Bifurcations in the dynamic system of the mechanic processing in metal-cutting tools

Vilor Zakovorotny

Full Time Professor of Don State Technical University
(DSTU), 344010 Gagarin square 1, Rostov-on-Don,
RUSSIA, vzakovorotny@dstu.edu.ru, http://donstu.ru/en

Abstract: The nonlinear dynamics in processing of metal-cutting tools is considered. The author proposes the mathematical model that includes two spatial elastic subsystems. They interact through the dynamic connection, which is a force in the model of coordinates of the system state. The simultaneous excitation sources and nonlinear damping vibrations of the cutting tool are taken into consideration in the model. This approach allows defining the following system properties: branching equilibrium points; the conditions under which generated limit cycles and invariant tori; the formation of chaotic dynamics. The dynamic subsystem tool is presented as the linear dynamical system in the normal plane. The main attention is paid to the analysis of bifurcations in the space of parameters of dynamic equations, as well as in the space of control parameters. General patterns of loss of stability of equilibrium of the system are considered. The role of acceleration of gyroscopic and circulation forces is specified. The results are important for the analysis of motion of mechanical systems that interact with various environments (processes of friction, hydrodynamic, aerodynamic, etc.).

Key-Words: - processing, metal cutting, dynamic system, asymptotic stability, invariant manifolds, and bifurcation.

1 Introduction

Each machine has subsystems that perform relative movement. They interact with various environments (aerodynamic, hydrodynamic, tribological, technological, etc.). In order to describe its dynamics it is necessary to describe the interactions of mechanical forces, which depend on the coordinate system state [1]-[4]. In the area of intersection between the tool and the work piece a unique environment is formed. Its properties are characterized by nonlinearity, temporal variability of the parameters of equations and periodic disturbances, including kinematic disturbances [4]-[14]. In this system, the loss of stability of the equilibrium point may be possible, which is seen in the moving coordinate system, the motion of which is determined by the trajectories of the control elements of the machine [3], [17]. If the equilibrium is unstable, in its vicinity there are various sets (limit cycles, invariant tori, chaotic attractors, etc...) [3], [10]-[16]. These sets characterize stationary changes of the strength and deformation properties that directly affect the cutting process and the quality of the work piece [18] - [21].

Cutting forces depend on the area, which is formed by the intersection of the tool and the work piece. That means that they depend on the trajectories of the executive parts of the machine, tool geometry and elastic deformation displacement of the tool and the work piece [3] - [10]. The regularities of transformations of area to forces are defined by many physical processes which changes happen in time. Therefore, phase shifts between the forces and deformations happen [3], [8], [9]. In many cases, especially in the process of milling, it is necessary to take into account that the parameters in the equation are the periodic functions of time [10] - [16]. While studying the systems with periodically varying parameters, the equations of Mathieu - Hill and the Floquet theory are used [22], [23].

Typically, a simple scalar model with the confirmed existence of chaotic attractors are used [24 - 28].They are observed in an experimental way, and in those cases, where periodic changes of the parameters are missing in the obvious way [24]. In addition, the increase in cutting speed leads to a decrease of strength [25]. This dependence is described by the equation of van der Pol or Rayleigh [18]. These models take into account the dependence of the force on the speed, but they do not take into account the change in the square of the shear layer. In addition, many mathematical models take into account that the trajectory of the tool in the previous turnover of detail affects the cutting forces [27] - [28].

The main feature of the above-cited works is that they analyze only one mechanism of loss of stability, for example, the impact of the delay. At the same time, usually, the system is described as one oscillating link. The real system is considered as selective circuits and it has several interrelated mechanisms of loss of stability. The consideration of a spatial model of the elastic system which has a plurality of simultaneously existing sources of self-excitation, not only complicates the model, but it creates its new quality. For example, gyroscopic, no potential and accelerating forces are formed in the spatial model, in the equations in variations relative to the equilibrium point. They characterize the general properties and mechanisms of loss of stability of equilibrium.

The consideration of the numerous mechanisms of self-excitation helps to investigate many of the effects of nonlinear dynamics that were not considered previously. For example, it became possible to investigate the branching of the equilibrium points, the periodic formation of chaotic attractors when changing control parameters, the formation of invariant tori and their transformation into limit cycles, and so on. These new issues of dynamics of the process of cutting are considered in this article. Thus, the proposed research results characterize the development of the main provisions of the nonlinear dynamics of the cutting process in case, in which a spatial model and several simultaneously existing nonlinear sources of self-excitation are taken into account.

The article focuses on the problems of changes of topology of the phase space, depending on the system's parameters, i.e. system's bifurcations. These properties are common in the dynamics of the interactions of the mechanical system with the environment of any type, for example, tribological environment. According to the author, the general characteristics of vibrations of mechanical systems that interact with various environments represent the main research results of the article.

2 Basic mathematical model

The main properties of the system can be expanded if we use a simplified model of the process of free cutting. Without revealing the force structure, the dynamic equation can be written as [3], [9] (Fig. 1)

$$m\frac{d^2X}{dt^2}+h\frac{dX}{dt}+cX=F(t), \qquad (1)$$

where $m=\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$, $h=\begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \end{bmatrix}$, $c=\begin{bmatrix} c_{1,1} & c_{2,1} \\ c_{1,2} & c_{2,2} \end{bmatrix}$ - is the matrix of inertia, dissipative and elastic coefficients of the subsystem tool; $X=\{X_1, X_2\}^T$ - elastic deformation displacement vector in a plane normal to the cutting surface; $F(t)=\{F_1(t), F_2(t)\}^T$ - forces vector. Deformation changes are considered in the moving coordinate system moving in the direction of the work piece at a speed of support V_C. The angular velocity of rotation of the work piece $\Omega=const$, therefore, one turn time is $T=const$. Besides, $V_C=const$. Therefore, in the steady state the flow value is $V_C T=S_P=const$. Cutting force is generated as the sum of two components acting on the front face of the instrument $F^{(1)}$, and its rear face $F^{(2)}$, that is $F=F^{(1)}+F^{(2)}$, (Figure 1). Parameters m, h and c can be determined through the use of the methods described, for example, in [3], [9], [30]. It is possible to use, for example, methods of determining the parameters of the dynamic models presented in [30]. In modeling the forces we shall restrict to the following properties.

1. The module force is proportional to the square $S = t_P S_P$, and in this scheme $t_P = const$. Then

$$F_0 = \rho(S_P - X_1), \qquad (2)$$

where ρ - coefficient of rigidity $[кг / мм]$. Orientation of force in space is given by the coefficients (χ_1, χ_2) i.e. $F = \rho(S_P - X_1)\{\chi_1, \chi_2\}^T$.

2. The stiffness ρ depends on the cutting speed determined by the difference in the velocities of the work piece and elastic deformation shifts of the tool $(V - dX_2 / dt)$. Following [25], [29], the dependence of ρ from the velocity can be represented as

$$\rho = \rho_0 (1 + \mu e^{-\alpha_1 (V - dX_2 / dt)}) \qquad (3)$$

where μ - dimensionless coefficient; α_1 - coefficient in $[s / m]$.

3. There are phase shifts between deformation and forces. In addition, with increasing frequency module forces decreases. To simulate this feature it is convenient to introduce coordinates Y_1 and Y_2, i.e.

$$T_1 \frac{dY_1}{dt} + Y_1 = X_1; \; T_2 \frac{dY_2}{dt} + Y_2 = X_2. \qquad (4)$$

4. In the area of contact of the tool part with the rear face an additional force $F^{(2)} = \{F_1^{(2)}, F_2^{(2)}\}$ is forming (Fig. 1). They depend on the rear corner of the instrument α. They increase with a decrease of this angle. The value of the angle is evaluated not only by the geometry of the tool, but also by the direction of motion of the tool relative to the work piece surface, that is, for a given cutting speed – from the total speed $V_C - dX_1 / dt$. Their formation can also be represented in the form

$$\begin{cases} F_1^{(2)} = F^{(2)} (e^{\alpha_2 (V_C - dX_1 / dt)} - 1); \\ F_2^{(2)} = F^{(2)} k (e^{\alpha_2 (V_C - dX_1 / dt)} - 1), \end{cases} \qquad (5)$$

where α_2 - as the parameter characterizing the rate of rise of force depending on the speed of movement and speed of elastic deformation displacement in the feed direction; k - coefficient of friction. It should be noted that $\alpha_2 \rangle\rangle \alpha_1$. Presentation of a force $F^{(2)} = \{F_1^{(2)}, F_2^{(2)}\}$ as in (5) represents a nonlinear dissipation of processing. In the steady state in the case of stable equilibrium of forces, acting on the rear face of the tool, can be neglected.

5. In paragraphs 1 - 5 representation is based on the assumption that the deformation of the displacement is not accompanied by rotation. By reducing the angle (lower illustration in Fig. 1) contains a positive feedback, further excitation of the system. Moreover, it is non-linear. According to our data in the processing of chromium steel additional forces are approximated by the following expression

$$\Delta F_1(X_2) = \alpha (X_2)^3, \; \Delta F_2(X_2) = \beta X_2, \qquad (6)$$

where α - parameter representing the density; β - parameter representing the stiffness of the cutting process. Then the generalized dynamics equation is represented as

$$\begin{cases} m \frac{d^2 X_1}{dt^2} + h_{1,1} \frac{dX_1}{dt} + h_{2,1} \frac{dX_2}{dt} + c_{1,1} X_1 + c_{2,1} X_2 = \\ = \rho_0 \chi_1 (1 + \mu e^{-\alpha_1 (V - dX_2 / dt)}) \{S_P - Y_1\} + \\ + F^{(2)} (e^{\alpha_2 (V_C - dX_1 / dt)} - 1) + \alpha (Y_2)^3; \\ m \frac{d^2 X_2}{dt^2} + h_{1,2} \frac{dX_1}{dt} + h_{2,2} \frac{dX_2}{dt} + c_{1,2} X_1 + c_{2,2} X_2 = \\ = \rho_0 \chi_2 (1 + \mu e^{-\alpha_1 (V - dX_2 / dt)}) \{S_P - Y_1\} + \\ + F^{(2)} k (e^{\alpha_2 (V_C - dX_1 / dt)} - 1) + \beta Y_2; \\ T_1 \frac{dY_1}{dt} + Y_1 = X_1; \; T_2 \frac{dY_2}{dt} + Y_2 = X_2. \end{cases} \qquad (7)$$

System (7) is not a scalar. It has several simultaneously existing sources of self-excitation and it has two oscillators. In addition, the nonlinear dissipation takes place.

3 Bending deformation tools are missing

This is the case when in (7) $\alpha = \beta = 0$. Let's also say that $T_2 = 0$. The analysis is performed as follows. We shall analyze the properties of the system at constant values of control parameters. First, we will analyze the equilibrium point. Then, we shall consider the possibility of attracting sets in the neighborhood of equilibrium.

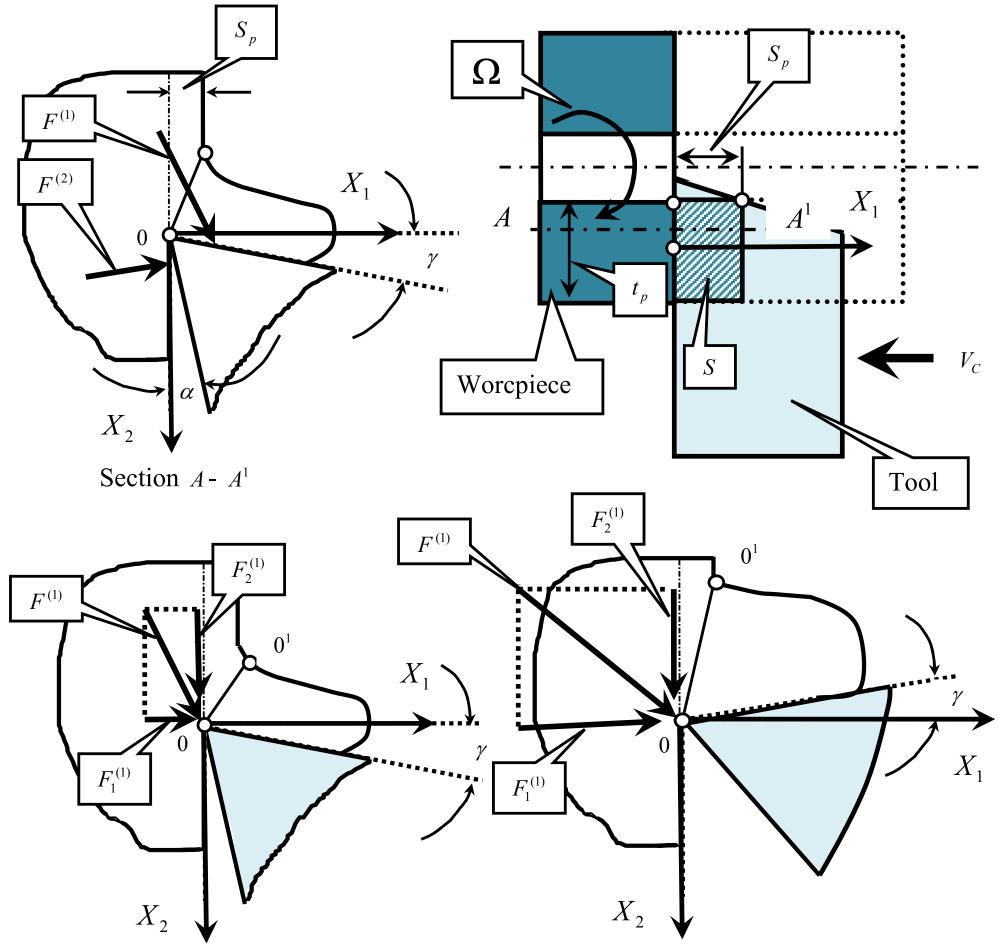

Fig. 1. The orientation of deformation displacements and forces acting on the cutting tool

3.1 Analysis of equilibrium

For the equilibrium point $X^* = \{X_1^*, X_2^*\}^T$ is valid

$$c_\Sigma X^* = F_\Sigma(S_P, V, V_C) \ , \qquad (8)$$

where

$$F_\Sigma = \{[\rho_0\chi_1(1+\mu e^{-\alpha_1 V})S_P + F^{(2)}(e^{\alpha_2 V_C}-1)], \\ [\rho_0\chi_2(1+\mu e^{-\alpha_1 V})S_P + F^{(2)}k(e^{\alpha_2 V_C}-1)]\}^T ;$$

$$c_\Sigma = \begin{bmatrix} c_{1,1}+\rho_0\chi_1(1+\mu e^{-\alpha_1(V)}) & c_{2,1} \\ c_{1,2}+\rho_0\chi_2(1+\mu e^{-\alpha_1(V)}) & c_{2,2} \end{bmatrix}.$$

The system (8) has a unique solution $X^* = \{X_1^*, X_2^*\}^T$. For further analysis it is necessary to consider the equations in variations with respect to (X_1^*, X_2^*) [32]. In equilibrium $X_1^* = Y_1^*$. After replacement $X_1 = X_1^* + x_1(t)$, $X_2 = X_2^* + x_2(t)$, $Y_1 = Y_1^* + y_1(t)$ from (7, 8), we obtain

$$\begin{cases} m\dfrac{d^2x_1}{dt^2} + h_{1,1}\dfrac{dx_1}{dt} + h_{2,1}\dfrac{dx_2}{dt} + c_{1,1}x_1 + c_{2,1}x_2 = \\ = \rho_0\chi_1\mu(S_P - X_1^*)e^{-\alpha_1 V}(e^{\alpha_1 dx_2/dt}-1) - \\ -\rho_0\chi_1(1+\mu e^{-\alpha_1(V-dx_2/dt)})y_1 + \\ + F^{(2)}e^{\alpha_2 V_C}(e^{-\alpha_2 dx_1/dt}-1); \\ m\dfrac{d^2x_2}{dt^2} + h_{1,2}\dfrac{dx_1}{dt} + h_{2,2}\dfrac{dx_2}{dt} + c_{1,2}x_1 + c_{2,2}x_2 = \\ = \rho_0\chi_2\mu(S_P - X_1^*)e^{-\alpha_1 V}(e^{\alpha_1 dx_2/dt}-1) - \\ -\rho_0\chi_2(1+\mu e^{-\alpha_1(V-dx_2/dt)})y_1 + \\ + F^{(2)}ke^{\alpha_2 V_C}(e^{-\alpha_2 dx_1/dt}-1); \\ T_1\dfrac{dy_1}{dt} + y_1 = x_1. \end{cases} \qquad (9)$$

Consequently, the linearized in the neighborhood of the equilibrium equation is

$$m_\Sigma\frac{d^2z}{dt^2} + h_\Sigma\frac{dz}{dt} + c_\Sigma z = 0 , \qquad (10)$$

where $m_\Sigma = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 0 \end{bmatrix}$;

$h_\Sigma = \begin{bmatrix} h_{1,1}+A_1 & h_{2,1}-A_2 & 0 \\ h_{1,2}+B_1 & h_{2,2}-B_2 & 0 \\ 0 & 0 & T_1 \end{bmatrix}$;

$c_\Sigma = \begin{bmatrix} c_{1,1} & c_{2,1} & A_3 \\ c_{1,2} & c_{2,2} & B_3 \\ -1 & 0 & 1 \end{bmatrix}$;

$x_1 = z_1$, $x_2 = z_2$, $y_1 = z_3$;

$A_1 = F^{(2)}\alpha_2 \exp(\alpha_2 V_C)$,

$A_2 = \rho_0 \chi_1 \mu \alpha_1 (S_P - X_1^*) \exp(-\alpha_1 V)$,

$A_3 = \rho_0 \chi_1 [1 + \mu \exp(-\alpha_1 V)]$,

$B_1 = F^{(2)}\alpha_2 k \exp(\alpha_2 V_C)$,

$B_2 = \rho_0 \chi_2 \mu \alpha_1 (S_P - X_1^*) \exp(-\alpha_1 V)$,

$B_3 = \rho_0 \chi_2 [1 + \mu \exp(-\alpha_1 V)]$.

The system (10) has constant parameters, so for the sustainability analysis it is sufficient to know the distribution of the roots of the characteristic polynomial. You can also select the stability area in the parameter space [33], which we will analyze in the consideration of bifurcations.

3.2 Bifurcation system

In (10) it is possible to have three steady states: an asymptotically stable equilibrium; a stable limit cycle; two-dimensional invariant TORUS. Let us consider a system whose parameters are given in the table. 1. The frequency of the system without the right part: $\omega_{0,1}^{(1)} = 1020 Hz$, $\omega_{0,2}^{(1)} = 345 Hz$. The main parameters affecting the stability of the: ρ_0, α_1, T_1. Note that ρ_0 can be varied by changing the width of the shear layer. Therefore, in this section α_1, α_2 and μ are assumed to be fixed. They are shown in the table. 2. Detailed description of the bifurcation is in the plane T_1, ρ_0 (Fig. 2). In the figure the gray portion of the area of asymptotic stability. The shaded area is the area of existence of two-dimensional invariant tori. White areas correspond to the formation of self-oscillations.

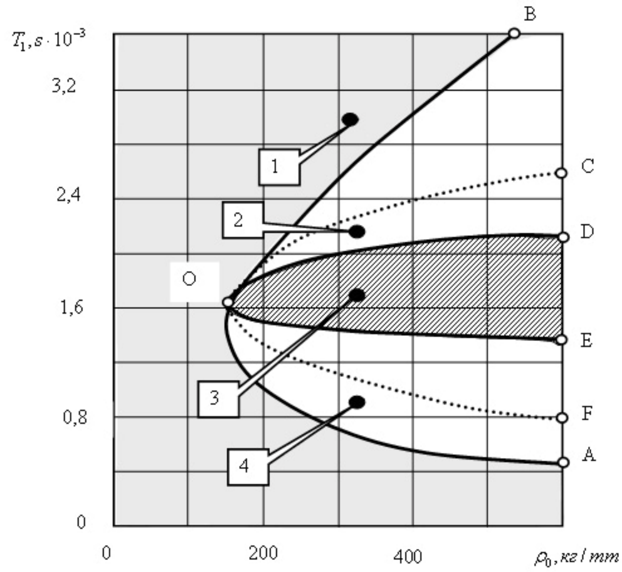

Fig.2.Bifurcation diagram in the plane T_1, ρ_0

Table 1.

$m, [кг \cdot c^2 / мм]$	$h, [кг \cdot c / мм]$	$c, [кг / мм]$
$\begin{bmatrix} 3{,}0 \cdot 10^{-3} & 0 \\ 0 & 3{,}0 \cdot 10^{-3} \end{bmatrix}$	$\begin{bmatrix} 5{,}0 & 1{,}2 \\ 1{,}2 & 1{,}5 \end{bmatrix}$	$\begin{bmatrix} 3000 & 50 \\ 50 & 360 \end{bmatrix}$

Table 2.

$\alpha_1 [c / мм]$	$\alpha_2 [c / мм]$	μ	$F^{(2)} [кг]$
$0{,}6 \cdot 10^{-3}$	0,5	0,5	5,0

Fig. 2 also shows the area of the D-decomposition. They divide the space into regions "B-O-C", "C-O-F», «F-O-A» and shaded areas where the equilibrium of the system is asymptotically stable. They differ in the number of roots of the characteristic polynomial of the linearized equations in the right half-plane. In this case, small deviations about the equilibrium point are considered. To highlight the scope, the characteristic equation of the system (10) with constant parameters was used. In the «B-O-C» and «F-O-A» there is one pair of complex conjugate roots with positive real roots in the «C-O-F» - two pairs of roots. The boundaries of the diagram of bifurcation, which produces two-dimensional invariant torus are displaced. Line "C-O" translates to a line "D-O" And "D-O" $\rightarrow$"E-O". This is due to changes in the parameters of the linearized along the trajectory equation. To confirm this, we can consider the temporal trajectory corresponding to the point "2" (Fig. 3).

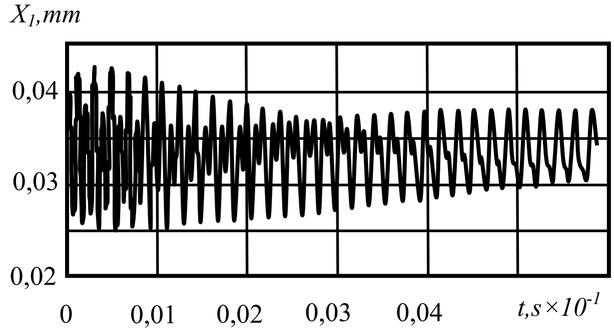

Fig. 3. The transformation of the torus in the limit cycle

We see the transformation of the torus in the limit cycle. Here on the interval (0 - 0,05) seconds high frequency oscillations are damped. Thor is converted into a limit cycle. A similar situation arises in the field of (E-O-F). However, in this case, the limit cycle is formed in the region of high frequencies. Note that high-frequency and low-frequency oscillations have different amplitudes. They are also oriented in different ways in space.

Fig. 4 shows examples of trajectories of stationary motions that correspond to points 1, 2, 3, 4 (Fig. 2). The left image of trajectories in-plane strain, the other in the projection of the deformation and strain rate on the phase plane. The line numbers correspond to selected points in Fig. 2, that is $\rho_0 = const$. Only T_1 varies.

As it can be seen, with increasing the T_1 system becomes unstable at first, due to phase shifts between the deformations and forces. With further increase it becomes sustainable because it significantly reduces the variable component forces. During the variation of the parameters we can identify areas in which the topology of the phase space remains unchanged and the lines of bifurcations in which topology changes. In a real system parameters vary due to evolutionary changes of the system associated with the irreversible transformation of input energy, resulting in, for example, the development of tool wear out, and, as a consequence - the change of the parameters of the dynamic link. In the manufacture of complex geometrical shapes along the trajectory of the tool relative to the work piece not only the parameters of the dynamic model of communication, but also the parameters of the subsystem of the tool and the work piece are changed.

Therefore, during processing, not only the displacement of the roots of the characteristic polynomial in areas of asymptotically stable equilibrium, but the change of topology of the phase space, i.e. the dynamic restructuring of the system are observed. The topology of the phase space is also significantly influenced by the parameter α_1 that, along with cutting speed, defines the second mechanism of loss of stability of the system. As for the coefficient α_2, it essentially limits the development of the periodic motions of the tool, but it does not affect the stability of equilibrium.

In the example, in a dynamic system there are three types of attractors: point, limit cycle or an invariant torus. Here, the elastic deformation of the tool always corresponds to the decrease of cutting forces. System properties fundamentally change if the deformation corresponds to the increase of cutting forces. Let's consider this case separately.

4 The influence of positive feedback

Elastic properties of interacting when cutting metals subsystems have different properties. In some cases, deformation of the work piece and the tool does not reduce forces, as in the example given above, and their increase. In this case, it contains a feedback loop that contributes to the loss of stability. This effect is observed when reducing the torsional rigidity of the work piece; when you change the orientation of the stiffness ellipse subsystems of the instrument; the same effect causes deformation of the bending tool.

From (7) it follows that the bending strain leads to the formation of a positive feedback, since the increase of deformation in the direction of cutting speed causes an increase in forces. This fundamentally changes the properties.

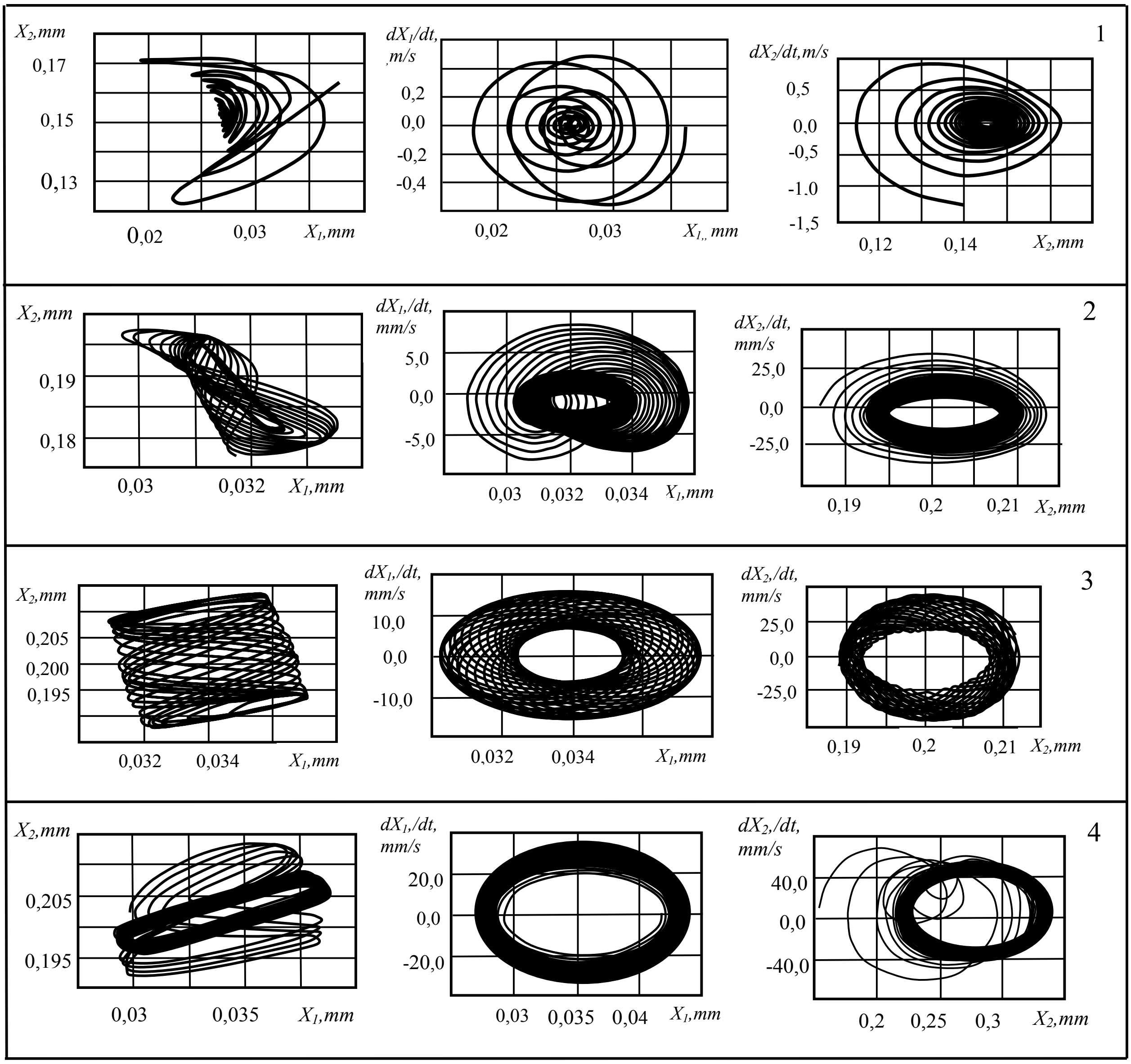

Fig. 4. Examples of trajectories in the sections of the phase space for the four points of the diagram in Fig. 2

1. Nonlinear dependence of force from the coordinates leads to branching of equilibration.

2. In the phase space coordinates of the system may periodically change from stable to unstable area. As a result, the system chaotic dynamics is produced.

Let us analyze the bifurcations of the system on a concrete example. Let us emphasize it is possible on the examples to define general properties of the system. Consider the system (7). The equilibrium is determined analytically. The trajectory is determined using the method of Runge-Kutta 4-th order. The application of this method was preliminary tested on examples with exact analytical solution. Thus the error in the calculation of the trajectories did not exceed 0.02%. It is on the two orders more exact than the exactitude of definition of parameters of dynamic model. Consider a system that has the following options:

$$m = \begin{bmatrix} 0,2\cdot 10^{-2} & 0 \\ 0 & 0,2\cdot 10^{-2} \end{bmatrix};$$

$$h = \begin{bmatrix} 1,2 & 0,8 \\ 0,8 & 2,0 \end{bmatrix}; c = \begin{bmatrix} 800 & 600 \\ 600 & 1000 \end{bmatrix}.$$

The dimension of the matrix elements, respectively: $[кg \cdot c^2 / мм]$, $[кg \cdot c / мм]$, $[кg / мм]$.

The parameters of the cutting process: $\chi_1 = 0{,}8$, $\chi_2 = 0{,}6$, $\rho_0 = 500\ kg/mm$, $S_P = 0{,}2\ mm$, $\alpha_1 = 0{,}6 \cdot 10^{-3}\ s/mm$, $T_1 = 0{,}002\ s$, $T_2 = 0{,}002 s$, $\mu = 0{,}5$, $F^{(2)}\alpha_2 k \exp(\alpha_2 V_C) \Rightarrow 0$, $\alpha = 200 kg/mm$.

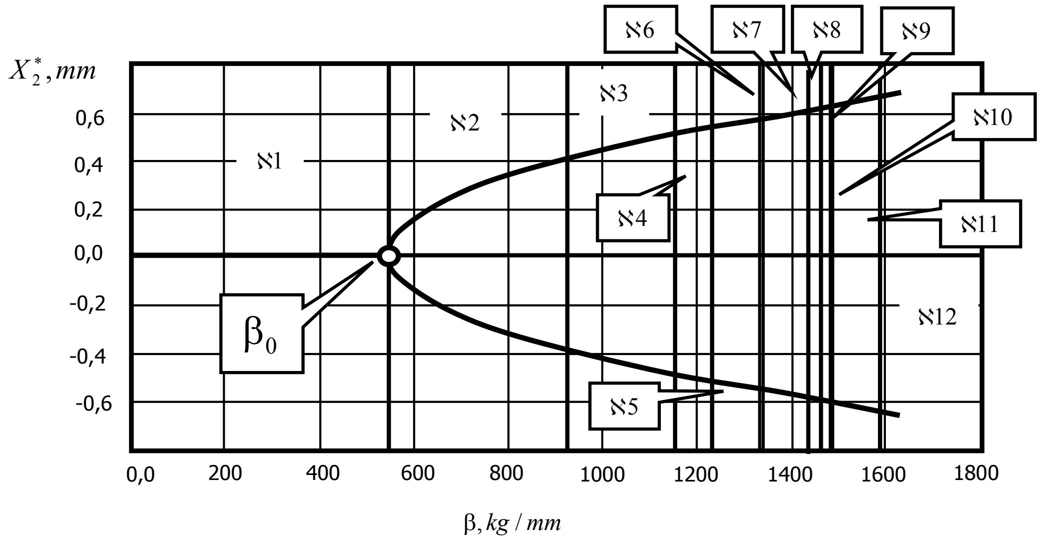

Fig. 5. The bifurcation diagram at varying β

From (7) we obtain the equation for the calculation of equilibrium of

$$A(X_2^*)^3 + BX_2^* + C = 0 \quad , \qquad (11)$$

where $A = (c_{1,2} + \rho_0 \chi_2^*)\alpha$;

$$B = c_{1,1}c_{2,2} - (c_{1,2})^2 + \rho_0(\chi_1^* c_{2,2} - \chi_2^* c_{1,2}) - -(c_{1,1} + \rho_0 \chi_1^*)\beta ;$$

$$C = S_P \rho_0 (\chi_1^* c_{1,2} - \chi_2^* c_{1,1}) ;$$

$$\chi_1^* = \chi_1 (1 + \mu e^{-\alpha_1 V}) ;$$

$$\chi_2^* = \chi_2 (1 + \mu e^{-\alpha_1 V}) .$$

In the present example $\chi_1^* c_{1,2} = \chi_2^* c_{1,1}$.

Fig. 5 shows the change of equilibrium in the system depending on the parameter β. First, as we increase the parameter β in the system is observed branching decisions. At point β_0, there is a bifurcation type plugs. Secondly, as we increase the parameter in a neighborhood of the equilibrium system are formed of different attractors. They, as a rule, are located symmetrically relative to the two extreme equilibriums. In the above illustration, the field with the same topology in the phase space indicated ($\aleph i$).The results of the numerical simulation allowed to determine the basic properties of the system in these areas. Let us characterize them.

Fig. 5 may be divided into several zones. Let us analyze them with increase of the parameter β. In the field to $\aleph 1$ a single equilibrium point is asymptotically stable. In area ($\aleph 2$), there are two asymptotically stable equilibrium points which have a limited field of attraction. Moreover, the stability of the equilibrium points is aperiodic. Then ($\aleph 3$) we can see Andronov-Hopf bifurcation, birth of orbitally asymptotically stable pair of limit cycles. Moreover, these limit cycles are symmetrical around the upper and lower equilibrium. They are arranged in a confined space located between two upper and lower branches.

With a further increase of β (set $\aleph 4$) a cascade of bifurcations of a period doubling can be seen. It is known that the period-doubling bifurcation characterizes one of the scenarios of birth of a chaotic attractor, considered by M. Feygenbaum [34]. Chaotic attractors are formed on the (set $\aleph 5$). Moreover, they are formed in the vicinity of the upper and lower equilibrium points, existing in a limited volume of the phase space. At the same time, they characterize attracting varieties. Further, by increasing the intensity of excitation, trajectories become orbital relative to all three equilibrium points. Firstly, the trajectory with a cascade of period-doubling is formed (set $\aleph 6$). Then, after the cascade of period doubling bifurcations, a chaotic attractor is newly formed (set $\aleph 9$). However, unlike the previously considered chaotic attractor, trajectory covers all three equilibrium points. Subsequently transformation cycles of stationary trajectories are periodically repeated (sets $\aleph 10$, $\aleph 11$). Finally, the system becomes unstable as a whole (set $\aleph 12$). However, in this case, a trajectories moving relative to a point of equilibrium, are in the spaces that do not overlap with each other.

Note that beginning with $\beta = 1200 \kappa g / mm$, the properties of the system are extremely sensitive to variations in this parameter. Some of the most typical examples of the projections of the phase trajectories on the plane $X_2 - dX_2/dt$ are shown in Fig. 6. Here it is necessary to pay attention to the fact that chaotic attractors at $\beta \in \aleph 4$ are formed only at two points in the area of equilibrium, while at $\beta \in \aleph 9$ they resemble the Lorenz attractor and have properties similar to this attractor.

In that case, if the condition $\chi_1^* c_{1,2} = \chi_2^* c_{1,1}$ is not met, then the system properties depending on the parameter β are becoming more diverse. In addition, they are not symmetric in respect to the equilibrium points. For example, in the neighborhood of one point of equilibrium may cause the formation of chaotic attractors, and in the vicinity of the second point is formed by a stable limit cycle. Then the properties of the system depend on the initial conditions. The domain of attraction of the attractors is limited and she has a complex configuration in the phase space. Dynamic properties of the system are sensitive to small variations of the parameters. Depending on the perturbations they can spontaneously change.

5 Discussion of the results

Dynamic cutting system is an example of the complex behavior of the system. Its properties depend on the characteristics of elasticity of the tool and work piece, as well as on the dependence of the forces from the coordinates. In studies of the dynamics of the cutting process is focused on the stability of equilibrium and generated during cutting of self-oscillations. In these studies only a single mechanism of self-excitation in the system is typically analyzed. In our study, first, we shall consider several sources of self-excitation, secondly, we shall analyze the case when the elastic deformations of the tool lead to an increase in cutting forces. This fundamentally changes the properties of the system. They complement the well-known properties of the nonlinear dynamics of cutting process and characterize new knowledge about the process of cutting materials. Let us list them.

1. Two sources of self-excitation system and two voting circuits with different frequencies lead to the formation in the vicinity of the equilibrium invariant tori. They exist in a limited parameter space of the system. When a certain combination of parameters of two-dimensional invariant of the tori is converted into limit cycles. We emphasize that it is consideration of the two sources of self-excitation leads to the formation of invariant tori, what distinguishes the above data from previous studies.

2. If the deformation of the tool is accompanied by bending of the cutting blade, it is produced in the system of nonlinear positive feedback. In this case, in a dynamic system of cutting, depending on the level of excitation it is possible branching of the equilibrium points. For the subject area that is of fundamental importance. First, the coordinate of the equilibrium point determines the current value of the size of the work piece. Secondly, in the process of cutting, for example, through the development of tool wear, changes of the parameter β can reach the point of bifurcation of equilibrium. In this case, an accurate prediction of changes in the geometry of the work piece is not possible. This circumstance has not previously been emphasized.

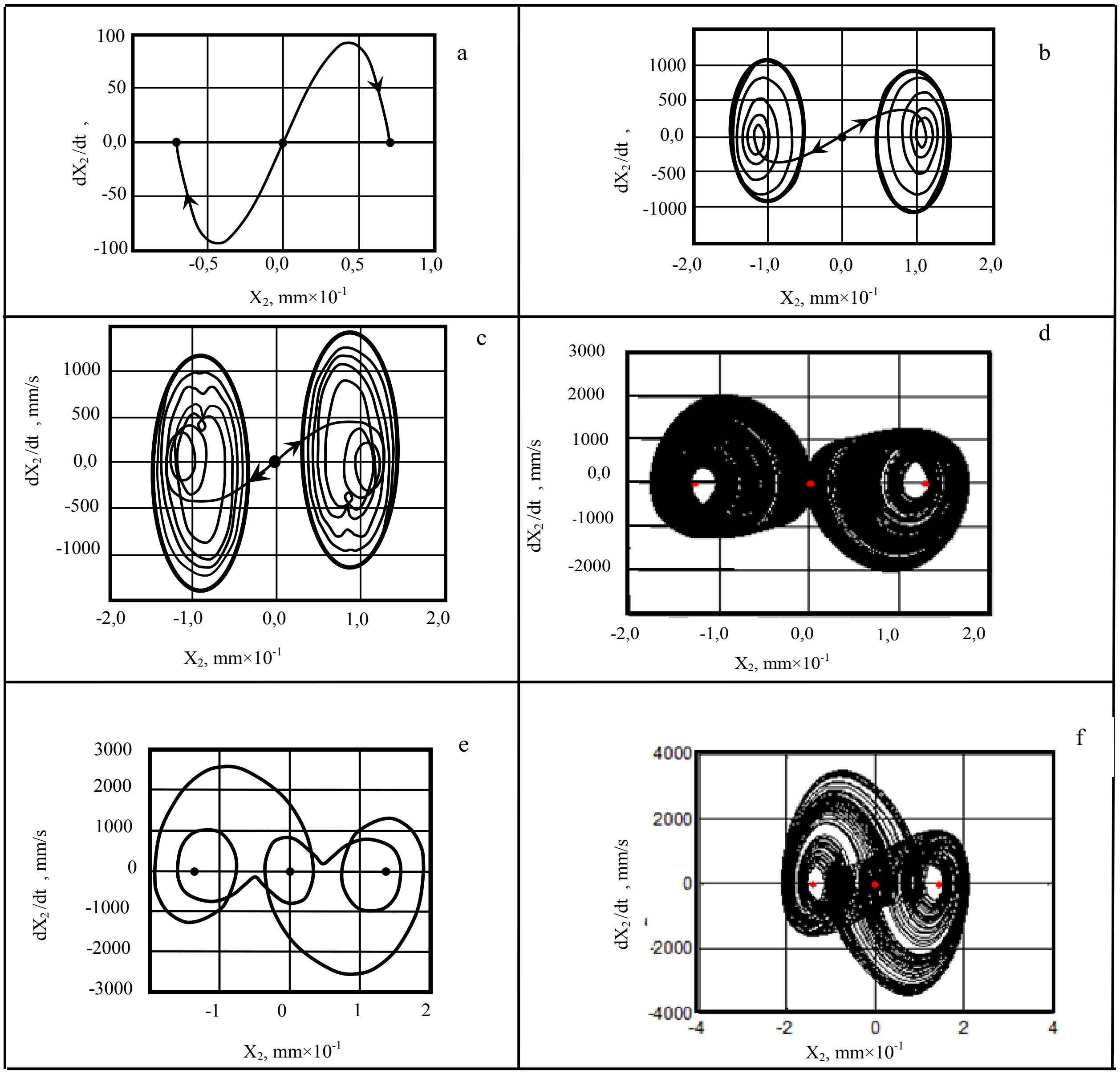

Fig. 6. Some examples of projections of stationary trajectories on the phase plane $X_2, dX_2/dt$ with increasing of parameter β: a - $\aleph 2$; b - $\aleph 3$; c - $\aleph 4$; d - $\aleph 5$; e - $\aleph 6$; f - $\aleph 9$

3. In addition to the branching of the equilibrium points depending on the parameter β observed change of attractors and the formation of chaotic oscillations. In addition, chaotic oscillations, as a rule, have a limited area around the point of equilibrium. It should be noted that the type of the attractor directly affects the properties of the metal cutting process. For example, in this model, nonlinear periodic oscillations cause the dynamic displacement of the point of equilibrium and directly affect the size of the workpiece. We emphasize that in the previous research the chaotic attractors were analyzed in systems with retarded arguments, and in the Grinding Process, taking into consideration the forces depend on the velocity [13] - [16]. In this article it is shown that chaotic behavior is possible due to the formation in the process of cutting the nonlinear positive feedback. Let us emphasize that the mechanism of a positive feedback does not matter.

An interesting feature of the system is periodic alternation of chaotic and regular oscillations with increasing feed. The formation of chaotic attractors is greatly influenced by the parameter α_2. Chaotic oscillations appear under the condition that

$\alpha_2 \rangle\rangle \alpha_1$. Fig. 6 shows the steps of converting regular fluctuations into chaotic ones.

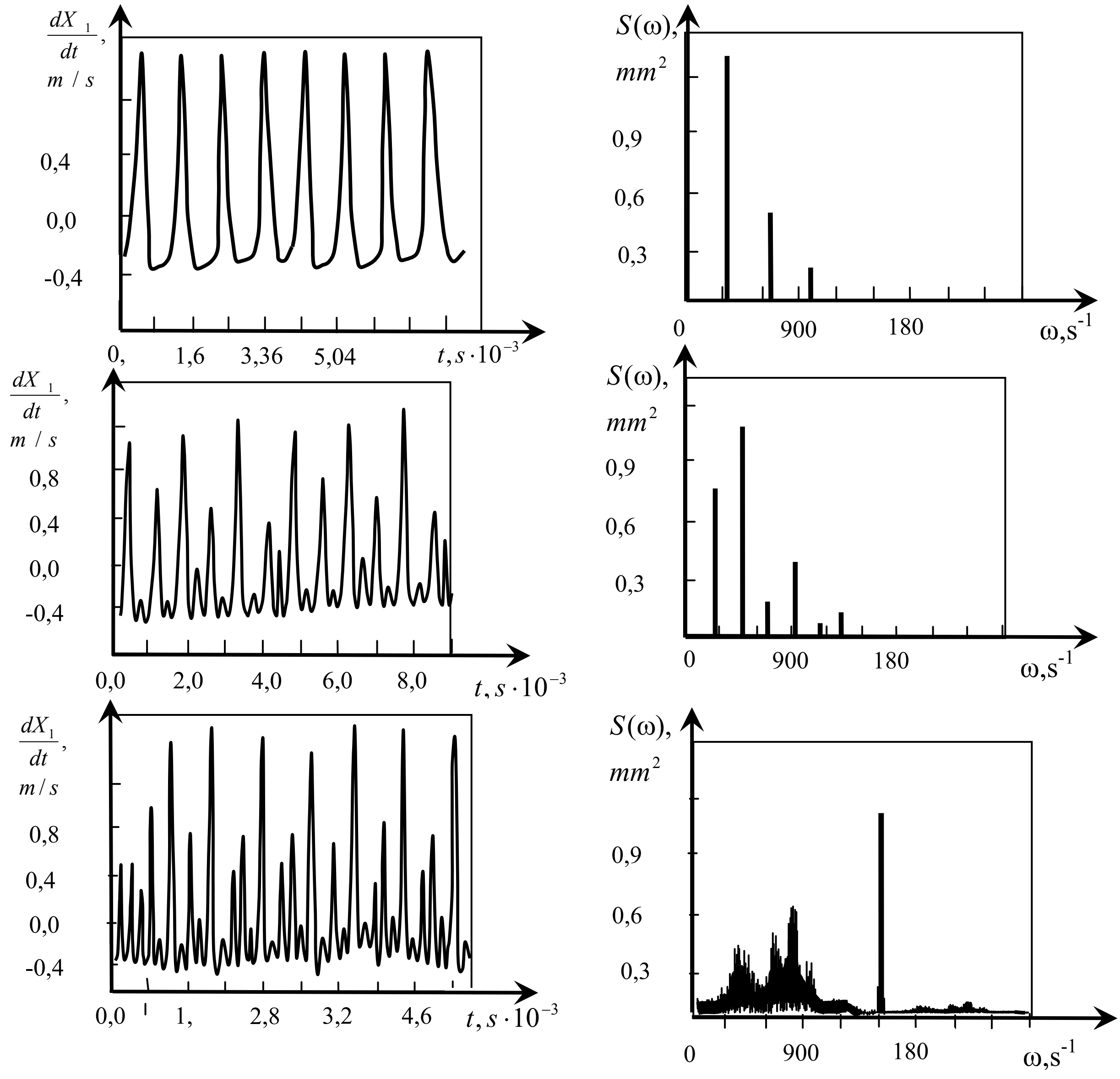

Fig. 7. Example of converting regular fluctuations in chaotic ones

Here, the time synchronization of excitation and decay leads to the formation of regular oscillations (upper illustration). The stiffness changes due to bending of the tool and, consequently, the frequency of the oscillator formed the main component of the cutting force. As a result, the time intervals, at which stability is been lost, go astray and the system generates chaos. In addition, the transition to chaos occurs through a cascade of bifurcations of period-doubling (see middle picture) (see Fig. 7).

The lack of self-excitation system in the high cutting speeds is associated with the fact that in this case two self-excitation sources of the system discussed in the article practically disappear. However, in this case, the parametric phenomena caused by periodic changes in parameters are of fundamental importance. However, this problem is not discussed in this article.

4. These materials also allow us to reveal general patterns of loss of stability of the trajectories of motion of mechanical systems interacting with a variety of mediums. These properties have not been disclosed to date. Let us reveal them the on the example of the stability of the system (7) for the case $T_1 = 0$. Then (10) is transformed into

$$m\frac{d^2x}{dt^2} + h_\Sigma \frac{dx}{dt} + c_\Sigma x = 0\,, \qquad (12)$$

where $m = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$;

$$h_\Sigma = \begin{bmatrix} h_{1,1} + A_1 & h_{2,1} - A_2 \\ h_{1,2} + B_1 & h_{2,2} - B_2 \end{bmatrix};$$

$$c_\Sigma = \begin{bmatrix} c_{1,1} + A_3 & c_{2,1} \\ c_{1,2} + B_3 & c_{2,2} \end{bmatrix}.$$

Matrix h_Σ and c_Σ are asymmetric, i.e., they can be represented as a sum of symmetric and skew-symmetric components. Thus is formed the acceleration, dissipative, gyroscopic and circulation components of the cutting force [35, 36]. The basic mechanism of the instability is associated with the formation of the accelerating force, which forms the symmetric part of the matrix of speed ratios. A skew-symmetric component of this matrix forms gyroscopic forces. They can't stabilize the equilibrium if it is unstable [35, 36].

Taking into account the retardation of forces with respect to the deformation displacements ($T_1 \neq 0$), the conversion of the symmetric part of the matrix of speed coefficients increases from a positive definite to negative definite. At the point of equilibrium in the traditional modes of cutting, forces acting on the rear face of the tool, can be neglected, i.e. $F^{(2)}\alpha_2 \exp(\alpha_2 V_C) \Rightarrow 0$. Then, the necessary condition for the stability of the system (12), is defined by the requirements

$$h_{1,1}h_{2,2} - \rho_0\chi_1\mu\alpha_1(S_P - X_1^*)h_{1,1}\exp(-\alpha_1 V) - \\ -[h_{1,2} - 0{,}5\rho_0\chi_2\mu\alpha_1(S_P - X_1^*)h_{1,1}\exp(-\alpha_1 V)]^2 \rangle 0 .$$

Thus, the loss of stability depends not only on ρ_0 and α_1, but also on the conditions S_P, V, as well as tool geometry which determines χ_1, χ_2. The matrix c_Σ can also be represented as a sum of symmetric and skew-symmetric components $c_\Sigma = c_\Sigma^{(c)} + c_\Sigma^{(k)}$. A skew-symmetric component

$$c_\Sigma^{(k)} = \begin{bmatrix} 0 & \Leftrightarrow \quad -0{,}5\rho_0\chi_2[1 + \mu\exp(-\alpha_1 V)] \\ 0{,}5\rho_0\chi_2[1 + \mu\exp(-\alpha_1 V)] & \Leftrightarrow \quad 0 \end{bmatrix}$$

as is known, forms a circulation force orthogonal to the direction of deformation of the offset. Therefore, the tool paths are always similar to an ellipse. This experimentally observed feature is observed by most of researchers.

It is necessary to take into account that these characteristics are quite common for systems that interact with various environments, such as the studies of dynamics of mechanical systems interacting with tribological environment. A more in-depth study of models of contact of interaction forces during cutting and friction in the coordinate system for analyzing the dynamic properties of the systems at the design stage is necessary. In addition, fluctuations of the parameters accompanying the process of cutting, actually characterize important information base that shows the direction of the dynamic monitoring of the state of the system dynamic parameters [37 - 40].

6 Conclusion

The analysis of the system in metal cutting tools gives us an example of a complex behavior. In this system depending on the parameters found the following effects:

- the branching equilibrium;
- the creation in local areas of self-oscillations and invariant tori;
- the formation of chaotic dynamics.

The above diversity affects the quality of the cutting process and output characteristics of the workpiece. However, it is not clear until now, for example, what is the impact of oscillations on the wear resistance of the tool. Until now there is conflicting experimental data about the effects of vibration on tool wear and quality indicators of the workpiece. There is no data on the influence of the chaotic dynamics of the cutting process. These issues require further research.

The branching properties of the system depend on non-linear dependency of force from the deformation tool offset and nonlinear elastic properties of the subsystems of the instrument. Therefore, further studies will be devoted to a thorough identification of the parameters of these nonlinear dependencies. For this we will use methods of experimental dynamics, proposed in our works [3], [9], [12].

The definition of the dynamic state of the system also defines a new information base for dynamic condition monitoring system directly in the cutting process [36] – [40]. The developed models and methods of analysis allow us to define the information model for the diagnosis of cutting process by measuring the vibration of the tool. The principles of building information models also characterize the subject of our further research. These results are very general, and methods of dynamic analysis of a segment may be extended in the case of the analysis of dynamics of mechanical systems that interact with various environments, for example, when studying the motion of mechanical systems with friction.

Acknowledgements

This paper was completed with the financial support by the Russian Foundation for Basic Research (Grant No. 14-08-00206).

References:

[1]A. Masoumi, F. Pellicano, F. S. Samani, M. Barbieri. Symmetry breaking and chaos-induced imbalance in planetary gears. Nonlinear Dynamics. Volume 80, Issue 1-2 (2015) 561-582.

[2]Q. Chang, L. Hou, Z. Sun, Y. Wei, Y. You. Nonlinear Modeling of Helical Gear Pair with Friction Force and Frictional Torque. WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, Volume 9 (2014) 264 – 274.

[3]V. Zakovorotny, M. Fleck. Dynamics of the cutting process. Synergetic approach. Rostov-on-Don: Publishing House "Terra" (2006), 876 (In Russian).

[4]V. Zakovorotny. Dynamics of tribosystems. Self-organization, evolution, Publishing Center of the Don State Technical University, (2003), 502 (In Russian).

[5]J. Tlusty, A. Polacek, C. Danek, & J. Spacek. Selbsterregte Schwingungen an Werkzeugmaschinen,VEB Verlag Technik, Berlin (1962).

[6]J. Tlusty. Manufacturing Processes and Equipment. Prentice Hall, NJ (2000).

[7]S.A. Tobias, Machine Tool Vibrations. Blackie, London, (1965).

[8]V.A. Kudinov. Dynamics of machine tools. Moscow, Publishing house "Engineering", (1967) 359 . (In Russian).

[9]V.L. Zakovorotny V.L., P. D. Tung, N. X. Tem. Mathematical modeling and parametric identification of the dynamic properties of the subsystem tool and work piece. News of Higher Educational Institutions. The North Caucasus region. Series: Engineering. № 2. (2011) 38-46. (In Russian).

[10]G. Stepan Delay-differential equation models for machine tool chatter, Nonlinear Dynamics of Material Processing and Manufacturing, ed. Moon, F. C. (John Wiley, NY), (1998) 165-192.

[11] G. Stepan, T. Insperge and R. Szalai. Delay, Parametric excitation, and the nonlinear dynamics of cutting processes. International Journal of Bifurcation and Chaos, Vol. 15, No. 9 (2005) 2783-2798.

[12] V.L. Zakovorotny V.L., H. P. Thy. Parametric self-excitation of the dynamic system of cutting. Bulletin DSTU, № 5-6 (74-75), (2013) 97 - 104. (In Russian).

[13]G. Litak G. Chaotic vibrations in a regenerative cutting process. Chaos, Solitons & Fractals (2002) 1531-1535.

[14]I. Grabec. Chaotic dynamics of the cutting process. J Mach Tools Manufact (1988) 19-32.

[15]M. Wiercigroch. Chaotic vibrations of a simple model of the machine tool-cutting system. ASME J. Vibr. Acoust. (1997); 119: 468-75.

[16]Nicolae-Doru Stanescu. Chaos in Grinding Process. WSEAS Transactions on Applied and Theoretical (2009), Issue 4, Volume 4, October, p.p. 195 – 204.

[17]V.L. Zakovorotny, A.D. Lukyanov, Dong Anh Nguyen, Pham Dinh Tung. Synergetic system controlled synthesis of dynamics of machine tools, taking into account the evolution of relations. Rostov-on-Don: Publishing House DSTU. (2008) 324 (In Russian).

[18]K. Robenack, D. Ahmed, S. Eckhardt, C. Gottlober. Peripheral Milling of Wooden Materials without Cutter-Marks – A Mechatronic Approach. WSEAS TRANSACTIONS on

APPLIED and THEORETICAL MECHANICS, issue 2, Volume 8, (2013) 170 -179.

[19]Wei Ji, X. Liu, L. Wang, S. Sun. Experimental evaluation of polycrystalline diamond (PCD) tool geometries at high feed rate in milling of titanium alloy TC11. The International Journal of Advanced Manufacturing Technology Issue 9-12, Volume 77, (2015) 1549-1555.

[20]X. Cui, J. Guo, J. Zhao, Y. Yan. Chip temperature and its effects on chip morphology, cutting forces, and surface roughness in high-speed face milling of hardened steel. The International Journal of Advanced Manufacturing Technology, Volume 77, Issue 9-12, (2015) 2209-2219.

[21]Y-C. Kao, N-T Nguyen, M-S. Chen, S-T. Su. A prediction method of cutting force coefficients with helix angle of flat-end cutter and its application in a virtual three-axis milling simulation system. The International Journal of Advanced Manufacturing Technology. Volume 77, Issue 9-12, (2015) 1793-1809.

[22]E. Mathieu. Mémoire sur le mouvement vibratoire d'une membrane de forme elliptique," J. Math.Pure Appl. 13, (1868) 137-203.

[23]M.G. Floquet. Equations différentielles linéaires à coefficients peridiques, Ann. Scientifiques de l'École Normale Supérieure 12, (1883) 47-89.

[24]J.G. Kabaldin. Self-organization and nonlinear dynamics in the processes of friction and wearout of the tool during cutting. Publishing center of Komsomolsk on Amur state technical University, (2003) 175. (In Russian).

[25] L.S. Murashkin, S.L. Murashkin. Applied nonlinear mechanics of metal cutting machines. Leningrad, publishing house "Engineering", (1977). 192. (In Russian).

[26] Van der Pol, F. & Strutt, MJO. On the stability of the solutions of Mathieus equation, Philos. Mag. J. Sci. 5, (1928), 18-38.

[27]WT. Corpus, WJ. Endres. Added stability lobes in machining processes that exhibit periodic time variation - Part 1: An analytical solution, J. Manuf. Sci. Engin. 126, (2004) 467-474.

[28]AM. Gouskov, SA Voronov. Paris, H. & SA Batzer Nonlinear dynamics of a machining system with two interdependent delays, Commun. Nonlin. Sci. Numer. Simul. 7, (2002) 207-221.

[29]J. Warminski, G. Litak G., MP Cartmell, R. Khanin, M. Wiercigroch Approximate analytical solutions for primary chatter in the non-linear metal cutting model. Journal of Sound and Vibration (2,003) 259 (4) 917-933.

[30]V.L. Zakovorotny, D.T. Pham, S.T. Nguyen. Modeling of deformation tool offset relative to the work piece during turning. Bulletin Don State Technical University. № 7, (2010) 1005-1015. (In Russian).

[31]J. Yongjun, J. Zhang, X. Guan**.** Theoretical Calculation and Experimental Analysis of the Rigid Body Modes of Power train Mounting System. WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, Issue 3, Volume 8, (2013) 193 – 201.

[32] A.M. Lyapunov. The general problem of stability of motion. Collected works. Vol. 2, Publishing house of the USSR Academy of Sciences. (1959) 36 – 67. (In Russian).

[33] D.R. Merkin. Introduction to the theory of stability of motion. Moscow, Publishing House "Science", (1971), 495. (In Russian).

[34]MJ. Feigenbaum. Quantitative universality for a class of nonlinear transformations. J. Stat. Phys., (1978), v. 19, 25 - 52.

[34] V.M. Lihadanov. On the influence of the structure of power in the resistance movement. Journal of applied mechanics and mathematics, vol. 38, (1974) 246 -253. (In Russian).

[35]V.M. Lihadanov. Stabilization of potential systems. Journal of applied

mechanics and mathematics, vol. 38, (1975), v. 39 53 - 58. (In Russian).

[36]V.L. Zakovorotny, E.V. Bordachev, M.I. Alekseichik. Dynamic monitoring of the status of the cutting process. STIN. 1998. № 12. 6-12. (In Russian).

[38]V.L. Zakovorotny, I.V. Ladnik. Building information model of a dynamic system of a machine tool for the diagnosis of the treatment process. Problems of mechanical engineering and reliability of the machines, №4 (1991) 75 - 79. (In Russian).

[39]V.L. Zakovorotny, E.V. Bordachev. Information support of a system of dynamic diagnostics of the tool wearout on the example of turning. Problems of mechanical engineering and reliability of the machines, №3, (1995) 95 - 101. (In Russian).

[40]J. Karandikar, T. McLeay, S. Turner, T. Schmitz. Tool wear monitoring using naïve Bayes classifiers. The International Journal of Advanced Manufacturing Technology, Volume 77, Issue 9-12, (2015) 1613-162