Detection of metal fibres in cementitious composites based on signal and image processing approaches

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Abstract: Mechanical properties of cementitious composites, reinforced by metal fibres, are conditioned by the volume fraction and distribution of directions of fibres. However, their reliable non-destructive or low-invasive experimental evaluation is a serious problem. The paper pays attention to four classes of such indirect methods. The first class relies on the planar X-ray imaging, with the discrete fast Fourier transform applied to image processing. The second one applies the magnetic approach, with certain electromagnetic alternative. The third one comes from the computed tomography, as an unique exact method for the detection of volume fraction without breaking the sample, with an information on (an)isotropy as a benefit. The last one is concentrated to the FEM modelling. Examples related to all sketched method from the experiments performed at the Brno University of Technology show the advantages and restrictions of particular approaches.

Key–Words: Cementitious composites, non-destructive testing, signal and image processing, computational simulation.

1 Introduction

Advanced building structures make frequently use of materials as silicate composites, reinforced by metal particles (e. g. steel-fibre-reinforced concrete), preventing the tension stresses and strains as sources of undesirable micro- an macro-cracking. Mechanical properties of such composites are determined by the choice of fibre properties and their volume fraction, location and orientation in the matrix, sensitive to the technological procedures (as special compaction) and to the early-age treatment – cf. [12], as well as by the bond / slip interface relations – cf. [3]. The employment of the destructive approach relies usually on the separation of particles, taken from the early-age matrix, alternatively obtained from the crushed part of the existing structure, in the laboratory; consequently the volume fraction of particles can be evaluated accurately, whereas any information related to the original orientation of particles is missing. Moreover, such experiments with many structures are not allowed by technical standards. This is a strong motivation for the employment of some reliable non- or (at least) semi-destructive measurement methods, applicable in situ, handling homogeneity and isotropy and detecting the volume fraction of fibres in the material structure.

Regardless of the significant progress in this research area in the last decade (for more historical remarks and references see [8]), no inexpensive, robust and reliable method is available, thus all identification approaches rely on a) some indirect measurements and b) non-trivial numerical analysis, to handle a corresponding inverse problem – typically ill-posed, unstable, etc., forcing artificial regularization. Since a) produce quite other information than needed volume fractions and directional distributions of fibres, typically digital images in pixels or voxels, or electromagnetic quantities detected on the specimen surface, some calibration relations are needed, motivated by the physical and geometrical similarity. Moreover, some reasonable algorithm for the evaluation of effective material properties, using the properties of matrix and particles and the geometrical configuration, as input data, is needed: from simple arguments from the mixture theory to complicated physical and mathematical homogenization techniques (which will be specified later, in connection with electromagnetic measurements).

In this paper we shall pay attention thanks to the research experience of the authors from BUT (Brno University of Technology), namely to four representative approaches:

1. the planar X-ray imaging, with the discrete fast Fourier transform applied to image processing,

2. the magnetic approach, utilizing the Hall probe and advanced considerations on material homogenization with certain electromagnetic alternative,

3. the computed tomography, as an unique exact method for the detection of volume fraction without breaking the sample, with an information on (an)isotropy as a benefit.

4. the finite element modelling, as a method for exact electromagnetic field modelling based on the random fibre generation and identification.

2 First class of methods: analysis of X-ray images

The radiographic approach, developed in [7], [22] for a rather large class of building materials, comes from the grey-scale planar images and some of their post-processing modifications, in particular:

1) the reduction of all fibres (whose length and thickness is known) to one-pixel thick black curves, followed by the simplified evaluation of their amount and orientation, by [7],

2) the application of the two-dimensional fast Fourier transform by [10] and [15], avoiding most artificial image changes, where the same as in 1) can be identified with a special diffraction process: for the Cartesian coordinates x, y and the grey level $f(x, y)$, related to a square image containing $N \times N$ pixels (with N tending to ∞ theoretically), with the associated image in the Fourier transforms $F(u, v)$ in the Cartesian coordinate system, the direct and inverse Fourier transforms can be evaluated using the formulae

$$F(u,v) = \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \exp(-2\pi i(ux+vy)/N) \,,$$

$$N^2 f(x,y) = \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(x,y) \exp(2\pi i(ux+vy)/N)$$

moreover the power spectrum $P(u, v) = |F(u, v)|^2$ contain the useful information, needed for the derivation of the histograms of fibre directions.

Figure 1 presents an example of such MATLAB-supported evaluation of fibre orientation in the fibre concrete specimen; the utilized X-ray equipment is shown on Figure 2. In general, the radiographic analysis gets useful results related to preferential orientations of fibres, although limited to data from planar images, even from several views to cubic specimens. The estimate of volume fraction of fibres is not very precise, at least in the comparison with destructive tests.

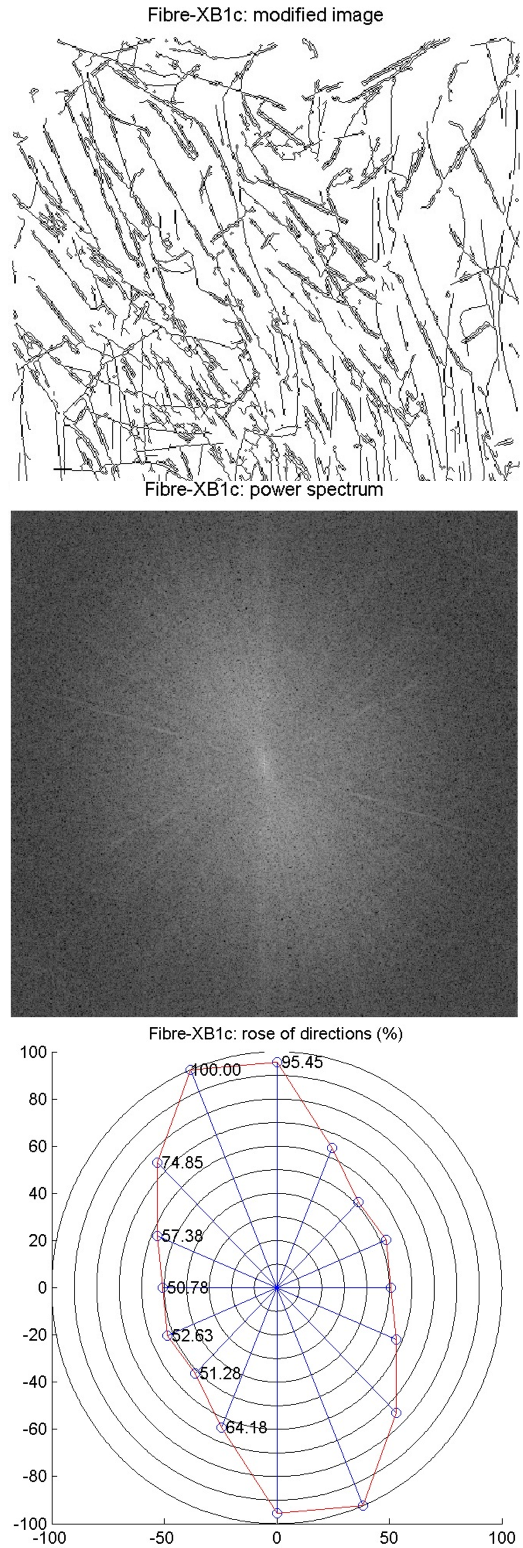

Figure 1: Evaluation of fibre orientation from the X-ray image (images from the top to bottom): original image, result of fibre localization, power spectrum P, resulting rose of fibre directions.

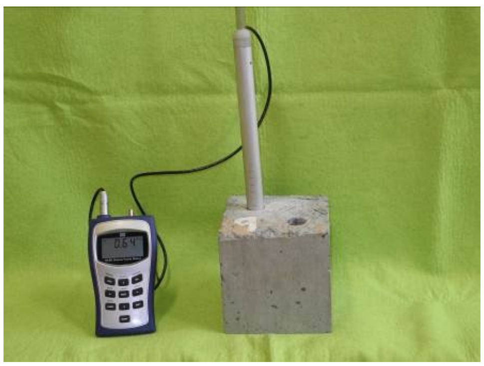

Figure 2: X-ray machine EcoRay HF 1040 with digital recording to PC equipment (top photo). PeMaSo-01 depth probe for magnetic measurements (bottom photo).

3 Second class of methods: numerical treatment of magnetic measurements

Magnetic measurements like [24] and [5] rely on the different values of relative permeability of fibres and a matrix, with possible alternative of electrical measurements and relative permittivity. The special experimental configuration usually tries to force a (nearly) stationary process, whose mathematical description works with a differential operator close to the classical Laplace one, to enable non-expensive software simulation. Figure 3 shows the geometrical configuration of such process numerical simulation of such process in COMSOL: the magnetic field is generated by several permanent magnets, located in the drilled hole (thus this method could be classified as low-invasive, not non-destructive completely), consequently the Hall effect based probe from Figure 2 detects the magnetic field strength. Figure 4 documents the numerical simulation of such experiment, applying the COMSOL supported planar finite / infinite element technique: the influence of the irregularities caused by an artificial hall seems to be not substantial. The comparative simulation, applying only selected functions of *pde* toolbox from MATLAB, leads to the same conclusion.

The crucial problem is now to implement a correct evaluation procedure for an effective relative permeability (or permittivity) using the incomplete data on the material microstructure and on relative permeability of fibres. For spherical particles the classical Maxwell-Garnett mixing formula is available; the generalization of [6] comes from the so-called Brugemann approach and the repeated usage of similar ellipsoids as reference volume elements, whereas [14] admits the presence of multiple scattering, important for high volume fractions of fibres. No additional physical assumption are needed, again for periodic spheres, in [13]: the auxiliary problem, referring to the mathematical theory of homogenization of elliptic operators, can be then analysed (including the existence and uniqueness of solution, the convergence of sequences of approximate solutions, etc.) using the two-scale and similar convergence theorems by [2]; the crucial (seemingly) explicit formula for the evaluation of an effective parameter value, comes from the method of oscillating test functions.

In [25] the difficulties with complex particle shapes are handled using the boundary integral approach, thanks to the knowledge of general solutions of the Laplace equation, with Heaviside characteristic functions of particles; [11] admits a priori anisotropic structures. Some generalizations are available using the least squares and conjugate gradient approaches – cf. [21]. Unfortunately, further substantial generalization of this approach (namely to non-periodic structure, avoiding all mixing tricks), lead to non-trivial (partially still open) problems of mathematical analysis, namely to the convergence using probability measures by [20], thus various alternative statistical approaches, as that with Sobol sensitivity indices and Monte Carlo simulations by [9], have been developed.

The unique material characteristics included here is the magnetic permeability μ [Vs/(Am)]; at least in the case of silicate composites used in civil engineering μ can be set to 1 for the pure matrix, but no relevant constant is guaranteed by the producers of ferromagnetic fibres. In practice, the dimensionless relative permeability $\mu_r = \mu/\mu_0$, using the well-known magnetic constant $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am), is usually considered; similarly the relative permeabilities μ_c for the matrix and μ_s for all fibres can be introduced. Fortunately, for a sufficiently slow volume fraction ξ of fibres ($\xi \leq 0.05$ in real experiments), following [6], under the assumption of random orientation of fibres, we obtain an explicit monotone and continuous dependence between μ and ξ

in the form

$$\xi = 1 - M\,\frac{\mu_s-\mu_r}{\mu_s-\mu_c}\left(\frac{\mu_c}{\mu_s}\right)^{3L(1-2L)(2-3L)}\,,$$

where the factors

$$M = \left(\frac{M_1}{M_2}\right)^{2(3L-1)^2/((2-3L)(1+3L))}\,,$$

$$L = \frac{\varsigma}{4\vartheta^3}\left(2\varsigma\vartheta + \ln\frac{\varsigma-\vartheta}{\varsigma+\vartheta}\right)$$

are determined using the ratio ς of lengths of a major and (both) minor axes of ellipsoidal particles (clearly $\varsigma > 1$) for the simplifying notation $\vartheta = \sqrt{\varsigma^2-1}$ and $M_1 = (1+3L)\mu_c+(2-3L)\mu_s$, $M_2 = (1+3L)\mu_r+(2-3L)\mu_s$. In particular, for a (theoretically) infinite length and zero diameter of particles we receive $L = 1/3$. Unfortunately, all attempts to generalize this result for more complicated distributions of fibre directions lead to unpleasant non-analytical integrals, with the duty of their non-trivial numerical evaluations.

Figure 5 documents the least squares based identification of μ_r for 3 input data sets with assumed $\mu_r = 1$ for pure concrete and uncertain μ_s in all other cases, using the above sketched formulae for an isotropic medium. the specimens (unlike the situation in situ) were prepared with exact volume fractions of fibres 0.5 %, 1 % and 1.5 %. Other experiments with comparable results have been performed by the authors' team with magnetic field induced by an electric coil. Moreover, [4] presents a totally non-destructive equipment, applicable to the surface of a specimen (thus preferring fibres close to such surface). All these result seem to give good estimates of volume fractions (whose improvement using more advanced mathematical analysis is possible), but the differentiating between system and random errors in distributions of fibre directions is difficult.

4 Third class of methods: computed tomography

A new approach to non-destructive analysis of structures of cementitious composites, motivated by [16], [23] and [1], has been offered by the computed tomography (X-ray CT), generating 3-dimensional images from large series (slices) of 2-dimensional radiographic images taken around a single axis of rotation. The modern industrial tomograph, presented on Figure 6, has been recently installed in the Central European Institute CEITEC of BUT. Unlike most tomographs for medical applications, an analysed specimen is fixed on the manipulation table of the tomograph, between the

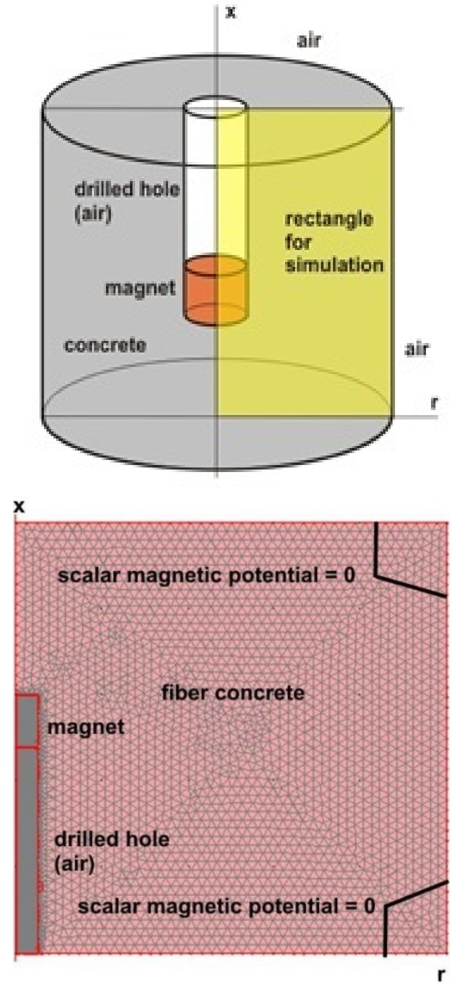

Figure 3: Radially symmetric geometrical arrangement of the magnetic experiment (top) and computational simplification, including finite / infinite element mesh (bottom).

radiation source and the surface radiation detector, compound from a matrix of mini-detectors. During the rotation of the table the surface detector records successive changes of X-ray radiation; consequently the specialized computer software is needed to analyse the inner structure of a specimen.

Several types of fibre concrete specimens have been tested using this equipment: Figure 7 shows the cubic specimen, similar to that from Figure 1, and demonstrates the ability of the specialized software to recognize all fibres completely unlike all approximate estimates from separate planar images. Consequently various forms of histograms or graphical or 3-dimensional roses of directions similar to 2-dimensional ones from Figure 1 can be created. However, this is rather time consuming, expensive and not applicable to the fibre concrete structures in situ. Nevertheless, this seems to be a useful method to obtain a reliable reference basis for all numerical simulation attempts with random po-

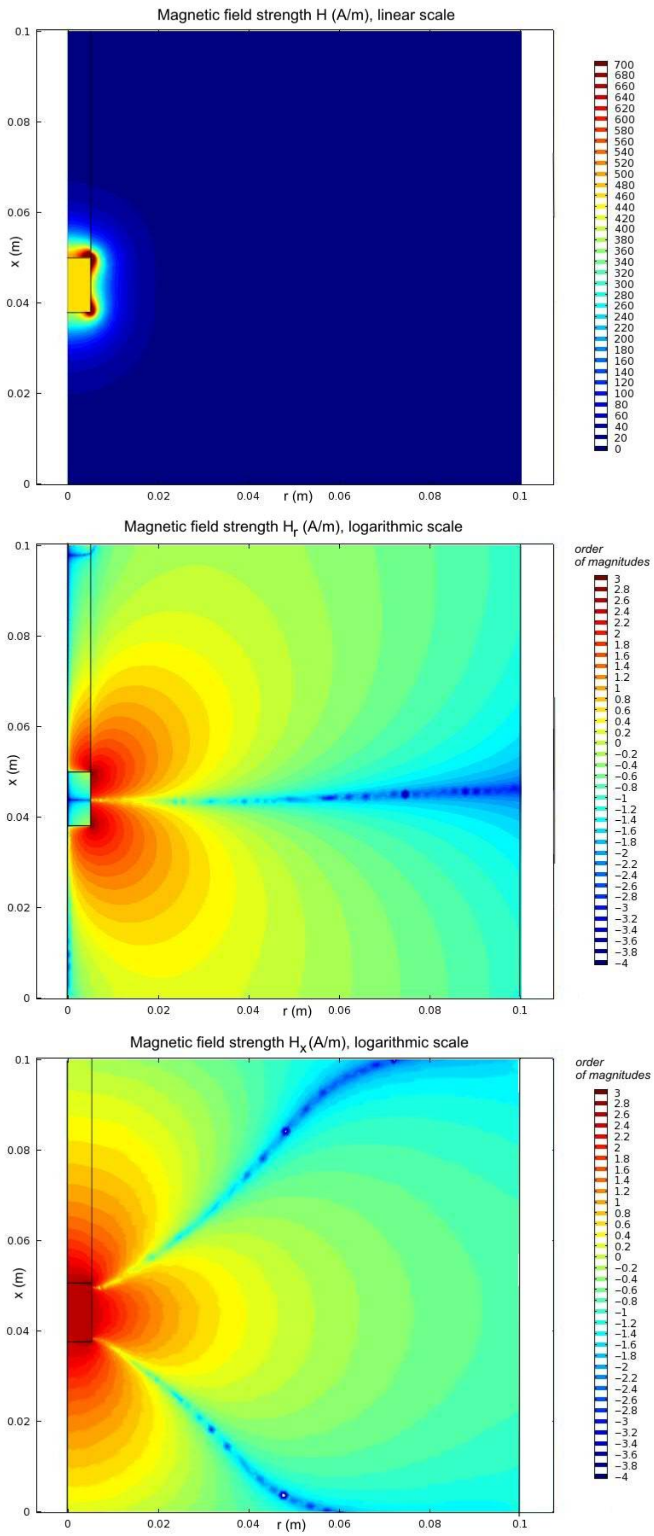

Figure 4: Results of COMSOL based on finite element simulation of stationary magnetic field strength.

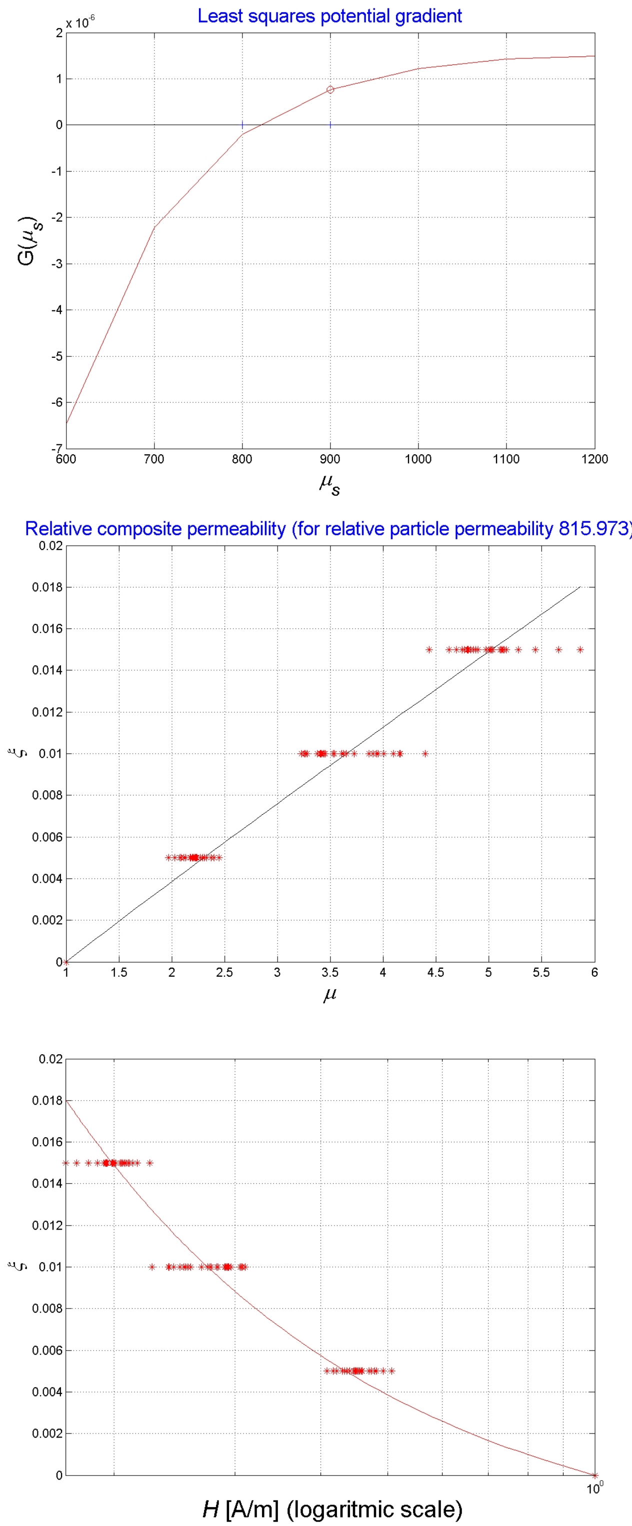

Figure 5: Application of the least squares technique to the identification of parameters ξ and μ_s from magnetic measurements.

sitions and orientation of fibres.

Figure 6: Tomograph GE phoenix,v|tome|x L 240 (top photo) and a cylindrical specimen fixed in its manipulator (bottom photo).

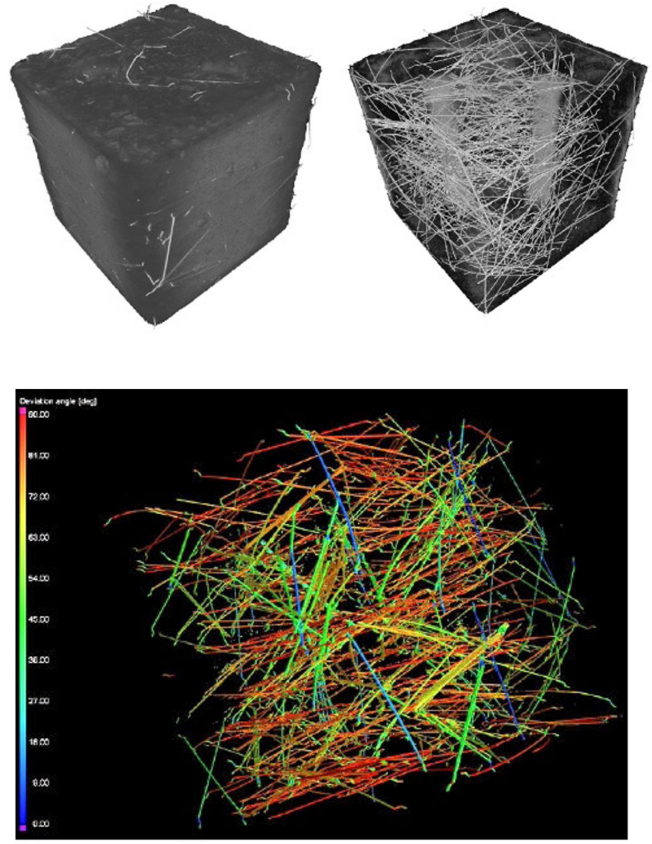

Figure 7: Cubic fibre concrete specimen, edge length 150 mm, required X-ray tube voltage 300 kV (images from the left to the right): axonometric view on its surface, axonometric view inside its structure, axonometric projection of separated fibres in the cube specimen.

5 Fourth class of methods: FEM modelling

The current trend of work in micromechaninics addresses the industry requirements to decrease the dependence on experimental work, and complement it with new numerical and/or analytical processes capable of providing quickly and efficiently the same information. A highly attractive process to simulate the real behaviour of composite is through finite element analysis. For that, a representative volume element (RVE) of the materials needs to be defined and am equivalent random distribution of fibres generated.

The first issue concerning the use of RVE is its dimension. The RVE cannot be too large as this would endanger the possibility to numerically to analyse it; however, it cannot be too small either as it could not be representative of composite material, see [26]. Trias et al. [27] demonstrates that for long fibre composites a value of 50x the fibre radius should be used.

The second issue involving the use of an RVE is the spatial arrangement of reinforcements which normally is not periodic and is highly dependent upon manufacturing process. [28] using homogenization theory concluded that distribution of reinforcements in the RVE does not affect to macroscopic response, but it significantly affects the microscopic stress distribution and following damage in the matrix.

Good review of some numerical methods for the finite mesh generation can be found in [29] and [30]. Digital image analysis provides a perfect replica of the real composite, but can be extremely time and resource consuming as it requires specific software and hardware. To generate a random distribution of fibres is coupled with a statistical analysis and verified by sets of experiments.

6 Conclusion

This paper should be understood as the comparative study to the most promising non-destructive approaches to macroscopic identification of content and random location of fibres in the structure of cementitious composites. However, all approaches have strong restrictions: serious obstacles to get some reasonable estimate of volume fraction of fibres, as the most requested parameter, in the first case, expensive and fastidious experimental setting in the third case, interpretable as the more sophisticated upgrade of the first one, both technical and computational difficulties in the second case.

For the successful computational detection of volume fraction and preferential orientation of fibres, making use of their ferromagnetic properties, both under laboratory and in situ conditions, the crucial point of all considerations is the development of a homogenization procedure, specific to the analysed class of materials, including its formal verification and its validity range. This leads to non-trivial problems of both physical and mathematical analysis, whose validation seems to be available thanks to the progress in the image processing techniques.

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