Probability Calculation for Natural frequency of Anisotropic Stator Core of Large Turbo-Generator

YIXUAN WANG
College of Mechanical and Electrical Engineering, Xi'an Polytechnic University
19 Jinhua South Road ,Xi'an City,P.R.China E-mail：wyingping886@126.com

Abstract: - Techniques for vibration reduction and design of stator core require knowledge of its modal frequencies, which depend on the geometry shapes, dimensions and material properties. The material properties of stator core are mainly related to its laminated process and assembly as well as the operating temperature of turbo-generator. It is found that the generally accepted value of material properties is not valid for the stator core with laminated silicon steel slice and operating at about 110-120 Celsius. This paper simply summarizes the method of determining elastic constants of stator core material, and then introduces a simple and effective method for the calculation of the modal frequencies of stator core. The material model of stator core is thought anisotropic. The elastic modulus, Poisson's ratio, and mass density are all considered as uncertainty variables, obeying Gaussain distribution, and changing in the possible range; and then ANSYS workbench 14.5 program is used to determine the resonant frequencies and corresponding probability of stator core ,and response surfaces of natural frequencies vs. elastic constants by statistical method (Six Sigma Analysis). The results have an important guiding role for the laminated structure design, manufacture process, the fixedness and the vibration isolation design of stator core.

Key-Words: - Large turbo-Generator, Anisotropic Stator Core, Material Properties, Natural Frequency, Six Sigma Analysis

1 Introduction

1.1 Previous Works

Vibration and acoustic noise in the large turbo-generator is mainly caused by the oval deformation of the stator lamination stack due to its radial magnetic attraction to the rotor [5, 6, 9] .There are several ways of determining stator core resonant frequencies and mode shapes of the large turbo-generator: analytical calculations[7,8,10,18], numerical computation[1,2,11,12] and experimental measurements[17].

The paper [15] proposes a measurement method of the Young modulus, shear modulus, mass density for stator core of switched reluctance motor. However, the Young modulus value of stainless steel ($E=2.07\times10^{11}N/m^2$) has been widely used by researchers in small electric machine vibrations area.

The fact is that the stator lamination steel of a large turbo-generator is different from stainless steel; in addition, the material properties may change during core lamination, manufacturing, and vary with operation temperature of turbo-generator. In fact, the Young modulus value of stator core varies with exciting current, temperature and structure shape. In other words, the elastic constants and mass density of the stator core is actually uncertain variables, distributed in an interval, subject to a statistical distribution law; so the natural frequency value for the linear system should be subject to the same distribution. If Finite Element Method (FEM) is used to model, the elements of mass matrix and stiffness matrix are random variables, and the conventional method cannot be used for the calculation of natural frequency of stator core, but the method of probability statistics; for stator core of large turbine generator, the literature in this area are rare. Because of the complexity of the material characteristics of stator core, in the calculation of the natural frequency of the stator core of large turbine generator, the all researchers consider that the material constants are unchanged. This paper is an in-depth research.

In the author's paper[3, 4],we consider that the stator core for large turbo-generator is isotropic body ,so the calculation of mechanical characteristic needs only the elastic modulus and Poisson's ratio, and material properties is considered as uncertain variables .

In fact, the stator core assembly of large turbo generator is an anisotropic body. A new nonlinear anisotropic model for soft magnetic materials is proposed for determining electro-magnetic properties of electromagnetic device [13]and mechanical properties for permanent synchronous motor[14],but these methods are only applicable to a small motor, and the properties are all constant. For 125MW large hydro-generator, the papers [16, 17] consider that the stator core is orthotropic body, and could be simplified as transversely isotropic body, but mechanical properties are considered as constant.

1.2 The Works of This Paper

On the basis of author's paper[3,4], this paper consider that stator core of 1000MW large turbo-generator is simplified as orthotropic anisotropic body, and the elastic constants are uncertain variables, distributed in an interval, subject to a statistical distribution law(normal distribution); the natural frequency and vibration mode are calculated by means of Six sigma analysis method on ANSYS Workbench 14.5.

Because of considering the change of the material constants and uncertainty in the process of the actual operation of generators, so ones can calculate the each order natural frequency of the stator core and the corresponding probability.

Compared with previous work, material model and calculation model in this paper is more complex, and more close to the operating condition of the large turbo generator, the result is more accurate ,reliable and practical. The increase in the amount of calculation is not much. These results have an important guiding role for the laminated structure design and manufacture process of stator iron core, and the fixedness and the vibration isolation design of stator core.

2 Structure and Simplified Mechanics Model of Stator Core

2.1 Structure of Stator Core

The inner stator of large turbo-generator consists of stator winding, stator core, inner stator cage and locating rib, etc., as shown in Fig.1. As the result of the laminating and assembly technology of stator core, the stator iron cores in the turbo generator are generally laminated with punched fan sheets of nonlinear soft-magnetic silicon steel, and constituted by the dislocation seam laminated body(see also Fig.2 and Fig.3). The pretightening force is applied to two ends of stator core through clamp plate, clamp ring and long through bolt. The material coordinate system of stator core is shown in Fig.3 In the paper, the stator core is modeled as laminated structure in ANSYS (see also Fig.3) and 3D solid in ANSYS workbench (see also Fig.4) respectively, so that it's mechanical characteristic can be calculated easily.

2.2 Constraint conditions of Stator Core

The fixedness of stator core in stator frame is divided into three cases. First the stator core is directly stacked through the positioning ribs (generally consist of about eighteen), and these is directly fixed on ring plate of stator frame middle wall (see also Fig.14 in [1]).So the stator core is considered as fixed along the axial of locating ribs. The fixed type is only used for small capacity turbo-generator.

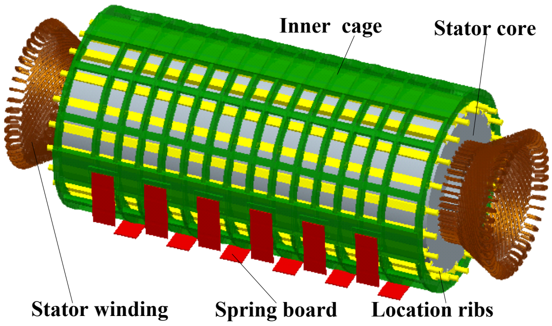


Fig.1 Turbo-generator inner stator assembly

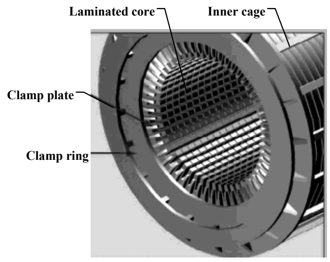
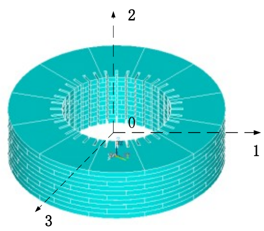


Fig.2. Laminated structure of stator core



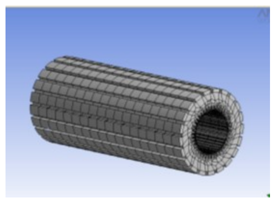


Fig.3Laminated core in ANSYS Fig.4 Solid core in workbench

Second the stator core is directly stacked through the positioning ribs, and formed as a whole. In order to reduce the vibration by the stator core transferring to the frame and foundation, flexible coupling or the isolation device is used between the frame and the stator core, which a combination of horizontal axial spring plate is usually used as the vibration isolation structure and fixed at the back of locating ribs, and then connect with stator frame (see also Fig.14 in [1]). So the stator core is considered as supported along the axial spring plate. The supported type is normally used for middle capacity turbo-generator (below 300MW).

Third the stator core is stacked in the inner base formed by the positioning ribs, their forming as a whole called as inner stator, and then into the stator frame. In order to reduce the vibration by the stator core transferring to the frame and foundation, flexible coupling or the isolation device is used between the frame and the inner stator, which a combination of tangential vertical spring plates is usually used as the vibration isolation structure(see also Fig.19 in [1]). So the stator core is considered as supported along tangential vertical spring plates .The supported type is generally used for large capacity turbo-generator (above 300MW).

As shown above, the boundary conditions of stator core are complex. Our study is for stator core of large turbo-generator, so the stator core is considered as supported by elastic plate. The simplified mechanics model is a thick circular cylindrical shell resting on an elastic foundation and in certain cases can be viewed as resting on distributed elastic springs that have spring rates k1, k2, and k3 and act in three different directions [12]. The paper doesn't discuss the situation, in other thesis of the author the problem will research in depth.

In this paper, the stator core is considered as free-free boundary conditions, there are several reasons, one is because natural frequencies of a thick circular cylindrical shell resting on an elastic foundation can be calculated as follows[12],

$$\omega_F = \frac{\omega_0 \rho h + k}{\rho h + \frac{1}{3}\rho_F h_F} \tag{1}$$

Where ω_F are the natural frequencies of the shell on an elastic foundation, and ω_0 are the natural frequencies of the same shell without the elastic foundation. Thus, as expected, the natural frequencies increase with k, with the larger percentual increase for the lower natural frequencies. The mass of the elastic foundation tends to lower all natural frequencies, but not enough for a typical foundation that it would override the stiffness effect. When an elastic foundation is neglected, the difference of natural frequencies of stator core should be small. The other is that stator core includes more modal information under free-free boundary conditions according to the Rayleigh constraint law of characteristic value of a system. The other reason is that the experiment modal analysis of stator core is easily done under free-free boundary conditions, so that compared with the calculated results.

2.3 Material Models of Stator Core

As above, the structure and technology of stator core is very complicated. Generally, the material properties of stator core are non-homogenous anisotropic， changes with operating temperature and uncertainty variables. Otherwise in paper [16,17], the stator core is considered as transversely isotropy, the calculation results of modal frequencies were compared with isotropic material ones ,it showed that the difference of natural frequencies should be small . In this paper, material properties of stator core are handled by three cases, one is considered as homogenous isotropic and certainty constant [3, 4], second case is homogenous isotropic and uncertainty variables, and third case is that stator core of large turbo generator is simplified as orthotropic body and uncertainty variables.

2.4 Anisotropic Material Models of Stator Core

An isotropic body is one for which every material property in all directions at a point is the same. An anisotropic body is one that has different values of a material property in different directions at a point; i.e., material properties are direction-dependent.

In general, the stator core is considered as anisotropic material. The linear constitutive model for infinitesimal deformation is referred to as the generalized Hooke's law. Suppose that the reference configuration has a (residual) stress state of σ^0. Then if the stress components are assumed to be linear functions of the components of strain, then the most general form of the linear constitutive equations for infinitesimal deformations is [20]

$$\sigma_i = C_{ij}\varepsilon_j + \sigma_j^0 \quad i,j = 1,2......6 \tag{2}$$

Where C_{ij} is termed stiffness matrix, σ_i stress column vector, ε_j strain column vector, and σ_j^0 a residual stress state. In matrix notation, Eq. (2) can be written as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} + \begin{Bmatrix} \sigma_1^0 \\ \sigma_2^0 \\ \sigma_3^0 \\ \sigma_4^0 \\ \sigma_5^0 \\ \sigma_6^0 \end{Bmatrix} \tag{3}$$

If we also assume that the material is hyper elastic, and initial stress σ_i^0 is omitted, the coefficient C_{ij} must be symmetric

($C_{ij}=C_{ij}$) .Hence, we have 21 independent stiffness coefficients for the most general elastic material.

When the elastic coefficients at a point have the same value for every pair of coordinate systems which are the mirror images of each other with respect to a plane, the material is called a monoclinic material. Thus out of 21 material parameters, we only have 13 independent parameters, as indicated below.

$$[C]=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{21} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{31} & C_{32} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{54} & C_{55} & 0 \\ C_{61} & C_{62} & C_{63} & 0 & 0 & C_{66} \end{bmatrix} \quad (4)$$

Since the laminated stator core is formed by a silicon material sheet, having an axis of symmetry properties, i.e., having three mutually perpendicular plane of symmetry. When three mutually orthogonal planes of material symmetry exist, the number of elastic coefficients is reduced to 9 using arguments similar to those given for single material symmetry plane, and such materials are called orthotropic. The stress-strain relations for an orthotropic material take the form

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}=\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix} \quad (5)$$

Most simple mechanical property characterization tests are performed with a known load or stress. Hence, it is convenient to write the inverse of relations in (5). The strain-stress relations of an orthotropic material are given by

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}=\begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix}\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (6)$$

Where S_{ij} are the compliance coefficients ($[C]=[S]^{-1}$)

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}=\begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (7)$$

where E_1, E_2, E_3 are Young's moduli in 1, 2, and 3 material directions, respectively, ν_{ij} is Poisson's ratio, defined as the ratio of transverse strain in the j th direction to the axial strain in the i th direction when stressed in the ith direction, and G_{23}, G_{13}, G_{12} are shear moduli in the 2-3, 1-3, and 1-2 planes, respectively. Since the compliance matrix [S] is the inverse of the stiffness matrix [C] and the inverse of a symmetric matrix is symmetric, it follows that the compliance matrix [S] is also a symmetric matrix. This in turn implies that the following reciprocal relations hold

$$\frac{\nu_{21}}{E_2}=\frac{\nu_{12}}{E_1};\ \frac{\nu_{31}}{E_3}=\frac{\nu_{13}}{E_1};\ \frac{\nu_{32}}{E_3}=\frac{\nu_{23}}{E_2}$$

or, in short

$$\frac{\nu_{ij}}{E_i}=\frac{\nu_{ji}}{E_j} \quad \text{(no sum on i , j)} \quad (8)$$

For i, j = 1, 2, 3. The 9 independent material coefficients for an orthotropic material are

E_1, E_2, E_3, G_{23}, G_{13}, G_{12}, ν_{12}, ν_{13}, ν_{23}

The stator core assembly is a kind of axis symmetry material, in the 1-3plane can be considered isotropic, and obey generalized Hooke's law. So that

$$E_1=E_3 \quad (9)$$

And E_1, G_{13} and ν_{13} meet the following formula

$$G_{13}=\frac{E_1}{2(1+\nu_{13})} \quad (10)$$

So

$$\nu_{31}=\nu_{13} \quad (11)$$

$$\nu_{12}=\nu_{32} \quad (12)$$

$$\nu_{21}=\nu_{23} \quad (13)$$

$$G_{12}=G_{23} \quad (14)$$

The 5 independent material coefficients for an orthotropic stator core material are

E_1, E_2, G_{12}, ν_{12}, ν_{13}

Sometimes, in order to simplify the analysis, we think that the stator core is isotropic material. When there exist no preferred directions in the material (i.e., the material has infinite number of planes of material symmetry), the number of independent elastic coefficients reduces to 2. Such materials are called isotropic. For isotropic materials we have

$$\left.\begin{aligned} &E_1 = E_2 = E_3 = E \\ &G_{12} = G_{13} = G_{23} = G \\ &\nu_{12} = \nu_{23} = \nu_{13} = \nu \\ &G = \frac{E}{2(1+\nu)} \end{aligned}\right\} \quad (15)$$

Most simple mechanical-property characterization tests are performed with a known load or stress. The method for determining the elastic constants of stator core can also see paper [16, 17].

2.5 Mechanics models of Stator Core

In the paper, first stator core is considered as homogenous isotropic and orthotropic thick circular cylindrical shell respectively, and is free at two ends; the material properties of stator core are constant and isotropic and orthotropic respectively; the FEM software, ANSYS or ABAQUS, is used to establish the FEM model of thick shell and 3D solid separately. Second stator core is considered as homogenous isotropic and orthotropic thick circular cylindrical shell respectively, and is free at two ends; the material properties of stator core are uncertainty variables, obeying Gaussain distribution; ANSYS /workbench is used to establish the FEM model of thick shell, and calculate the cumulative distribution function (CDF) of the natural frequencies of stator core, and response surfaces of natural frequencies vs. elastic constants by statistical method (Six Sigma Analysis).

3 Idealization Mechanics Model and Analysis of Stator Core

3.1 Thin Shell Models of Stator Core

The stator core is simplified as homogenous isotropic and orthotropic thin circular cylindrical shell respectively, and is free at two ends; the material properties of stator core are constant; a simplified shell equation is used as theory solution or exact solution [12]. This simplification has a large error, the practical were rarely used.

3.2 Thick Shell or 3D Solid Models of Stator Core

The stator core is simplified as homogenous isotropic and orthotropic thick circular cylindrical shell respectively, and is free at two ends; the material properties of stator core are constant; the energy method, such as Rayleigh-Ritz method[12,18], is used to calculate natural frequencies; in this case, the mode shape function must be assumed by means of experience. The paper does not in-depth discussion.

3.3 FEM Models of Stator Core

On the basis of above analysis, ANSYS and ABAQUS software are used to build the FEM models of the stator core of 1000MW turbo-generator that adopt thick shell element and 3D solid element respectively, and solve their natural frequencies and modes.

Specific parameters of the stator core of 1000MW turbo-generator are as follows:

Isotropic material models:

Young's Modulus E=$(1.2\sim1.8)\times10^5$ N/mm^2

Poisson's Ratio : $\nu = 0.25\text{-}0.35$

Density: $\rho = (0.9\sim1.1)\times7.85\times10^3 \text{kg/m}^3$

In the calculation of nature frequency, the elastic constants values are as follows:

$$\text{E}=1.5\times10^5\text{N}/\text{mm}^2$$
$$\nu = 0.3$$
$$\rho = 7.85\times10^{-6}\text{kg}/\text{mm}^3$$

Orthotropic material models:

$$E_1 = E_3 = (1.3\sim1.8)\times10^5 N/mm^2$$
$$E_2 = (0.5\sim1.0)\times10^5 N/mm^2$$
$$G_{12} = G_{23} = (0.9-1.2)\times10^5 N/mm^2$$
$$G_{13} = \frac{E_1}{2(1+\nu_{13})} = (0.5-0.69)\times10^5 N/mm^2$$
$$\nu_{21} = \nu_{23} = 0.31\sim0.37$$
$$\nu_{12} = \nu_{32} = 0.31\sim0.37$$
$$\nu_{31} = \nu_{13} = 0.3-0.32$$

In the calculation of nature frequency, the elastic constants values are as follows:

$$E_1 = E_3 = 1.69\times10^5 N/mm^2$$
$$E_2 = 0.937\times10^5 N/mm^2$$
$$\nu_{21} = \nu_{23} = 0.34$$
$$\nu_{31} = \nu_{13} = 0.3$$
$$G_{12} = G_{23} = 0.952\times10^5 N/mm^2$$

$$G_{13} = \frac{E_1}{2(1+\nu_{13})} = 0.65 \times 10^5 N/mm^2$$

Number of stator slots is 36; Outer diameter of stator core 2945 mm; Inner diameter of stator core 1471mm; Yoke inner diameter of stator core 1791mm; Length of stator core 8100 mm; Height of slot 160 mm;.

Parameters of equivalent cylindrical shell of the stator core of 1000MW turbo-generator are as follows: Radius of middle surface is 1184 mm; Thickness 577 mm; Length of shell 8100 mm; Material constants are above.

In the dynamics calculation of stator core, the shell181 and solid185 element are selected, isotropic and orthotropic material model is adopted respectively. In the ANSYS/Workbench software, the certainty dynamics equation is as follows:

$$\left([K] - \omega^2 [M]\right)\{u\} = 0 \quad (16)$$

Where:

[M], [K] is mass matrix and stiffness matrix respectively. $\omega, \{u\}$ is natural frequency and mode shape respectively. Obviously, these relate with geometric dimensions and material parameters. In our problem, the geometric dimensions are constant, and material parameters are variation.

The results of natural frequencies are shown in Table 1, and the mode shapes for isotropic stator core can also see [1-3]. The results show that thick shell model and the 3D solid model can be used as the calculation of the stator core model. The results of the two models differ little, comparatively speaking, a thick shell model is simpler, and fewer elements, nodes and the number of degrees of freedom, less computer resources and consumption time required, faster speed of calculation. This model is ideal for scanning to optimize the design; and 3D solid model only as a reference. The modal experiments of stator core prototype is quite difficult, so far we have not seen the relevant literature about these of 1000MW generators. We just use a different model (the number of nodes and elements) for comparison, and compare with the calculation results of different software platforms, so that the accuracy of calculations is ensured. The FEM solution of thick shell model have high precision as comparison with the results [17, 18], and is available for stator core.

Table 1.Natural Frequencies for Stator Core (Hz)

	FEM solution			
Freq.	Thick shell		3D solid	
Oder	Isotropic	Orthotropic	Isotropic	Orthotropic
1-6	0	0	0	0
7	142.16	150.91	144.5	155.83
8	142.16	150.92	144.5	155.83
9	168.85	179.41	167.34	182.54
10	201.4	221.88	206.68	229.12
11	201.5	221.88	210.61	233.36
12	204.77	226.47	211.51	235.31
13	204.77	226.47	215.06	248.25
14	230.69	255.62	240.41	264.27
15	263.45	255.62	266.6	270.42
16	263.45	279.98	272.3	280.23

4 Dynamics statistical model and calculation of stator core

4.1 Six Sigma Analysis Method [19]

In traditional deterministic analyses, uncertainties are either ignored or accounted for by applying conservative assumptions. Uncertainties would be typically ignored if the input parameter has no effect on the behavior of the component under investigation. In this case, only the mean values or some nominal values are used in the analysis. However, in some situations, the influences of uncertainties exist, and cannot be neglected, as for the material properties of stator core, for which the scatter is not usually ignored.

A Six Sigma Analysis uses statistical distribution functions (such as the Gaussian or normal distribution, the uniform distribution, etc.) to describe uncertain parameters, and allows us to determine the extent to which uncertainties in the model affect the results of an analysis.

In a deterministic analysis, computer models are described with specific numerical and deterministic values. The accuracy of a deterministic analysis depends upon the assumptions and input values used for the analysis.

While scatter and uncertainty naturally occur in every aspect of dynamics analysis of stator core, deterministic analyses do not take them into account. To deal with uncertainties and scatter, use Six Sigma Analysis, which we can use to answer the following questions: when the input variables of a finite element model are subject to scatter, how large is the scatter of the output parameters? How robust are the output parameters? When the output is subject to scatter due to the variation of the input variables, then what is the probability that a design criterion given for the output parameters is no longer met? How large is the probability that an unexpected and unwanted event takes place (i.e., what is the failure probability)? Which input variables contribute the most to the scatter of an output parameter and to the failure probability? What are the sensitivities of the output parameter with respect to the input variables?

Six Sigma Analysis can be used to determine the effect of one or more variables on the outcome of the analysis. In addition to the Six Sigma Analysis techniques available, ANSYS Workbench offers a set of strategic tools to enhance the efficiency of the Six Sigma Analysis process.

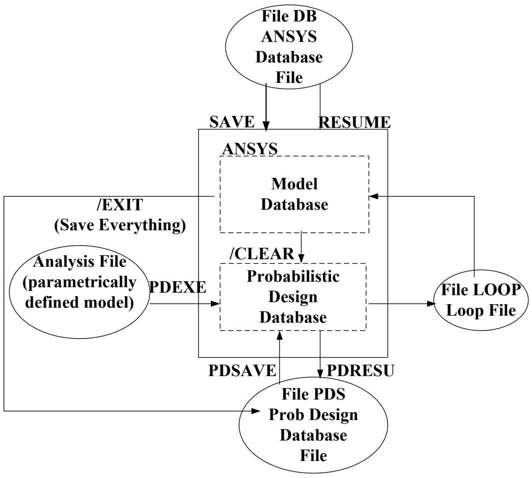


Fig. 5 Probabilistic design data flow

The Fig.5 shows the flow of information during a probabilistic design analysis. Note that the analysis file must exist as a separate entity, and that the probabilistic design database is not part of the ANSYS model database.

4.2 The Selection of FEM Model and Parameters Range

As mentioned above, the FEM modeled by thick shell element has good precision and excellent computational efficiency. Therefore, in ANSYS workbench 14.5 program it is used to determine the resonant frequencies curves of stator core vibration or response surfaces of natural frequencies vs. elastic constants .The elastic modulus, Poisson's ratio, and mass density are selected as input variables and changed in the possible range. The ranges of parameters are shown in Table 2.

Natural frequencies are selected as output or response parameters; in practical computation, any a frequency concerned with can be selected for response parameters, here only 10^{th}, 12th and 14^{th} natural frequencies are available. The boundary conditions of cylindrical shell are free at both ends. The geometric parameters are the same as above and fixedness.

4.3 The Design of Experiment

4.3.1 Design parameters

The Input and response parameters are all uncertainty variables, the distribution type are all Gaussain（or normal）, the distribution limit are all from negative infinity to positive infinity ,the skewness and kurtosis coefficients are all null, and the others have been given in Table 2. The stator core model for isotropic has 15 design points, and 150 design points for orthotropic.

Table 2. Continuous input uncertainty parameter definitions

Name	Ranges	Mean	Standard Deviation
ν	0.25-0.35	0.3	1.5e-002
E	(1.268-1.732)e11	1.5e+011	7.5e+009
ρ	6637.1-9062.9	7850	392.5
ν_{12}	0.29-0.39	0.34	0.017
ν_{13}	0.25-0.35	0.3	0.015
ν_{23}	0.29-0.39	0.34	0.017
E_1	(1.43-1.95)e11	1.691e11	8.455e9
E_2	(2.6-3.7)e11	9.37e10	4.685e9
E_3	(1.43-1.95)e11	1.691e11	8.455e9
G_{23}	(0.878-1.2)e11	1.0385e11	5.19e9
G_{13}	(5.5-7.5)e10	6.5e10	3.25e9
G_{12}	(0.878-1.2)e11	1.0385e11	5.19e9

Table3. Response Parameters Calculated For Isotropic

Name	Minimum	Maximum	Mean	Standard Deviation
10th Freq.	173. 77	229. 64	201. 14	7. 4078
12th Freq.	177. 57	233. 05	204. 87	7. 5368
14th Freq.	200. 41	262. 17	230. 8	8. 4872

Table4.Response Parameters Calculated For Orthotropic

Name	Minimum	Maximum	Mean	Standard Deviation
10th Freq.	191.08	254.36	.222.03	7.9485
12th Freq.	197.97	257.22	226.66	7.6973
14th Freq.	228.3	285.25	255.86	7.996

4.3.2 Six sigma analysis input parameter charts

The probability density function (PDF) and cumulative distribution function (CDF) of Poisson's Ratio, Young's Modulus and density have gained respectively. Here only these of Young's Modulus for isotropic are given in Fig6.

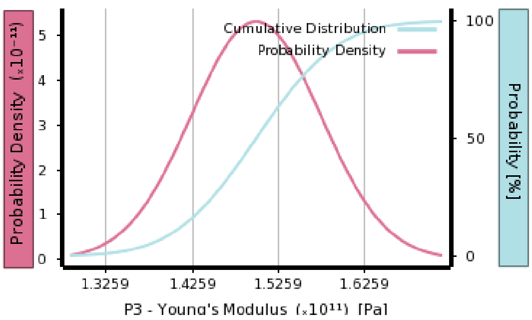


Fig.6 Young's Modulus PDF and CDF for isotropic

4.3.3 Six sigma analysis for isotropic stator core

The size of SSA Sample Set 1 is 10000, and statistical summary is given in Table3.

Histogram & CDF of response parameters have been gained respectively. Here only these of three response parameters for isotropic are given in Fig.7, Fig.8and Fig.9. Table5, Table6 and Table7 give response parameters(frequencies) and corresponding probability for isotropic respectively. Fig.7 shows the probability that the 10th natural frequency will remain less than a limit value of 2.1E+2 (Hz) is about 90%, which also means that there is a 10% probability that the 10th natural frequency will exceed the limit value of 2.1E+2 (Hz).

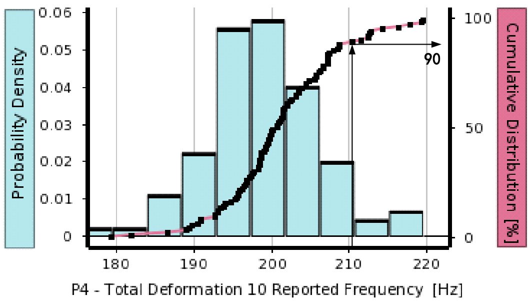


Fig.7 10th Frequency Histogram and CDF

Table5. Probability of 10th Frequency

10thFreq (Hz)	Probability	Sigma Level
179.44	0.0069075	-2.462
181.95	0.016217	-2.139
184.46	0.021446	-2.0248
186.97	0.027824	-1.9138
189.48	0.045827	-1.6867
191.99	0.09192	-1.329
194.51	0.16113	-0.98984
197.02	0.26787	-0.61926
199.53	0.42106	-0.19919
202.04	0.60436	0.26465
204.55	0.70142	0.5285
207.06	0.7897	0.80537
209.57	0.88773	1.2145
212.08	0.90853	1.3318
214.6	0.95457	1.6909
217.11	0.96772	1.8482
219.62	0.99309	2.462

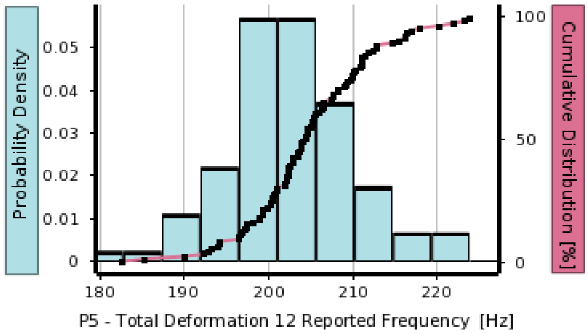


Fig.8 12th Frequency Histogram and CDF

Table6. Probability of 12th Frequency

12thFreq (Hz)	Probability	Sigma Level
182.79	0.0069075	-2.462
185.36	0.016106	-2.1418
187.93	0.021369	-2.0263
190.49	0.027772	-1.9146
193.06	0.048048	-1.6641
195.63	0.092081	-1.328
198.19	0.16594	-0.97034
200.76	0.27418	-0.60023
203.33	0.43001	-0.17635
205.89	0.60977	0.27871
208.46	0.70455	0.53755
211.02	0.79555	0.82585
213.59	0.88751	1.2134
216.16	0.91149	1.35
218.72	0.95632	1.7095
221.29	0.96905	1.867
223.86	0.99309	2.462

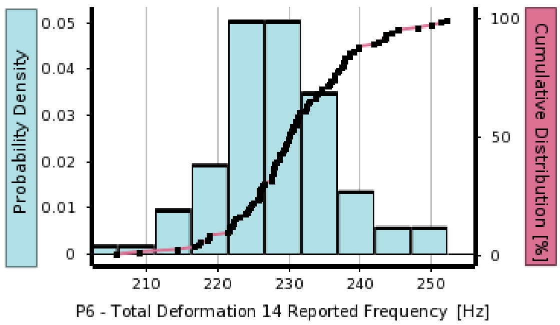


Fig.9 14th Frequency Histogram and CDF

Table7. Probability of 14th Frequency

14thFreq (Hz)	Probability	Sigma Level
205.95	0.0069075	-2.462
208.85	0.016033	-2.1436
211.74	0.021321	-2.0272
214.64	0.027717	-1.9155
217.53	0.049606	-1.6487
220.43	0.092098	-1.3279
223.32	0.16772	-0.96322
226.22	0.28382	-0.57153
229.12	0.43212	-0.17099
232.01	0.60967	0.27845
234.91	0.70479	0.53823
237.80	0.80134	0.84643
240.70	0.8874	1.2128
243.59	0.91788	1.3909
246.49	0.95705	1.7174
249.38	0.96958	1.8747
252.28	0.99309	2.462

From Table 1, 3, 5, 6 and 7, and Fig.7,Fig.8 and Fig.9,we know that when the input parameters are uncertainty and dispersion, natural frequencies are also dispersed, and the calculation values of natural frequencies under deterministic input parameters is approximately equal mean value of natural frequencies calculated with the uncertainty input parameters. The distribution range of the natural frequency is large. When the probability is more than 0.9, the natural frequencies has four values.

4.3.4 Six sigma analysis for orthotropic stator core

The size of SSA Sample Set 1 is 10000, and statistical summary is given in Table4.

Histogram & CDF of response parameters (10^{th}, 12th and 14^{th} nature frequencies) have been gained respectively. Here only these of three response parameters for orthotropic are given in Fig.10, Fig.11and Fig.12. Table8, Table9 and Table10 give response parameters(frequencies) and corresponding probability for orthotropic respectively.

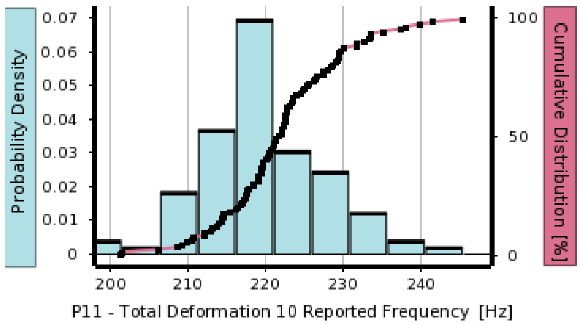


Fig.10 10th Frequency Histogram and CDF

Table8. Probability of 10th Frequency

10^{th}Freq (Hz)	Probability	Sigma Level
201.46	0.0069075	-2.462
204.21	0.021641	-2.021
206.96	0.028967	-1.8962
209.7	0.052516	-1.6209
212.45	0.098724	-1.2889
215.2	0.18296	-0.90413
217.95	0.28292	-0.57418
220.69	0.4392	-0.15301
223.44	0.64207	0.36399
226.19	0.73274	0.62111
228.94	0.81032	0.87908
231.68	0.88347	1.1925
234.43	0.93847	1.5421
237.18	0.95211	1.6657
239.93	0.97315	1.9293
242.67	0.98664	2.2155
245.42	0.99309	2.462

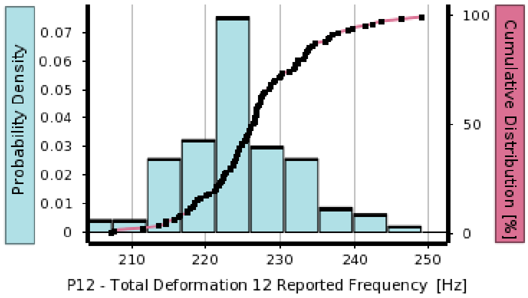


Fig.11 12th Frequency Histogram and CDF

Table9. Probability of 12th Frequency

12^{th}Freq (Hz)	Probability	Sigma Level
207.36	0.0069075	-2.462
209.97	0.022219	-2.0099
212.58	0.031083	-1.8651
215.19	0.060495	-1.5506
217.8	0.11276	-1.212
220.41	0.18534	-0.8952
223.02	0.2897	-0.55426
225.63	0.45388	-0.11587
228.24	0.65909	0.40998
230.85	0.73922	0.64096
233.46	0.81815	0.90835
236.07	0.88233	1.1867
238.68	0.92956	1.4725
241.29	0.95219	1.6665
243.9	0.97442	1.9502
246.51	0.98401	2.1446
249.12	0.99309	2.462

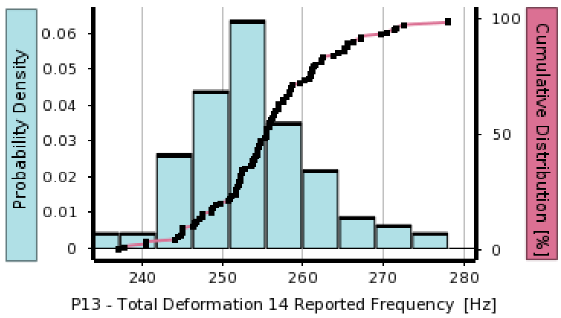


Fig.12 14th Frequency Histogram and CDF

Table10. Probability of 14th Frequency

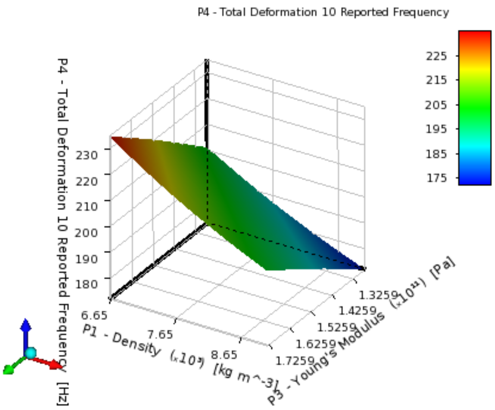
14^{th}Freq (Hz)	Probability	Sigma Level
237.15	0.0069075	-2.462
239.71	0.023442	-1.9874
242.27	0.041268	-1.7362
244.83	0.064152	-1.5208
247.39	0.14364	-1.0641
249.95	0.20688	-0.8173
252.51	0.33395	-0.42904
255.07	0.47946	-0.051502
257.63	0.64708	0.37745
260.19	0.73194	0.6187
262.75	0.8357	0.97692
265.31	0.87388	1.1449
267.87	0.92567	1.4443
270.43	0.94189	1.5708
272.99	0.97424	1.9471
275.55	0.97918	2.0372
278.11	0.99309	2.462

From Table 4, 8, 9 and 10, and Fig.10, Fig.11 and Fig.12, we know that when the input parameters are uncertainty and dispersion, natural frequencies are also dispersed. When the uncertainty input parameters obey the normal distribution condition, comparing the calculation results of isotropic and anisotropic stator core model, we obtain a conclusion: The distribution range of the natural frequency is larger, and the calculated values are also larger. When the probability is more than 0.9, the natural frequencies has five values. These have an important guiding role for the laminated structure design and manufacture process of stator iron core, and the fixedness and the vibration isolation design of stator core

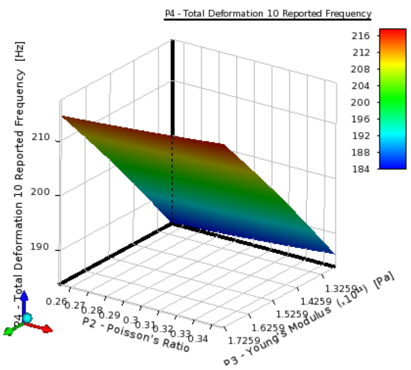
4.4 The Application of Response Surfaces

4.4.1 Response Surfaces for isotropic stator core

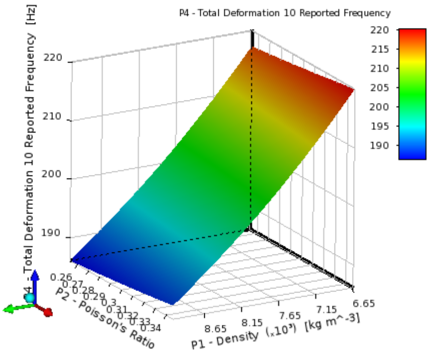
Response surface of the output parameters vs. input parameters have been calculated, and here response surface of 10th frequency vs. Poisson's Ratio, Young's Modulus and density are given in Fig.13.



(a) 10th frequency vs. density and Young's Modulus



(b)10th frequency vs. Poisson's Ratio and Young's Modulus



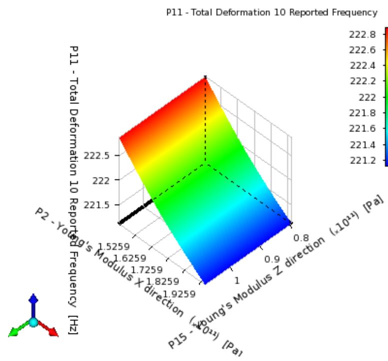
(c) 10th frequency Poisson's Ratio vs. and density

Fig.17 Response surface of 10th frequency

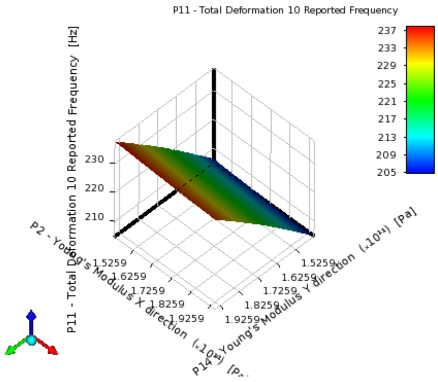
Fig.13 (a) shows that 10th natural frequency increases as the density decreases and young modulus increases. Fig.13 (b) shows that 10th natural frequency increases as the Poisson's Ratio and young modulus increase. Fig.13 (c) shows that 10th natural frequency increases as the Poisson's Ratio increase and density decreases. The Poisson's Ratio has little effect on natural frequencies.

4.4.2 Response Surfaces for orthotropic stator core

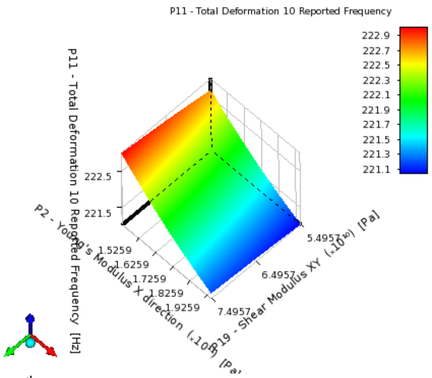
Response surface of the output parameters vs. input parameters have been calculated. The input parameters include nine elastic constants and mass density. Among the nine elastic constants, five are independent, so that there are a lot of response surface, and here response surface of 10th frequency vs. X,Y,Z direction Young's Modulus and XY,YZ,XZ plane shear Modulus are given in Fig.14 respectively.



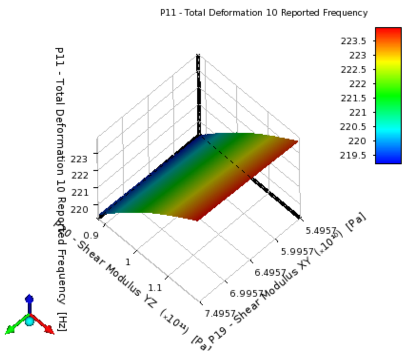
(a) 10th frequency vs. X and Z direction Young's Modulus



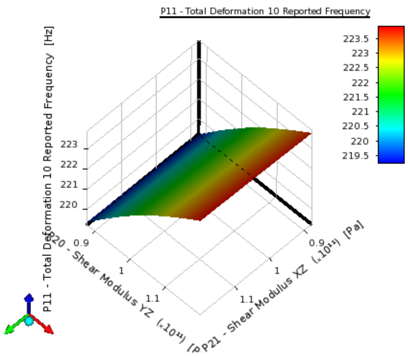
(b) 10th frequency vs. X and Y direction Young's Modulus



(c) 10th frequency vs. X direction Young's Modulus and XY plane shear Modulus



(d) 10th frequency vs. YZ and XY plane shear Modulus



(e) 10th frequency vs. YZ and XZ plane shear Modulus

Fig. 14 Response surface of 10th frequency vs. elastic modulus

Fig.14 (a) ,(b) and (c) shows that 10th natural frequency increases as the X direction Young's Modulus decreases and Y direction young modulus increases, and Z direction young modulus has little effect on natural frequencies. Fig.14 (c), (d) and (e) shows that 10th natural frequency increases as the YZ plane shear Modulus, and XY and XZ plane shear Modulus increase has little effect on natural frequencies.

5 Conclusions

The simple and effective method for the calculation of the modal frequencies and modes of stator core has been introduced, and the calculation has proved the FEM solution of thick shell model is simple, effective and accurate. The elastic constants of stator core material are changed in the possible range, and are considered uncertainty variables of obeying Gaussian distribution. We have obtained the resonant frequencies curves of stator core vibration or response surfaces of natural frequencies vs. elastic constants, and the natural frequencies and corresponding probability for isotropic and orthotropic stator core respectively. Comparing the calculation results of isotropic and anisotropic stator core model, we know that the distribution range of the natural frequency is larger, and the calculated values are also larger. The results of curves and surfaces are very useful and helpful in the fixedness and the vibration isolation design of stator core. For the characteristic improvement and operation management of the existing machine, the development and manufacture of new large turbine-generator, the results are also very important, and valuable.

Acknowledgements

This work was supported by Shaanxi Province Natural Science Fund (2004K06-G25) and Shaanxi Province Colleges and Universities Science Fund (07JK260; 11JK0882; 2013JC17), China.

References:

[1] Wang Yixuan, Wang Ying, Modal analysis of stator system of large turbo-generator, *International Review of Electrical Engineering,* Vol.8,No.2,March - April 2013,pp.681-689.

[2] Yixuan Wang, Ying Wang, Virtual Prototype and Modal Analysis of Stator System of Large

Turbo-Generator, *Applied Mechanics and Materials,* Vols. 190-191 ,2012,pp. 232-236.

[3] Yixuan Wang, Ying Wang, Material Properties and Natural Frequency of Stator Core of Large Turbo-Generator, *Advanced Materials Research,* Vols. 322, 2011, pp. 81-84.

[4] Yixuan Wang, Ying Wang, Research on Dynamic Characteristics of Stator core of Large Turbo-Generator, *Proceeding of Asia-Pacific Power and Energy Engineering Conference (APPEEC 2010)* March 28-31, 2010, Chengdu, china.

[5] Wang Yixuan, Wang Ying, Liu Xin and Qiu Haifei, Electromagnetic Design and Dynamic Analysis of Large Turbo-Generator, *Journal of Energy and Power Engineering,*. Vol.3,No.12, 2009,pp.19-28.

[6] Wang Yixuan, Wang Ying, etc., Dynamic Design and Simulation Analysis of 1000MW Large Turbo Generator, *Proceeding of the 2009 IEEE ICMA*, changchun, China, 2009 , vol.3, pp.1650-1655.

[7] WangYixuan, Zhu Jimei., Dynamic optimization design of stator winding end baskets of large turbo-generator, *Mechanical Science and Technology for Aerospace Engineering*, Vol.27, No.2, 2008, pp. 205-208.

[8] Wang Yixuan, Zhu Jimei, Research on virtual prototyping of stator winding end baskets of large turbo-generator, *Proceeding of Symposium on Global Manufacturing &Simulation Technology of the 21st Century, Guiyang*, International Academic Publishers, China, 2004, pp.67-70.

[9] Wang Yixuan, Zhu Jimei, Research on numerical simulation model of stator winding end baskets of large turbo-generator, *Chinese Journal of Mechanical Engineering*, Vol.41, No.9 , 2005, pp.217-222.

[10] Wang Yixuan, Dynamical modelling of stator winding end baskets of large turbo generator, *Journal of Northwest Institute of Textile Science and Technology,* Vol.13, No.4, 1999,pp.339-343.

[11] Marco Amabili, *Nonlinear vibrations and stability of shells and plates*, Cambridge University Press, Cambridge, UK, 2008, pp.298-308.

[12] Werner Soedel, *Vibrations of shells and plates,* Third Edition, Marcel Dekker, Inc. New York, USA, 2004, pp.184-189.

[13] Hideharu Hiwaki, Hiroshi Murakami etc., Reducing iron loss by decreasing stress in stator core of permanent synchronous motor, *IEEJ Trans. IA*, Vol.128,No.7, 2006,pp.852-856.

[14] D.Lin, P.Zhou, Z.Badics, etc., A New Nonlinear Anisotropic Model for Soft Magnetic Materials, CSY0084 of COMPUMAG '2005, 2005, pp.1-5.

[15] Zhangjun Tang,, Pragasen Pillay, Avoki M. Omekanda,Chen Li and Cetin Cetinkaya, Young's Modulus for Laminated Machine Structures With Particular Reference to Switched Reluctance Motor Vibrations, *IEEE Trans. Ind. Applicat.*, Vol.40,No.3, 2004, pp.748–754.

[16] Qiu wengan, Zhang yuncheng, etc., The computed elastic constant of stator core, *Large Electric Machine Technology*,Vol.16,No.1, 1986,pp.41-44.

[17] Luo gongfu, Research on computing method of anisotropy of stator core of hydro generator, *Eastern Electric Machine*, Vol.25, No.1,1998,pp.47-53.

[18] S.P.Verma, R.K.Singal and K.Willams, Vibration behaviour of stators of electrical machines, Part Ⅰ：Theoretical study, Part Ⅱ: Experimental study, *Journal of sound and vibration,* Vol.115, No.1, 1987,pp.1-23.

[19] Six Sigma Analysis Design Xplorer Help Release 14.5, Documentation *for ANSYS Workbench*，2013.

[20] J.N.Reddy, *Mechanics of Laminated Plates and Composite Shells Theory and Analysis (second edition),* CRC Press LLC, Florida, USA ，2004，pp.24-28.