdot x}{L} \qquad (1)$$

Behaviour of imperfect glass columns was published in [1]. Equation (1) describe (according to the second order theory) dependency of deformation $f(w_0)$ of axially loaded (by centric force N) imperfect (initial geometrical imperfection w_0) column on own size of initial imperfection considering the sinusoidal shape. Flexural deformation $f(w_0)$ increases with increasing amplitude of initial imperfection w_0. N is Euler's critical force, L is critical length of column and x is ordinate of point of interest.

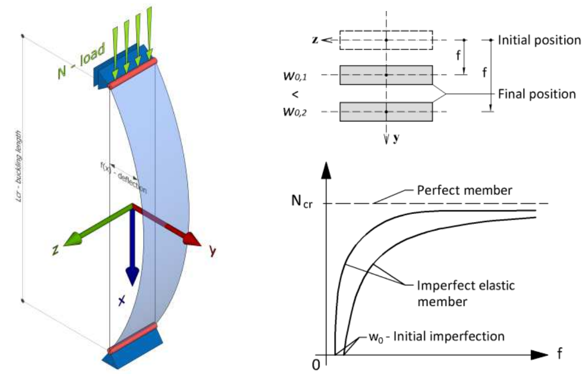


Fig. 1.*Flexural buckling of slender beam, load-deflection curve of perfect and imperfect member*

2 Methodology

Three types of initial imperfections are distinguished according to their origin: geometrical imperfections (geometrical curvature of column), construction imperfections (actual point of load application etc.) and physical imperfections (residual stresses, inhomogeneity of the material). Geometrical imperfections are relatively easily measurable and observable before placing the member in the structure.

2.1 Geometrical Imperfections of Glass Members and Normative Documents

The differences of nominal dimensions are generated during manufacturing processes and they should be lower than values given by product standards. Following tolerances should be checked: (i) glass pane thickness, (ii) glass pane length, wide and rectangularity, (iii) edge deformations due to vertical production (does not apply on float glass) and (iv) planarity (flatness).

Geometrical deformations (curvature) are the results of glass tempering (manufacturing process of fully tempered glass or heat strengthened glass). The size of deformations depends on type of glass (coated glass, patterned glass etc.), on glass dimensions and aspect ratio, on nominal thickness and on type of tempering process (vertical or horizontal).

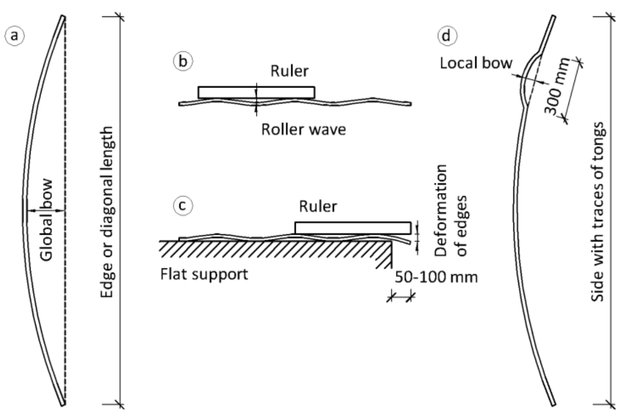


Fig. 2 *Glass pane deformations types*

Four types of deformations are follows: (a) overall bow, (b) roller wave (only for horizontally tempered glass), (c) curvature of edges (only for horizontally tempered glass) and (d) local bow (only for horizontally tempered glass) – see Fig. 2.

Glass type	Position at tempering	Fully tempered glass and heat strengthened glass	
		Global bow	Local bow
		mm/m	mm/300 mm
Float glass without coating	Horizontal	3.0	0.3
Others	Horizontal	4.0	0.5
All	Vertical	5.0	1.0

Table 1

The maximum allowable values of the overall bow deflection according to the EN 1863-1 and EN 12150-1 are: (i) 3.0 mm/m for float glass horizontally tempered, (ii) 4.0 mm/m for horizontally tempered glass (another types) and (iii) 5.0 mm/m for vertically tempered glass (all types).

2.2 Specimens

Initial geometrical imperfections (global bow) were measured on specimens listed in Table 2.

Specimen	Designation	Description	Glass	Foil	Pcs
T1 – T3	ESG 12	Safety glass	FTG	-	3
T4 – T6	VG 66.2	Laminated glass	ANG	PVB	3
T7 – T12	VSG 66.3	Laminated safety glass	FTG	EVASAFE	6
T13 – T15	VSG 444.33	Laminated safety glass	FTG	EVASAFE	3

Table 2

Every specimens made of soda lime silica glass has the same dimensions – length was 1500 mm and width was 150 mm. Specimens was from monolithic glass or double laminated glass or triple laminated glass (made from annealed glass or fully tempered glass) and sum of the glass panes thicknesses was 12 mm for all specimens.

2.3 Imperfections Measurement Method

Overall bow shape of glass specimens was analyzed used mechanical measuring system Carl Zeiss. Measured specimen was fixed in vertical position (due to dead load deflections elimination) and supported by two timber blocks (in according to EN 1863-1 and EN 12150-1) and stability was ensured by steel stand.

2.4 Measurement Device Description

Measuring device Carl Zeiss was composed from height-adjustable guiding rail situated on two supporting threaded columns, which was independently height-adjustable by two locking screws. Wagon with mechanical deflection sensor (Carl Zeiss 003 19 with accuracy 0.01 mm) was moving on guiding rail (x axis). Wagon was able to move with mechanical indicator in vertical (*z* axis) and lateral (*y* axis). Measuring set up was tuned so that the tip of indicator watches was situated 10 mm under measured specimen edge. Measuring set up is plotted in Fig. 3.

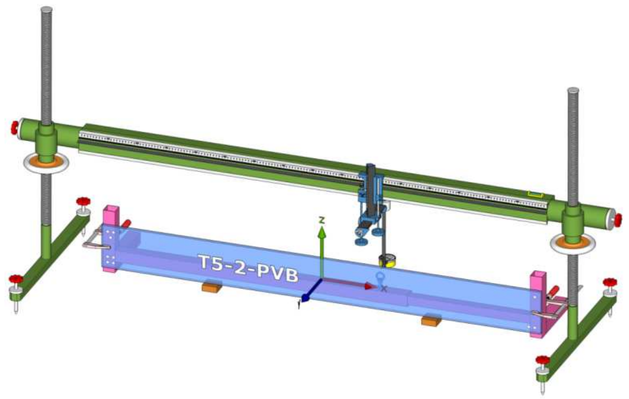


Fig. 3 *Measurement set-up*

Because of elimination of measurement errors due to measuring set up geometrical imperfections (guiding rail) every specimen and both its edges were measured in two locations (positive location and negative location) and in two positions (position 1 and position 2. Lateral horizontal deflection (*y* coordinate) was measured in steps of 50 mm. Both edges were measured two times for both locations and positions of specimen.

In the first step of evaluation the measured coordinates was transferred to new coordinate system, so that coordinate $y = 0$ at both ends of specimen. In the second step the average coordinates was made from two measuring – to received coordinates of deflections (imperfections) $u_0(x)$. In the last step the evaluated deflections was corrected by the value of guiding rail imperfections

2.5 Measuring Device Imperfection

Measuring set up itself was subjected to implicit errors – precision of mechanical deflection device, deviations of the theoretical parallel placement of specimen and test set up and geometrical imperfections of the guiding rail. The first two errors are not significant and were not taken into account. Errors due to shape imperfections of the guiding rail were significant and they were taken into account by compensating method described in Fig. 3.

The geometrical imperfection of the guiding rail was deducted from measuring the initial shape imperfections of the same glass specimen twice: once in the conventional position ($u_{0,uncorr,1}$) and once in the mirrored position ($u_{0,uncorr,2}$).

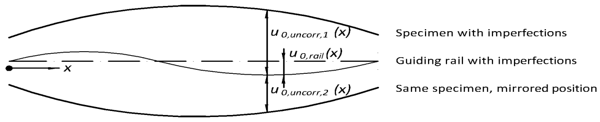


Fig. 4 *Evaluation of guiding rail initial imperfections*

Using this principle, the shape imperfections of the guiding rail $u_{0,rail}(x)$ was determined with (2). Repeating this process several times, the imperfections of the guiding rail could be reproduced with high accuracy. The resulting corrected geometrical imperfection $u_0(x)$ was obtained using (3).

$$u_{0,rail}(x) = \frac{u_{0,uncorr,1}(x) - u_{0,uncorr,2}(x)}{2} \tag{2}$$

$$u_0(x) = u_{0,uncorr}(x) \pm u_{0,rail}(x) \tag{3}$$

Shape imperfections of the guiding rail are plotted in Fig. 5. To evaluation of initial imperfections of all specimens the mean value of guiding rail imperfections was taken into account. The shape of initial imperfections from all measuring is similar, but on the other hand the amplitudes are relatively different. Mean value in the point of maximum deflection (close to the midspan) was 0.46 mm with accuracy ± 0.11 mm.

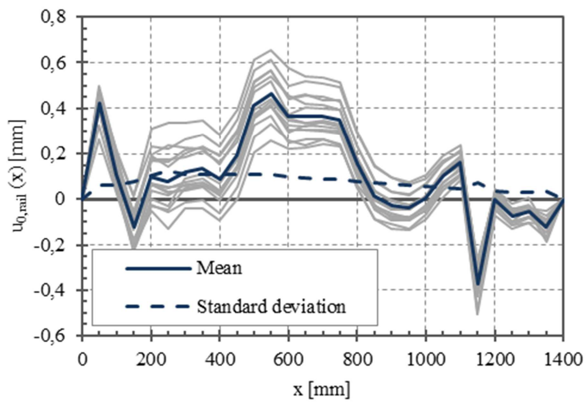


Fig. 5 *Guiding rail imperfections*

3 RESULTS AND DISCUSSION

Evaluation of the measurement was carried out according to the approach first presented by Belis et al in [2]. The shapes of initial imperfections of all specimens are plotted in graphs according to the glass composition (see Tab. 1) in Appendix.

Imperfections of all specimens are evaluated with positive sign.

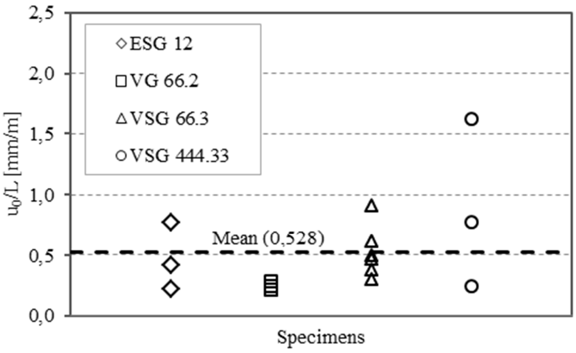


Fig. 6 *Amplitudes of initial imperfections of tested specimens*

The value of initial imperfection u_0/L is representing maximum relative amplitude, which is not generally situated in the half of length of specimen. Imperfections are plotted in Fig. 6.

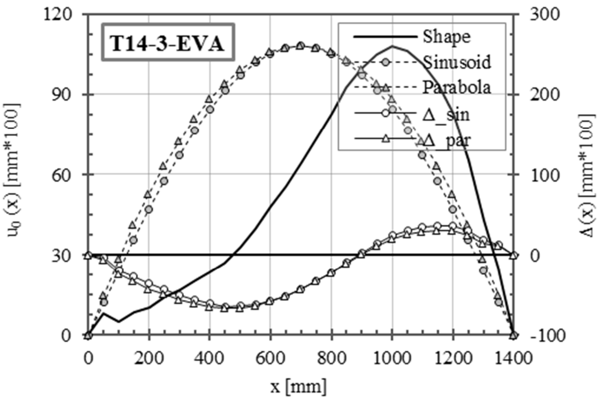
3.1 Overall Bow Shape

From imperfection shapes plotted in Fig. 7 is clear that initial imperfection shape should be approximated by second order parabola (4) or half sinusoidal wave (5):

$$a^{par}(x) = u_0 - \frac{u_0}{(0{,}5 \cdot L)^2} \cdot x^2 \tag{4}$$

$$a^{\sin}(x) = u_0 \cdot \cos\left(\frac{\pi \cdot x}{L}\right) \tag{5}$$

Overall bow shape of typical (most of the samples) and untypical (only unique case) specimen is plotted in Fig. 7. In the graph there approximation curves (sinusoid and parabola) are plotted. On secondary vertical axis there is plotted error of approximations.



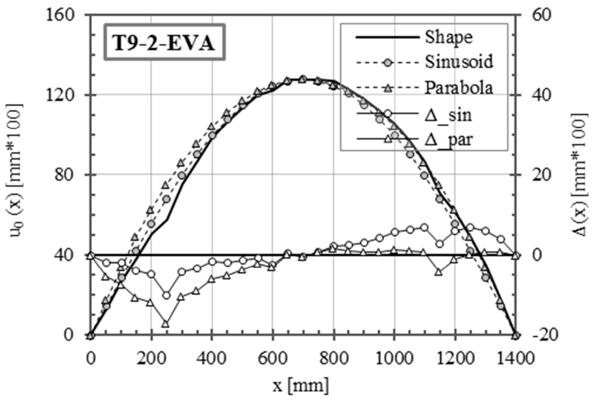


Fig. 7 *Typical and untypical specimen: Shapes of imperfection and their approximation*

To quantitatively describe the approximation quality of the initial imperfection shape, a fitting error (fe^{par} for the parabola and fe^{sin} for the sinusoid) is defined as the ratio of the maximal error (Δ^{par} for the parabola and Δ^{sin} for the sinusoid) and the maximal initial imperfection u_0:

$$fe^{par} = \Delta^{par} / u_0 \cdot 100 \tag{6}$$

$$fe^{\sin} = \Delta^{\sin} / u_0 \cdot 100 \tag{7}$$

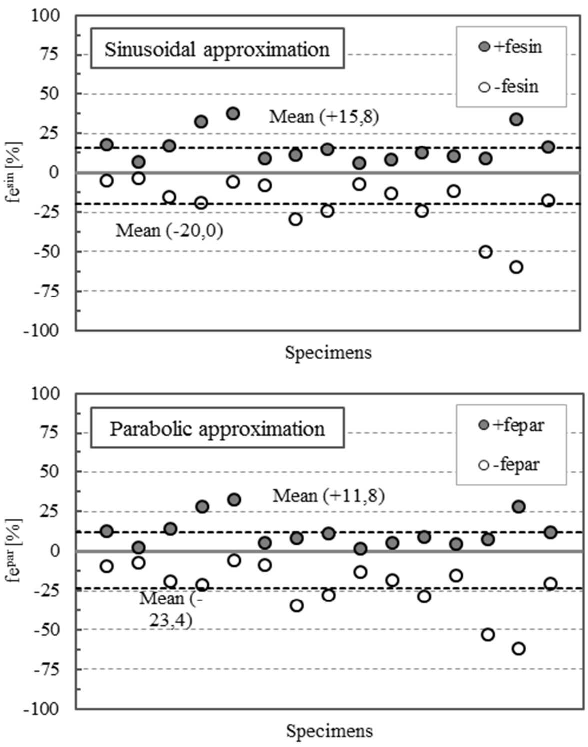


Fig. 8 *Fitting errors and mean values; approximation by sinusoid and parabola*

The error of approximation at position x is defined by (8) and (9).

$$\Delta^{par}(x) = u_0(x) - a^{par}(x) \tag{8}$$

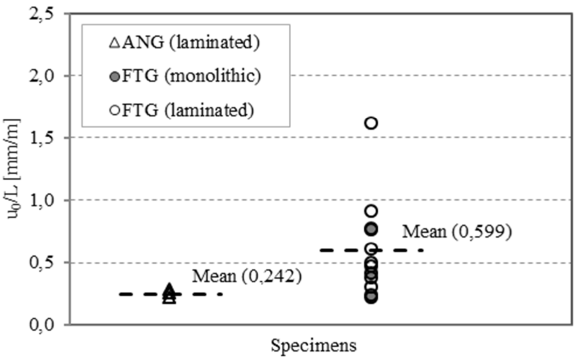
$$\Delta^{\sin}(x) = u_0(x) - a^{\sin}(x) \tag{9}$$

The values of fitting errors fe^{par} and fe^{sin} are plotted in graphs in Fig. 8.

The mean value of the positive fitting error is higher for a sinusoidal approximation (+15.8 %) than for parabolic approximation (+11.8 %). In the case of negative fitting error it is oppositely: parabola (-23.4 %) and sinusoid (-20.0 %). Adding and subtracting of absolute values of fitting errors for parabolic approximation (11.8 + 23.4 = 35.2 % or 23.4 – 11.8 = 11.6 %) and sinusoid approximation (15.8 + 20.0 = 35.8 % and 20.0 – 15.8 = 4.2 %) leads to the conclusion, that the sinusoidal approximation is more stable. Both functions can be used for implementing initial imperfections in numerical models. However, preference is given to the sinusoidal function because it is corresponding to the eigen-mode, which is often adopted as initial geometrical imperfection in buckling analysis.

3.2 Amplitude of Imperfection

Influence of glass type is illustrated on Fig. 9 upper. Initial imperfection of annealed glass (mean 0.242 mm/m) is significantly less than for fully tempered glass (mean 0.599 mm/m).Influence of lamination is plotted in Fig. 9 bottom. From comparison of mean values of imperfections for monolithic glass (0.468 mm/m) and laminated glass (0.543 mm/m) it should be noted that this influence is negligible.



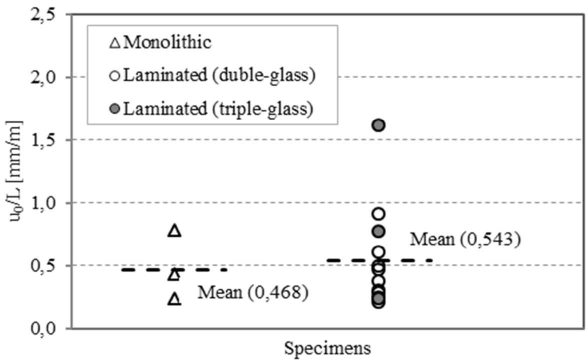


Fig. 9 *Influences on imperfections; glass type effect (upper); lamination effect (bottom)*

3.3 Characteristic value of initial imperfection

For practice design it is necessary to know the characteristic value of initial imperfection $(u_0/L)_k$, which entering into buckling analysis. Generally, characteristic value is considered as 5% quantile.

3.3.1 Normal Distribution

The results of initial imperfections measuring ware evaluated such that all imperfection values ware positive. Actually the curvature might be convex or concave and mean value of large population is theoretically equal to zero.

Available data set of 15 absolute values is assumed as truncated normal distribution with truncation at zero, in agreement with the Probabilistic Model Code [3] by JCSS (The Joint Committee on Structural Safety). This truncated normal distribution is equivalent to a normal distribution which corresponds to a modified population. More specifically, the available data set was considered twice, whereas one time it was given a negative sign and the other it kept a positive sign. Consequently, a new population of 30 imperfections values was obtained with a mean equal to zero and with normal distribution, Fig. 10. For new population the characteristic value was calculated.

In general, 5% quantile is calculated by (10), where μ is mean value and σ is standard deviation.

$$5\%\ quantile = \mu + 1.645 \cdot \sigma \tag{10}$$

Because of a “doubled” population (to get correct data set), 2.5% quantile on each tail of the normal distribution was calculated using (11):

$$\begin{aligned} &5\%\ quantile_{truncated\ dist.} = \\ &= 2.5\%\ quantile_{untruncated\ distr.} = \mu + 1.96 \cdot \sigma \end{aligned} \tag{11}$$

The characteristic value of initial imperfection based on this method is $(u_0/L)_k$ = 1.276 mm/m.

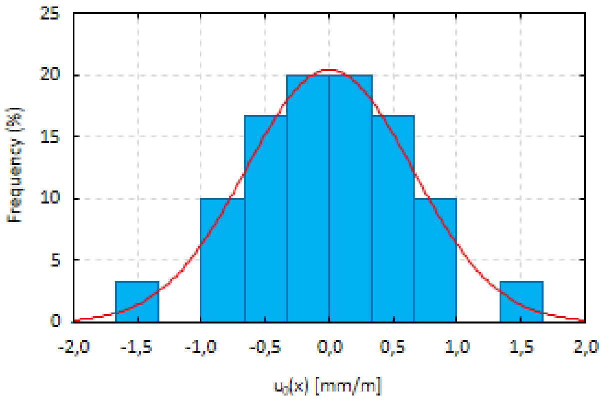


Fig. 10 *Normal distribution of doubled population*

3.3.2 Log-normal Distribution

An alternative approach is to analyze directly the asymmetric probability density function based on original 15 imperfection measurement values only. Histogram of original data set with log-normal distribution function is illustrated in Fig 11. Characteristic value - 5% quantile calculated by STATISTICA software [4] is $(u_0/L)_k$ = 1.186 mm/m.

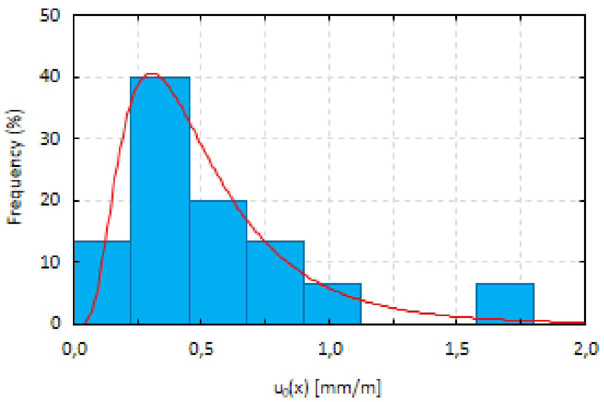


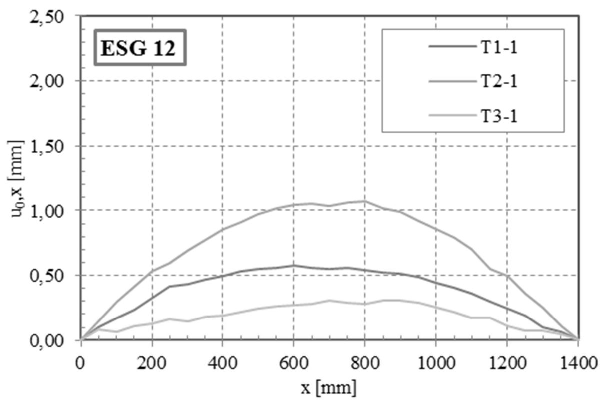
Fig. 11 *Log-normal distribution of original population*

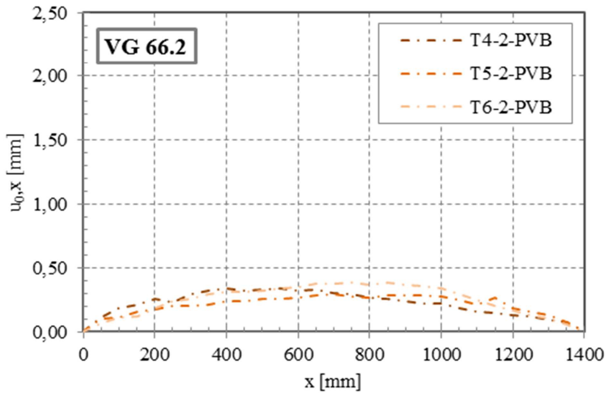
4 Conclusions

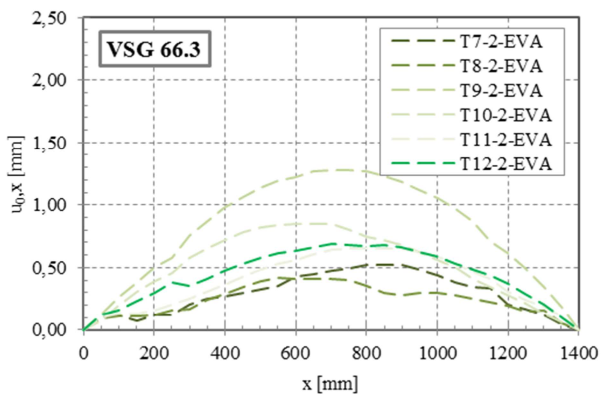
Characteristic value of initial imperfection 1.276 mm/m is corresponding to curvature $L/800$ (for both normal and log-normal distribution). This value is less than allowable values according to the product standards: 3 mm/m for horizontally tempered float glass. From study performed by Belis et al [2] resulted characteristic value $(u_0/L)_k$ = 2.5 mm/m ($L/400$).

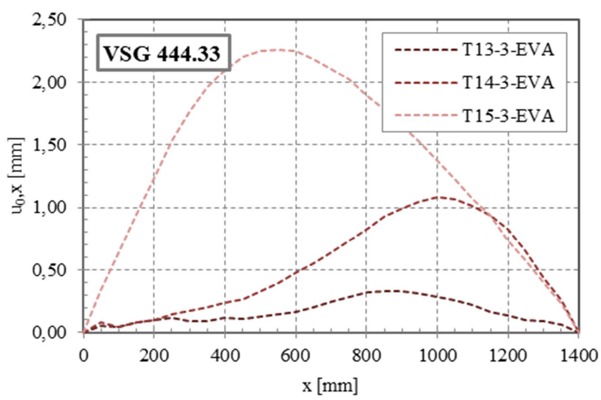
Appendix:

Shapes of glass specimens sorted according to the glass composition are plotted in graphs.









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