Hall current and thermal radiation effect on MHD convective flow of an elastico- viscous fluid in a rotating porous channel

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Abstract: - A theoretical analysis is carried out to study the heat and mass transfer effects on elastico-viscous fluid flow in a vertical porous channel with rotation and Hall current. The two porous plates are subjected respectively to constant injection and suction velocity. A magnetic field of uniform strength is applied in the direction perpendicular to the plates. The induced magnetic field is neglected due to the assumption of small magnetic Reynolds number. The elastico-viscous fluid flow is characterized by Walters liquid (Model $B^{/}$).The analytical solutions to the coupled non-linear equations governing the motion are obtained by regular perturbation technique. The effects of rotation, buoyancy force, magnetic field, thermal radiation and heat generation parameters on resultant velocity and temperature fields are analyzed and illustrated graphically in possible cases. The importance of the problem can be seen in cooling of electronic components of a nuclear reactor, bed thermal storage and heat sink in the turbine blades.

Key-Words: - Elastico-viscous, Hall current, Walters liquid (Model $B^{/}$), perturbation, MHD, radiation.

1 Introduction

Hydromagnetic free convective channel flow has paramount importance (because of its potential applications) in a number of engineering processes. To be more specific MHD generators and accelerators design, heat treated materials travelling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber are some practical examples of channel flow.

The effects of Hall current cannot be neglected as the conducting fluid when it is an ionized gas, and applied field strength is strong then the electron cyclotron frequency $\omega = eM/m$ (where e, B, and m denote the electron charge, the applied magnetic field, and mass of an electron, respectively.) exceeds the collision frequency so that the electron makes cyclotron orbit between the collisions which will divert in a direction perpendicular to the magnetic and electric fields directions. Thus, if an electric field is applied perpendicular to the magnetic field then whole current will not pass along the electric field. This phenomena of flow of the electric current across an electric field with magnetic fields is known as Hall effect, and accordingly this current is known as Hall current [26]. So it is very necessary to study the effect of Hall current in many industrial processes.

flat plate. Influence of magnetic field and thermal radiation by natural convection past vertical

El-Hakiem [9] has studied MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity. Oscillating plate temperature effects on a flow past an infinite vertical porous plate with constant suction and embedded in a porous medium have been discussed by Jaiswal and Soundalgekar [15]. Singh et al. [24] have analyzed a periodic solution of oscillatory Couette flow through porous medium in rotating system. Gupta [10] has explained hydromagnetic flow past a porous flat plate with hall effects. Hall effects on the hydromagnetic flow past an infinite porous flat plate has considered by Jana et al. [14]. Pop et al. [22] have measured hall effects on magnetohydrodynamic free convection about a semi-infinite vertical flat plate. Hossain et al. [11] have discussed the effect of radiation on free convection from a porous vertical plate. Effects of radiation and variable viscosity on a MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow has been investigated by Seddeek [23]. Ibrahim et al. [12] have analyzed radiating effect on chemically reacting magnetohydrodynamic (MHD) boundary layer flow of heat and mass transfer through a porous vertical cone subjected to variable surface heat flux have studied by Palani and Kim [21]. Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ion-slip currents have been examined by Aboeldahab and El Aziz [3]. Makinde and Chinyoka [17] have calculated numerical study of unsteady hydromagnetic generalized Couette flow of a reactive third-grade fluid with asymmetric convective cooling. Combined effects of Joule heating and chemical reaction on unsteady magnetohydrodynmic mixed convection of a viscous dissipating fluid over a vertical plate in porous media with thermal radiation have analyzed by Pal and Talukdar [19]. Pal and Talukdar [20] have recently studied influence of Hall current and thermal radiation on MHD convective Heat and mass transfer in a rotating porous channel with chemical reaction.

The applications of the mechanisms of elastico-viscous fluid flows in modern technology and industries have attracted the researchers in a large scale. Authors like Kelly *et al.* [16], Abel et al. [1], Sonth et al. [25], Abel et al. [2], Choudhury et al. [4, 5, 6, 7, 8] have analyzed some problems of physical interest in this field.

The purpose of the present study is to analyze the effects of hall current and thermal radiation with first-order chemical reaction on elastico-viscous fluid on a rotating porous channel with suction and injection.

The constitutive equation for Walters liquid (Model B´) is

$$\sigma_{ik} = -pg_{ik} + \sigma'_{ik} \quad , \sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \quad (1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v_i is the velocity vector, the contravarient form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}_{,m} - v^k_{,m} e^{im} - v^i_{,m} e^{mk} \quad (2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \quad (3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \ and \ k_0 = \int_0^\infty \tau N(\tau)d\tau \quad (4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [27, 28]. This idealized model is a valid approximation of Walters liquid (Model B´) taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau)d\tau, n \geq 2 \quad (5)$$

have been neglected.

2 Problem Formulation

We consider unidirectional oscillatory free convective flow of a visco-elastic incompressible and electrically conducting fluid between two insulating infinite vertical permeable plates separated by a distance d. A constant injection velocity w_0 is applied at the stationary plate $\bar{z} = 0$. Also, a constant suction velocity w_0 is applied at the plate $\bar{z} = d$, which oscillates in its own plane with a velocity $\bar{U}(\bar{t})$ about a nonzero constant mean velocity U_0. The channel rotates as a rigid body with angular velocity $\bar{\Omega}$ about the $\bar{z}$ –axis perpendicular to the planes of the plates. A strong transverse magnetic field of uniform strength H_0 is applied along the axis of rotation by neglecting induced electric and magnetic fields. The fluid is assumed to be a gray, emitting, and absorbing, but nonscattering medium. The radiative heat flux term can be simplified by using Rosseland approximation. It is also assumed that the chemically reactive species undergo first-order irreversible chemical reaction.

The solenoidal relation for the magnetic field $\vec{\nabla}.\vec{H} = 0$, where $\vec{H} = (\bar{H}_x, \bar{H}_y, \bar{H}_z)$ gives $\bar{H}_z = H_0$(constant) everywhere in the flow field, which gives $\vec{H} = (0,0,H_0)$. If $(\bar{J}_x, \bar{J}_y, \bar{J}_z)$ are the component of electric current density $\vec{J}$, then the equation of conservation of electric charge $\vec{\nabla}.\vec{J} = 0$ gives $\bar{J}_z =$ constant. This constant is zero, that is $\bar{J}_z = 0$ everywhere in the flow since the plate is electrically non-conducting. The generalized Ohm's law, in the absence of the electric field [18], is of the form

$$\vec{J} + \frac{\omega_e \tau_e}{H_0}\left(\vec{J} \times \vec{H}\right) = \sigma\left(\mu_e \vec{V} \times \vec{H} + \frac{1}{en_e}\nabla p_e\right) \quad (6)$$

where $\vec{V}, \sigma, \mu_e, \omega_e, \tau_e, e, n_e$ and p_e are the velocity, the electrical conductivity, the magnetic permeability, the cyclotron frequency, the electron collision time, the electric charge, the number density of the electron, and the electron pressure, respectively. Under the usual assumption, the electron pressure (for a weakly ionized gas), the thermoelectric pressure, and ion slip are negligible, so we have from the Ohm's law

$$\bar{J}_x + \omega_e \tau_e \bar{J}_y = \sigma \mu_e H_0 \bar{v},$$
$$\bar{J}_y - \omega_e \tau_e \bar{J}_x = -\sigma \mu_e H_0 \bar{u} \quad (7)$$

from which we obtain that

$$\bar{J}_x = \frac{\sigma \mu_e H_0 (m\bar{u} + \bar{v})}{1 + m^2},$$
$$\bar{J}_y = \frac{\sigma \mu_e H_0 (m\bar{v} - \bar{u})}{1 + m^2} \quad (8)$$

Since the plates are infinite in extent, all the physical quantities except the pressure depend only on $\bar{z}$ and $\bar{t}$. The physical configuration of the problem is shown in Figure 1. A Cartesian coordinate system is assumed and $\bar{z}$ -axis is taken normal to the plates, while $\bar{x}$ -and $\bar{y}$ -axes are in the upward and perpendicular directions on the plate $\bar{z} = 0$ (origin) , respectively. The velocity components $\bar{u}$, $\bar{v}$, $\bar{w}$ are in the $\bar{x}$- , $\bar{y}$- , $\bar{z}$ -directions, respectively. The governing equations in the rotating system in presence of Hall current, thermal radiation, and chemical reaction are given by the following equations:

$$\frac{\partial \bar{w}}{\partial \bar{z}} = 0 \Rightarrow \bar{w} = w_0 \quad (9)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + w_0 \frac{\partial \bar{u}}{\partial \bar{z}} - 2\bar{\Omega}\bar{v} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{k_0}{\rho}\left(\frac{\partial^3 \bar{u}}{\partial \bar{t}\,\partial \bar{z}^2} + w_0 \frac{\partial^3 \bar{u}}{\partial \bar{z}^3}\right) + g_0\beta(\bar{T} - T_d) + g_0\beta^*(\bar{C} - C_d) + \frac{H_0}{\rho}\bar{J}_y \quad (10)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + w_0 \frac{\partial \bar{v}}{\partial \bar{z}} + 2\bar{\Omega}\bar{u} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{y}} + \nu \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} - \frac{k_0}{\rho}\left(\frac{\partial^3 \bar{v}}{\partial \bar{t}\,\partial \bar{z}^2} + w_0 \frac{\partial^3 \bar{v}}{\partial \bar{z}^3}\right) - \frac{H_0}{\rho}\bar{J}_x \quad (11)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + w_0 \frac{\partial \bar{T}}{\partial \bar{z}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{z}^2} - \frac{Q_0}{\rho C_p}(\bar{T} - T_d) - \frac{1}{\rho C_p}\frac{\partial \bar{q}_r}{\partial \bar{z}} \quad (12)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + w_0 \frac{\partial \bar{C}}{\partial \bar{z}} = D_m \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} - k_1(\bar{C} - C_d) \quad (13)$$

The initial and boundary conditions are

$$\bar{u} = \bar{v} = 0, \quad \bar{T} = T_0 + \varepsilon(T_0 - T_d)\cos\bar{\omega}\bar{t}, \quad \bar{C} = C_0 + \varepsilon(C_0 - C_d)\cos\bar{\omega}\bar{t} \quad \text{at } \bar{z} = 0$$
$$\bar{u} = \bar{U}(\bar{t}) = U_0(1 + \varepsilon\cos\bar{\omega}\bar{t}), \ \bar{v} = 0, \bar{T} = T_d, \ \bar{C} = C_d \quad \text{at } \bar{z} = d \quad (14)$$

We introduce the following dimensionless variables

$$\eta = \frac{\bar{z}}{d}, u = \frac{\bar{u}}{U_0}, v = \frac{\bar{v}}{U_0}, t = \frac{\bar{\omega}}{\bar{t}}, \omega = \frac{\bar{\omega}d^2}{\nu}, \Omega = \frac{\bar{\Omega}d^2}{\nu},$$
$$\lambda = \frac{w_0 d}{\nu}, \theta = \frac{\bar{T} - T_d}{T_0 - T_d}, \emptyset = \frac{\bar{C} - C_d}{C_0 - C_d} \quad (15)$$

We combine the equations (10) and (11) and take q=u+iv. Then the dimensionless governing equations are:

$$\omega\frac{\partial q}{\partial t} + \lambda\frac{\partial q}{\partial \eta} = \frac{\partial^2 q}{\partial \eta^2} + \omega\frac{\partial U}{\partial t} - 2i\Omega(q - U) - k\left(\frac{\omega}{\lambda}\frac{\partial^3 q}{\partial \eta^2\,\partial t} + \frac{\partial^3 q}{\partial \eta^3}\right) + \lambda^2(Gr\theta + Gm\emptyset) - \frac{M^2(1 + im)}{1 + m^2}(q - U) \quad (16)$$

$$\omega\frac{\partial \theta}{\partial t} + \lambda\frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\left(1 + \frac{4}{3R}\right)\frac{\partial^2 \theta}{\partial \eta^2} - \frac{Q_H}{Pr}\theta \quad (17)$$

$$\omega\frac{\partial \emptyset}{\partial t} + \lambda\frac{\partial \emptyset}{\partial \eta} = \frac{1}{Sc}\frac{\partial^2 \emptyset}{\partial \eta^2} - \xi\emptyset \quad (18)$$

where $Gr = g_0\beta(T_0 - T_d)/U_0 w_0^2$ is the modified thermal Grashof number, $Gm = g_0\beta^*\nu(C_0 - C_d)/U_0 w_0^2$ is the modified solutal Grashof number, $Pr = \nu\rho C_p/\kappa$ is the Prandtl number, $M = H_0 d\sqrt{\sigma/\mu}$ is the Hartmann number, $Qh = Q_0 d^2/\kappa$ is the heat source parameter, $R = \kappa k^*/4\sigma^* T_d$ is the radiation parameter, $Sc = \nu/D_m$ is the Schmidt number, and $\xi = k_1 d^2/\nu$ is the reaction parameter.

The modified boundary conditions (14) can be expressed in complex form as

$$q = 0, \quad \theta = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right), \quad \emptyset = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right), \quad \text{at } \eta = 0$$
$$q = U(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right), \theta = 0, \ \emptyset = 0 \quad \text{at } \eta = 1 \quad (19)$$

3 Method of solution

The set of partial differential equations (2.11)-(2.13) cannot be solved in closed form. So to solve these equations we assume that

$$\Re(\eta, t) = \Re_0(\eta) + \frac{\varepsilon}{2}\left(\Re_1(\eta)e^{it} + \Re_2(\eta)e^{-it}\right) \quad (20)$$

where $\Re$ stands for q or θ or $\emptyset$ and $\varepsilon \ll 1$ which is a perturbation parameter. Substituting (20) into equations (16)-(18) and comparing the harmonic and non-harmonic terms, we obtain the following ordinary differential equations:

$$q_0'' - \lambda q_0' - Sq_0 - kq_0''' = -S - \lambda^2(Gr\theta_0 + Gm\emptyset_0) \quad (21)$$

$$q_1'' - \lambda q_1' - (S + i\omega)q_1 - k\left(\frac{\omega i}{\lambda}q_1'' + q_1'''\right) = -(S + i\omega) - \lambda^2(Gr\theta_1 + Gm\emptyset_1) \quad (22)$$

$$q_2'' - \lambda q_2' - (S - i\omega)q_2 - k\left(q_2''' - \frac{\omega i}{\lambda}q_2''\right) = -(S - i\omega) - \lambda^2(Gr\theta_2 + Gm\emptyset_2) \quad (23)$$

$$\theta_0'' - \frac{3\lambda PrR}{3R + 4}\theta_0' - \frac{3RQh}{3R + 4}\theta_0 = 0 \quad (24)$$

$$\theta_1'' - \frac{3\lambda PrR}{3R + 4}\theta_1' - (i\omega Pr + Qh)\frac{3R}{3R + 4}\theta_1 = 0 \quad (25)$$

$$\theta_2'' - \frac{3\lambda PrR}{3R+4}\theta_2' + (i\omega Pr - Qh)\frac{3R}{3R+4}\theta_2 = 0 \quad (26)$$
$$\phi_0'' - \lambda Sc\phi_0' - \xi Sc\phi_0 = 0 \quad (27)$$
$$\phi_1'' - \lambda Sc\phi_1' - Sc(i\omega + \xi)\phi_1 = 0 \quad (28)$$
$$\phi_2'' - \lambda Sc\phi_2' + Sc(i\omega - \xi)\phi_2 = 0 \quad (29)$$

The transformed boundary conditions are

$$q_0 = 0, q_1 = 0, q_2 = 0, \theta_0 = 1, \theta_1 = 1, \theta_2 = 1, \phi_0 = 1, \phi_1 = 1, \phi_2 = 1 \text{ at } \eta = 0$$
$$q_0 = 1, q_1 = 1, q_2 = 1, \theta_0 = 0, \theta_1 = 0, \theta_2 = 0, \phi_0 = 0, \phi_1 = 0, \phi_2 = 0 \text{ at } \eta = 1 \quad (30)$$

where

$S = \frac{M^2(1+im)}{(1+m^2)} + 2i\Omega$ and dashes denote the derivatives w.r.t. η.

Solutions of (24)-(29) under boundary condition (30) are

$$\phi_0 = C_1e^{A_1\eta} + C_2e^{A_2\eta} \quad (31)$$
$$\phi_1 = C_3e^{A_3\eta} + C_4e^{A_4\eta} \quad (3.2)$$
$$\phi_2 = C_5e^{A_5\eta} + C_6e^{A_6\eta} \quad (33)$$
$$\theta_0 = C_7e^{A_7\eta} + C_8e^{A_8\eta} \quad (34)$$
$$\theta_1 = C_9e^{A_9\eta} + C_{10}e^{A_{10}\eta} \quad (35)$$
$$\theta_2 = C_{11}e^{A_{11}\eta} + C_{12}e^{A_{12}\eta} \quad (36)$$

Again the set of partial differential equations (21)-(23) can't be solved in closed form. So we have used multi-parameter perturbation method to solve these equations. We assume that

$$q_0 = q_{00} + kq_{01} \quad (37)$$
$$q_1 = q_{10} + kq_{11} \quad (38)$$
$$q_2 = q_{20} + kq_{21} \quad (39)$$

where $k \ll 1$, for small shear rate [13].

Substituting (37)-(39) in the equations (21)-(23) and equating the coefficients of the same degree terms and neglecting terms of $O(k^2)$, the following differential equations are obtained:

$$q''_{00} - \lambda q'_{00} - Sq_{00} = -S - \lambda^2(Gr\theta_0 + Gm\phi_0) \quad (40)$$
$$q''_{10} - \lambda q'_{10} - (S + i\omega)q_{10} = -(S + i\omega) - \lambda^2(Gr\theta_1 + Gm\phi_1) \quad (41)$$
$$q''_{20} - \lambda q'_{20} - (S - i\omega)q_{20} = -(S - i\omega) - \lambda^2(Gr\theta_2 + Gm\phi_2) \quad (42)$$
$$q''_{01} - \lambda q'_{01} - Sq_{01} = q'''_{00} \quad (43)$$
$$q''_{11} - \lambda q'_{11} - (S + i\omega)q_{11} = \frac{i\omega}{\lambda}q''_{10} + q'''_{10} \quad (44)$$
$$q''_{21} - \lambda q'_{21} - (S - i\omega)q_{21} = q'''_{20} - \frac{i\omega}{\lambda}q''_{20} \quad (45)$$

corresponding boundary conditions are

$$q_{00} = 0, q_{01} = 0, q_{10} = 0, q_{11} = 0, q_{20} = 0, q_{21} = 0 \text{ at } \eta = 0$$
$$q_{00} = 1, q_{01} = 0, q_{10} = 0, q_{11} = 0, q_{20} = 1, q_{21} = 0 \text{ at } \eta = 1 \quad (46)$$

The solutions of equations (40)-(45) under the boundary conditions (46) are

$$q_{00} = 1 + C_{13}e^{A_{13}\eta} + C_{14}e^{A_{14}\eta} + B_1e^{A_1\eta} + B_2e^{A_2\eta} + B_3e^{A_7\eta} + B_4e^{A_8\eta} \quad (47)$$
$$q_{01} = C_{19}e^{A_{13}\eta} + C_{20}e^{A_{14}\eta} + B_{13}e^{A_{13}\eta} + B_{14}e^{A_{14}\eta} + B_{15}e^{A_1\eta} + B_{16}e^{A_2\eta} + B_{17}e^{A_7\eta} + B_{18}e^{A_8\eta} \quad (48)$$
$$q_{10} = 1 + C_{15}e^{A_{15}\eta} + C_{16}e^{A_{16}\eta} + B_5e^{A_3\eta} + B_6e^{A_4\eta} + B_7e^{A_9\eta} + B_8e^{A_{10}\eta} \quad (49)$$
$$q_{11} = H_1e^{A_{15}\eta} + H_2e^{A_{16}\eta} + B_{19}e^{A_3\eta} + B_{20}e^{A_4\eta} + B_{21}e^{A_9\eta} + B_{22}e^{A_{10}\eta} \quad (50)$$
$$q_{20} = 1 + C_{17}e^{A_{17}\eta} + C_{18}e^{A_{18}\eta} + B_9e^{A_{11}\eta} + B_{10}e^{A_{12}\eta} + B_{11}e^{A_5\eta} + B_{12}e^{A_6\eta} \quad (51)$$
$$q_{21} = H_3e^{A_{18}\eta} + H_4e^{A_{17}\eta} + B_{23}e^{A_{11}\eta} + B_{24}e^{A_{12}\eta} + B_{25}e^{A_5\eta} + B_{26}e^{A_6\eta} \quad (52)$$

4 Amplitude and phase difference due to steady and unsteady flow

Equation (47) and (48) correspond to the steady part, which gives u_0 as primary and v_0 as secondary velocity components. The amplitude (resultant velocity) and phase difference due to these primary and secondary velocities for the steady flow are given by

$$R_0 = \sqrt{u_0^2 + v_0^2}, \quad \alpha_0 = \tan^{-1}\left(\frac{v_0}{u_0}\right) \quad (53)$$

where $u_0(\eta) + iv_0(\eta) = q_0(\eta)$

Equations (49)-(52) give the unsteady part of the flow. Thus, unsteady primary and secondary velocity components $u_1(\eta)$ and $v_1(\eta)$, respectively, for the fluctuating flow can be obtained from the following:

$$u_1(\eta, t) = [\mathrm{Real}\, q_1(\eta) + \mathrm{Real} q_2(\eta)]\cos t - [\mathrm{Im}\, q_1(\eta) - \mathrm{Im} q_2(\eta)]\sin t \quad (54)$$
$$v_1(\eta, t) = [\mathrm{Real}\, q_1(\eta) - \mathrm{Real} q_2(\eta)]\sin t + [\mathrm{Im}\, q_1(\eta) + \mathrm{Im} q_2(\eta)]\cos t$$

The amplitude (resultant velocity) and the phase difference of the unsteady flow are given by

$$R_v = \sqrt{u_1^2 + v_1^2}, \quad \alpha_1 = \tan^{-1}\left(\frac{v_1}{u_1}\right), \quad (55)$$

where $u_1(\eta) + iv_1(\eta) = q_1(\eta)e^{it} + q_2(\eta)e^{-it}$

The amplitude (resultant velocity) and the phase difference

$$R_n = \sqrt{u^2 + v^2}, \quad \alpha = \tan^{-1}\left(\frac{v}{u}\right), \quad (56)$$

where u =Real part of q and v =Imaginary part of q.

5 Amplitude and phase difference of shear stresses due to steady and unsteady flow at the plate

The amplitude and phase difference of shear stresses at the stationary plate (η=0) for the steady flow can be obtained as

$$\tau_{0r} = \sqrt{\tau_{0x}{}^2 + \tau_{0y}{}^2}, \qquad \beta_0 = \tan^{-1}\left(\frac{\tau_{0y}}{\tau_{0x}}\right), \quad (57)$$

For the unsteady part of the flow, the amplitude and phase difference of shear stresses at the stationary plate $(\eta = 0)$ can be obtained as

$$\tau_{1r} = \sqrt{\tau_{1x}{}^2 + \tau_{1y}{}^2}, \qquad \beta_1 = \tan^{-1}\left(\frac{\tau_{1y}}{\tau_{1x}}\right), \quad (58)$$

where

$$\tau_{1x} + i\tau_{1y} = \left(\frac{\partial q_1}{\partial \eta}\right)_{\eta=0} e^{it} + \left(\frac{\partial q_2}{\partial \eta}\right)_{\eta=0} e^{-it}$$

The amplitude and phase difference of shear stresses at the stationary plate (η=0) for the flow can be obtained as

$$\tau = \left(\frac{\partial q}{\partial \eta}\right)_{\eta=0} = \sqrt{\tau_x{}^2 + \tau_y{}^2}, \quad \beta_2 = \tan^{-1}\left(\frac{\tau_y}{\tau_x}\right), \quad (59)$$

where τ_x =Real part of $\left(\frac{\partial q}{\partial \eta}\right)_{\eta=0}$ and τ_y = Imaginary part of $\left(\frac{\partial q}{\partial \eta}\right)_{\eta=0}$

The Nusselt number is given by

$$Nu = -\left(1 + \frac{4}{3R}\right)\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} = N_x + iN_y \quad (60)$$

The rate of heat transfer (i.e. , heat flux) at the plate in terms of amplitude and phase is given by

$$\Theta = \sqrt{N_x^2 + N_y^2}, \quad \gamma = \tan^{-1}\left(\frac{N_y}{N_x}\right) \quad (61)$$

The Sherwood number is given by

$$Sh = \left(\frac{\partial \emptyset}{\partial \eta}\right)_{\eta=0} = M_x + iM_y \quad (62)$$

The rate of mass transfer (i.e., mass flux) at the plate in terms of amplitude and phase is given by

$$\Phi = \sqrt{M_x^2 + M_y^2}, \quad \delta = \tan^{-1}\left(\frac{M_y}{M_x}\right) \quad (63)$$

6 Results and discussion

The numerical values of the transient velocity, transient temperature, coefficient of skin friction and Nusselt number are computed for different parameters like modified Grashof number, Grashof number, Hartmann number, Prandtl number, Schmidt number, frequency parameter, suction and injection parameter, heat source parameter, chemical reaction parameter and radiation parameter. The velocity profile and the shearing stress at the plate are illustrated graphically for various values of flow parameters involved in the solution. The values of the parameters $Gr = 5, Gm = 5, M = 3, m = 1, \lambda = 1, Pr = 3, Sc = 4, \Omega = 5, \omega = 5, R = 1, Qh = 5, \xi = 0.1$ are kept fixed throughout the discussion.

The effects of visco-elastic parameter on the thermal, mass and hydrodynamic behaviours of buoyancy-induced flow in a rotating vertical channel have been explained in this study. The visco-elastic effect is exhibited through the non-dimensional parameter K. The corresponding results for Newtonian fluid are obtained by setting K=0. Temperature of the heated wall (left wall) at $\bar{z} = 0$ is a function of time as given in the boundary conditions and the cooled wall at $\bar{z} = d$ is maintained at a constant temperature. Further, it is assumed that the temperature difference is small enough so that the density changes of the fluid in the system will be small. When the injection/suction parameter λ is positive, fluid is injected through the hot wall into the channel and sucked out through the cold wall.

The profiles for resultant velocity R_n for the flow are shown in Figures 2-10 for various flow parameters. Figures 2 and 3 represent the variation of fluid velocity against Grashof number for mass transfer (Gm) and Grashof number for heat transfer (Gr). Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in both heat and mass transfer mechanisms. Gr characterizes the free convection parameter for heat transfer and Gm characterizes the free convection parameter for mass transfer. In both the cases, it is observed that fluid velocity boost up to a considerable amount within the stationary plate with the increase of Gm and Gr.

Figure 4 represent the velocity profile for various values of suction parameter λ. It is noticed that both non-Newtonian and Newtonian fluid first boost up to a considerable amount and then diminishes. With the rising value of suction parameter λ fluid velocity experiences an accelerating trend.

The effects of Hartmann number and hall parameter on fluid velocity have been cited in figure 5 and 6. An inclined trend is observed for both kind of fluid velocity with the growing nature of magnetic parameter and hall parameter. The maximum effect of both the parameters on visco-elastic fluid and Newtonian fluid is seen in the neighbourhood of the plate.

In Figure 7 it is embodied that with the increase in rotation parameter Ω resultant fluid velocity follows an increasing path for $\eta > 0.5$ for both simple as well as elastico-viscous fluid flow. This effect is due to the rotation effects being more dominant near the walls, so when Ω reaches high values, the secondary velocity component v decreases with increase in rotation parameter while approaching to the right plate.

Figures 8, 9, 10 illustrate the behaviour of fluid flow for various values of radiation parameter R, chemical reaction parmeter ξ and heat source parameter Qh. In all the cases with the enhancement of radiation, chemical and heat source parameter resultant velocity subdues and matching movement is observed for both kind of fluid flow mechanisms.

The study of skin friction experienced by the governing fluid flow gives the significance of the concerned problem. So knowing the velocity field, the shearing stress at the plate is obtained for various values of visco-elastic parameter. Figures 11-17 portray the nature of viscous drag formed during the motion of Newtonian and non-Newtonian fluids.

Figure 11, 12 and 13 depict that the magnitude of skin friction improved along with the amplified values of Grashof number for heat and mass transfer (Gr and Gm) and suction parameter λ for Newtonian as well as non-Newtonian cases.

But a complete reverse trend is observed in Figures 14, 15 and 16. Figures 14, 15 and 16 represent the viscous drag against various values of hall parameter m, heat source parameter Qh and radiation parameter R. In all the cases it is pragmatic that shearing stress profile follow a diminishing trend with the increasing values of those flow parameters.

Figure 17 describe the viscous drag against rotation parameter Ω. In this case also skin friction increases with the increase in rotation parameter.

Figures 18 and 19 illustrate the Nusselt number and mass concentration against ω. It is observed in both the cases that the rate of heat transfer and rate of mass transfer are not affected significantly during the changes made in visco-elasticity of the fluid flow.

7 Conclusions

The influence of radiation parameter and hall current on unsteady MHD heat and mass transfer of an oscillatory convective flow in a rotating vertical porous channel with injection is studied analytically. Computed results are presented to exhibit their dependence on the important physical parameters. We conclude following from the numerical results:

- The velocity profile shows an enhancement trend in the neighbourhood of the plate and then follows a decreasing path.
- The fluid is decelerated with the increasing values of visco elastic parameter in comparison with the Newtonian fluid.
- The fluid is decelerated with the increasing values of visco elastic parameter in comparison with the Newtonian fluid.
- The shearing stress formed at the plate is subdued with the growing trend of visco-elastic parameter for m, Qh, R.
- The skin friction increases with increase in Gr, Gm, λ and Ω.
- The rate of heat transfer and rate of mass transfer are not significantly affected by visco-elastic parameter.

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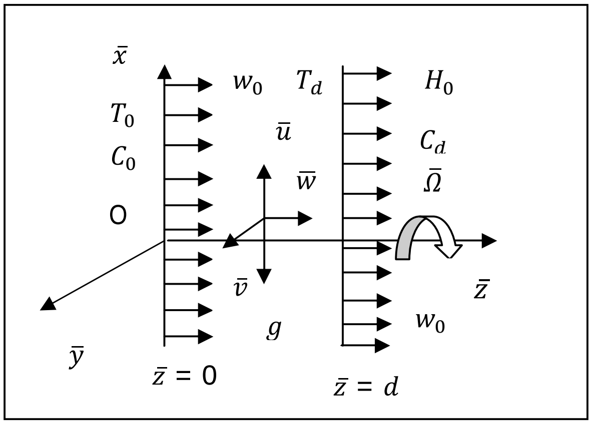

Figure 1: Physical configuration of the problem.

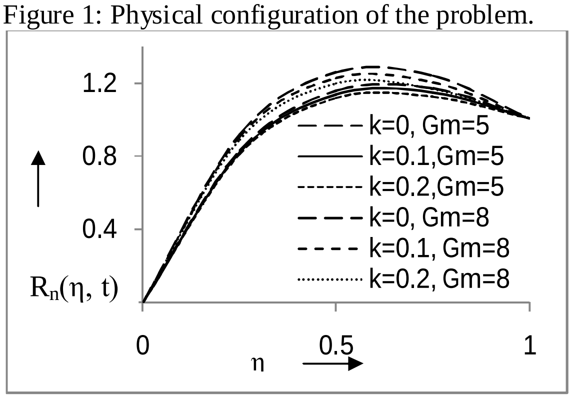

Figure 2: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of Gm.

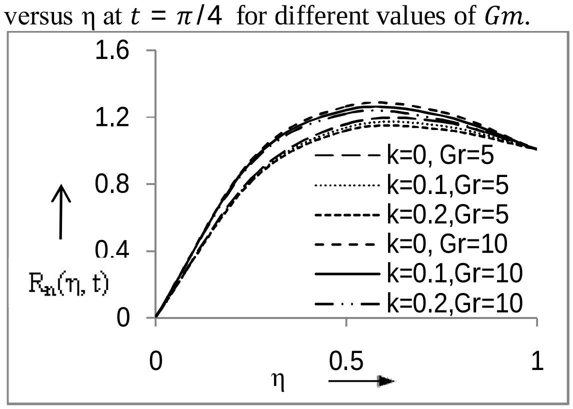

Figure 3: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of λ.

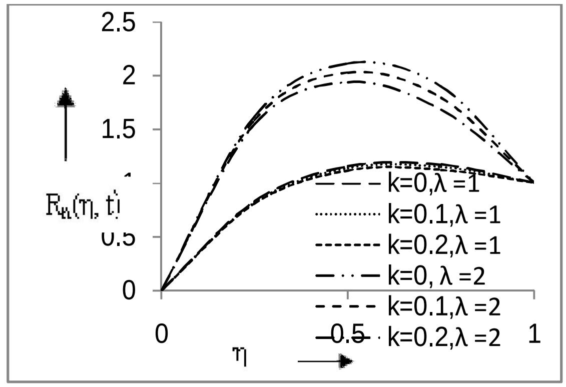

Figure 4: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of Gr.

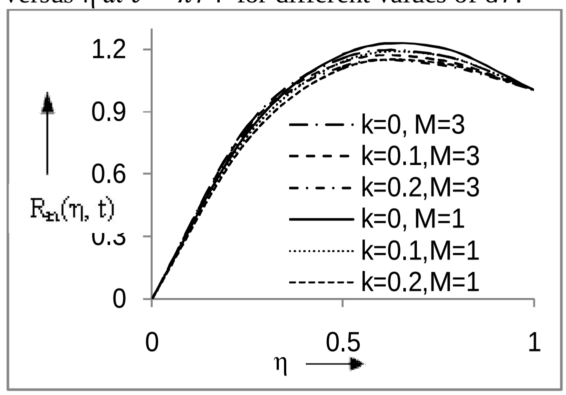

Figure 5: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of M.

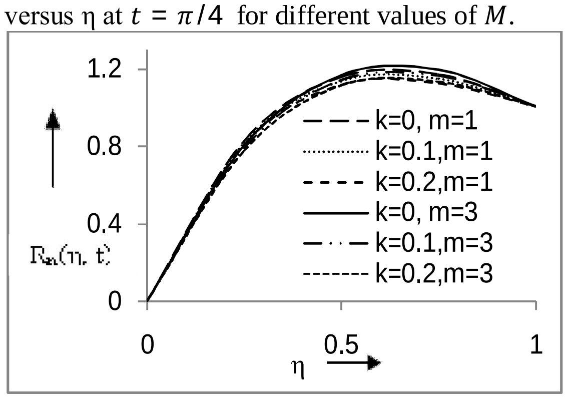

Figure 6: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of m.

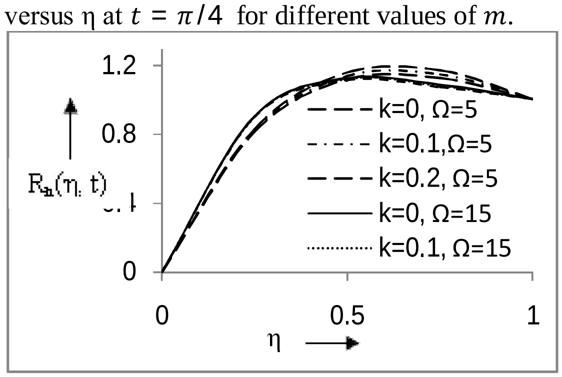

Figure 7: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of Ω.

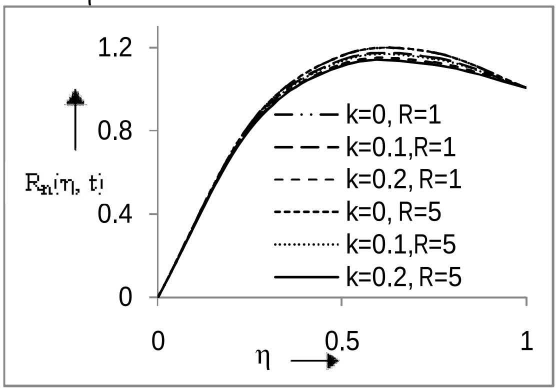

Figure 8: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of R.

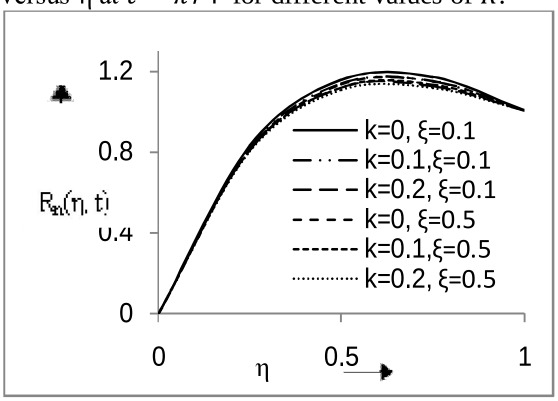

Figure 9: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of ξ.

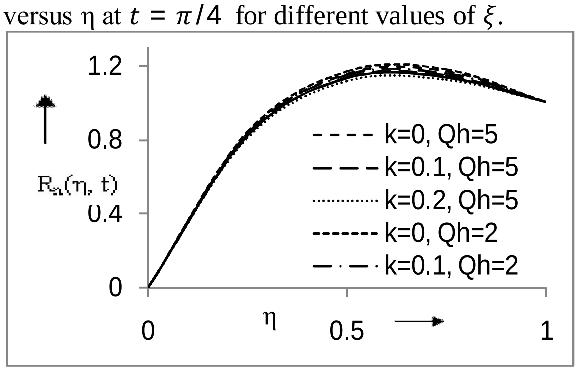

Figure 10: Resultant velocity R_n due to u and v versus η at $t = \pi/4$ for different values of Qh.

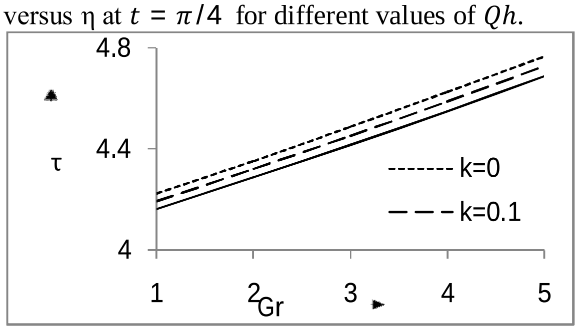

Figure 11: Variation of skin friction τ against Gr at $t = \pi/4$.

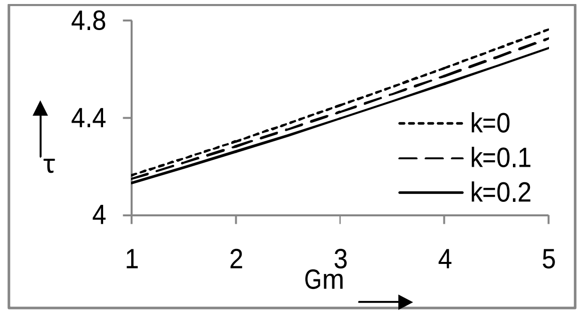

Figure 12: Variation of skin friction τ against Gm at $t = \pi/4$.

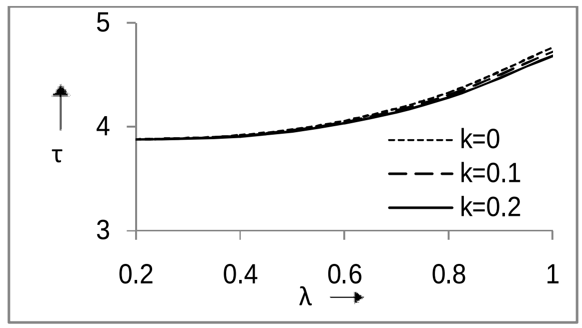

Figure 13: Variation of skin friction τ against λ at $t = \pi/4$.

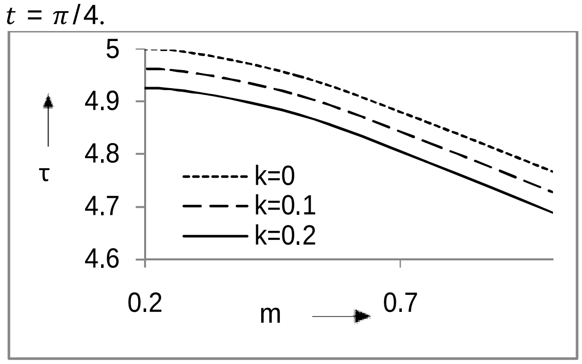

Figure 14: Variation of skin friction τ against m at $t = \pi/4$.

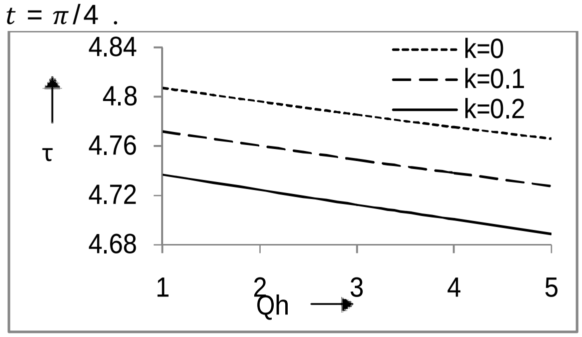

Figure 15: Variation of skin friction τ against Qh at $t = \pi/4$.

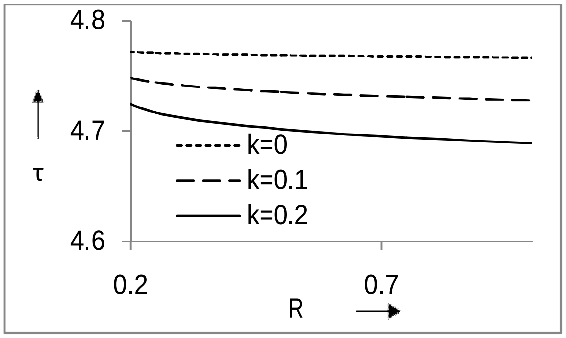

Figure 16: Variation of skin friction τ against R at $t = \pi/4$

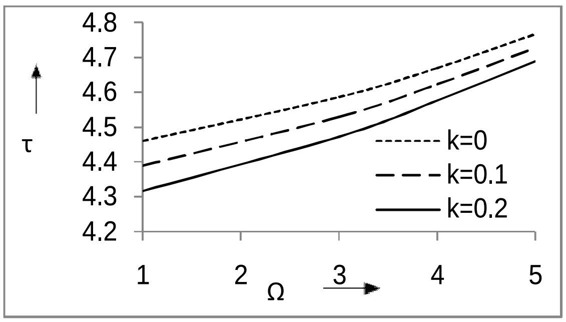

Figure 17: Variation of skin friction τ against Ω at $t = \pi/4$.

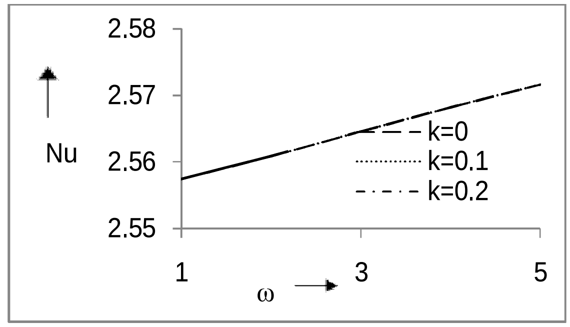

Figure 18: Variation of Nusselt number Nu against ω at $t = \pi/4$.

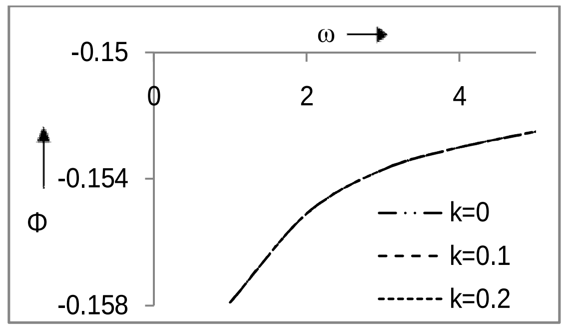

Figure 19: Variation of concentration profile against ω at $t = \pi/4$