Optimal Low-Thrust Trajectories to Reach the Asteroid Apophis

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Abstract: Apophis is considered by the Greeks as chaos, the God of destruction. For this reason, in current times, this name returns to scene to represent a destructive potential threat to the Earth. It is the name of an asteroid that orbits a region that represents a high risk of collision with the Earth. This asteroid was discovered in 2004 and received the initial name of 2004MN4. Then it was named Apophis, and several researchers and institutes are monitoring and designing missions with the goal of finding more information about the asteroid. The present research has the goal of finding optimal (in the sense of minimum fuel consumption) interplanetary missions, based in low thrust propulsion systems to send a spacecraft to Apophis. The spacecraft is assumed to leave the Earth from a Low Earth Orbit (LEO). An indirect optimization method is used in the simulations to find the trajectories. Two different engines will be used to determine the trajectories: the PPS1350 and the Phall 1. This is done to get some insights of the differences between a propulsion system that has a fixed magnitude for the force and one that has a magnitude that varies with the distance between the spacecraft and the Sun. Another reason to perform this research is to test the parameters of the Phall 1, that is an engine under development.

Key-Words: Astrodynamics, Celestial Mechanics, Space Trajectories, Low-Thrust, Solar Electric Propulsion.

1. Introduction

Approximately 1000 asteroids are currently known to have orbits that approach significantly the trajectory of the Earth in space, so constituting a potential threat to the planet. These asteroids are usually designated by the initials NEA (Near Earth Asteroid). The information about the asteroid used in the present simulations were collected from the JPL database, which is part of the NASA project "Near Earth Object Search Program" of the Jet Propulsion Laboratory (JPL), located in Pasadena, California.

Near-Earth Objects (NEOs) are comets and asteroids that were pulled by the gravitational forces of the planets to achieve orbits that pass by the vicinity of the Earth. They are mainly composed by water ice and dust particles. Opposite to comets, which were probably formed in the outer planetary system, most of the cold rocky asteroids were formed in the inner solar system, the majority of them in the regions between the orbits of Mars and Jupiter.

The main goal of the present paper is to find optimal trajectories for a spacecraft to reach the asteroid 2004MN4 (Apophis), which is part of the group of asteroids that have orbits near the Earth. The main reason to perform this study is that it may be necessary to reach this asteroid in order to explode it or to install an engine that is able to maneuver this asteroid to avoid a collision with the Earth in the future.

Regarding the orbital maneuvers, two different low thrust propulsion systems will be used. The first one uses an electric propellant that has been used to accomplish the maneuvers required by several satellites to keep them in orbits compatible with their missions. This type of propulsive system is the most used in missions toward asteroids.

Indirect optimization methods are suitable to find the most economical transfers when those low thrust trajectories are used. In this model, a limited force is applied during a finite time and it is necessary to integrate the equations of motion over time to follow the motion of the spacecraft. Several results exist in the literature for this model, like the works of Lawden (see references [1] and [2]). He developed one of the most used approaches, when considering this continuous assumptions for the thrust, the so called "primer-vector theory". Several other more recent references can be found working with the low thrust hypothesis, like references [3] to [12]. In the present paper, the theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary conditions is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated with the problem and to evaluate the optimal structure of the "switching function", that is a function that determines the instants where the spacecraft operates with the engine on or off.

A second usual alternative approach for orbital maneuvers uses the idea of an impulsive thrust, where the propulsive force is assumed to have an infinity magnitude and to be applied during a time that can be neglected. This option is very popular in the literature, mainly because it can be calculated and implemented very easily and many references used this approach. Some examples are shown in references [13] to [17].

Regarding missions to other bodies of the Solar System, in particular the Moon, the concept of gravitational capture has also been considered in several space maneuvers. In this type of maneuver the perturbation of a third-body generates a force that can be used to decrease the consumption of fuel. References [18] to [20] show this idea in more details.

Another idea that appears very often in the space program, in order to find alternatives to reduce fuel expenditure in interplanetary missions, is the so called close approach maneuvers. The main idea is to use a close approach between a spacecraft and a celestial body (the Moon, one planet, etc) to add or subtract energy from the spacecraft, getting the same effect of applying an impulse to it, but with zero cost in terms of fuel. Several missions to the planets, comets and asteroids used or are planning to use this concept to realize the mission. References [21] to [42] show more details, as well as missions using this technique.

In the present problem formulation, the spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and direction will be determined by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to make the spacecraft to leave from a Low Earth Orbit (LEO). After leaving the Earth's sphere of influence, the low thrust propulsive system is activated to control the interplanetary trajectory of the spacecraft. For the electric system, the available power is proportional to the inverse of the square of the distance from the Sun, while in the second test the magnitude is assumed to be constant. The electric propulsion system is assumed to have the force provided by the "PPS 1350 ion thrusters". After testing this propulsion system, the Phall 1 thruster that is under development at the University of Brasilia (Brazil), was also used for the simulations. These two choices were considered in order to show the possibility of reaching this important asteroid with the use of different engines and to know the transfer times and consumptions involved for different values of the magnitude of the thrust.

2. Solar Electric Propulsion (SEP)

The solar electric propulsion might be the best option for spacecraft missions in the future, due to its high specific impulses when compared to the chemical propulsion. Electric propellants are being extensively used to perform the orbital correction maneuvers of satellites that travel around the Earth. It is also been used as a primary propulsion system in missions toward other bodies of the Solar System. Both NASA and ESA have launched spacecrafts which used SEP (Solar Electric Propulsion) as the primary propulsion system, like NASA's DS1 and ESA's Smart-1 to the Moon and to the comet Borrelly. The efficient use of ion propulsion system fuel and electricity, which modern spacecraft are equipped, are used to travel farther, faster and cheaper than any other propulsion technology currently available.

3. Description of the Problem

The spacecraft is considered a point with variable mass m. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only the equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun's gravitational acceleration $\vec{g}(r)$ and by the propulsion system of the vehicle that has a thrust T.

Therefore, for this propulsion system, the available power and the magnitude of the thrust are assumed to vary with the inverse of the square of the distance of the spacecraft from the Sun. This thrust is the control force of the satellite during the heliocentric arcs. The approach used here is to try to get the minimum consumption of fuel, measured by the final mass of the spacecraft. Since the thrust appears linearly in the equations of motion, a bang-bang control, which consists of alternating ballistic arcs with arcs of maximum thrust, will give the solution. The trajectory, in general, is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, which optimal direction will be supplied by the optimization procedure.

To avoid numerical problems, the variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and the mass of the spacecraft at the beginning of the mission (when in the initial parking orbit) as values of reference. The boundary conditions are imposed at the junctions between the trajectory arcs.

So, the study initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_{\oplus}(t_i)$, that coincides with the position of the Earth, considering the velocity $\vec{v}_i$ as a free parameter. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_{\oplus}(t_i)$, assuming that a rocket thruster is used to make the spacecraft to leave the Low Earth Orbit (LEO) with an impulsive maneuver. The mass of the vehicle, when in this initial LEO, is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where , v_e and v_c are the escape and the circular velocity at the LEO radius [6].

In the next equations $\vec{r}_i$, $\vec{r}_{\oplus}$, $\vec{v}_i$ and $\vec{v}_{\oplus}$ represent the initial position of the spacecraft, the position of the Earth, the initial velocity of the spacecraft and the velocity of the Earth, respectively.

The initial mass (m_i) of the spacecraft at the Earth's sphere of influence can be written by the following relationship [3]:

$$m_f = m_i \exp\left(-\frac{\Delta V}{I_s g_0}\right) \qquad (1)$$

where,
m_f is the final mass of the spacecraft, I_s is the specific impulse of the engine, g_0 is the acceleration due to the gravity of the Earth at sea level and ΔV is the velocity increment produced by the electric propulsion.

At the final point the position and the velocity vectors of the spacecraft and the respective values for the asteroid have to coincide. Then, the theory of optimal control provides the control law and the necessary boundary conditions for optimality.

4. Optimization Procedures

Optimal control theory is used to maximize the spacecraft final mass, what is equivalent of minimizing the fuel consumed. The equations of motion are:

$$\begin{cases} \dot{\vec{r}} = \vec{v} \\ \dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \frac{\vec{T}}{c} \\ \dot{m} = -\frac{T}{c} \end{cases} \qquad (2)$$

where the vectors $\vec{r}$, $\vec{v}$ and $\vec{T}$ represents the position, velocity and the thrust of the spacecraft, respectively, m is the instantaneous mass of the spacecraft, $\vec{g}$ is the gravity force and c is the effective exhaust velocity of the rocket thruster.

Applying the theory of optimal control, the Hamiltonian function (*H*) is defined by [1]:

$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t \left(\vec{g} + \frac{\vec{T}}{m}\right) - \lambda_m \frac{T}{c} \tag{3}$$

where $\vec{\lambda}$ are the Lagrange multipliers and its subscripts indicate the related variable (position, velocity and mass).

An indirect optimization procedure is then used to maximize the payload mass, which is equivalent to minimize the fuel expenditure. According to the Pontryagin's Maximum Principle, the optimal control is the one that maximize the Hamiltonian *H*.

The maximum propulsion (T_{Max}), that is the nominal thrust T_o at 1 AU and the electrical power are given by (see reference [3]):

$$\begin{aligned} P_0 &= \frac{T_0 c}{2\eta} \\ T_{Max} &= \frac{T_0}{r^2} \end{aligned} \tag{4}$$

where P_o is the electrical power at 1 AU, T_o is the respective force, T_{Max} is the maximum level of the engine and η is a parameter that depends on the engine.

The optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange Equations). They are:

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\partial \vec{g}}{\partial \vec{r}} - S_f \frac{\partial T}{\partial \vec{r}} \tag{5}$$

$$\dot{\vec{\lambda}}_v^t = -\vec{\lambda}_r^t \tag{6}$$

$$\dot{\lambda}_r = \lambda_v^t \frac{\vec{T}}{m^2} \tag{7}$$

where S_f is defined by Equation (8).

The optimal control gives the thrust direction and magnitude of the thrust by:

$$\begin{cases} \vec{T} \parallel \vec{\lambda} \\ H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t \vec{g} + T\left(\frac{\lambda_v}{m} - \frac{\lambda_m}{c}\right) \\ S_f = \frac{\lambda_v}{m} - \frac{\lambda_m}{c} \end{cases} \tag{8}$$

$$T_{Max} = \begin{cases} \frac{T_0}{r^2} \rightarrow S_f > 0 \\ 0 \rightarrow S_f < 0 \end{cases} \tag{9}$$

The necessary optimal conditions are showed in more detail in reference [6]. Then, at the initial point, we have:

1. $\vec{r}_0 = \vec{r}_\oplus$;
2. $(\vec{v}_0 - \vec{v}_\oplus)^2 = \vec{v}_{\infty 0}^2$;
3. $\vec{\lambda}_{r_0}$ and λ_m are free;
4. the necessary optimal condition of the state is that $\vec{\lambda}_{v0}$ (primer vector) has to be parallel to the hyperbolic velocity;

At the final point we have:

1. $\vec{\lambda}_{v_f}$ has to be parallel to the hyperbolic velocity, $\vec{\lambda}_{r_f}$ has to be parallel to the radius vector and $\vec{\lambda}_{r_f}^t \vec{v}_f + \vec{\lambda}_{v_f}^t \vec{g} = 0$;
2. the final values of λ_{m_f} and H_f depends on the control model that was considered in the maneuver;
3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

5. Results

The asteroid Apophis has its keplerian elements shown in Table 1. Note that the combination of semi-major axis and eccentricity results in a perihelion of 0.746 AU, which is well inside the orbit of the Earth. This fact implies that there are two potential crossing points between the orbits.

Table 1– Keplerian Elements of the Asteroid Apophis

Name	Apophis
Epoch	54200
a	0.9222614 AU
e	0.19105942
i	3.33131º
Ω	126.38557º
ω	204.45915º
M	307.3630785º
r_a	1.098468 AU
r_p	0.746055 AU

5.1 Mission using the propulsion system PPS1350 – ESA

The characteristics of the spacecraft propulsion system, when using the PPS1350, are described below. The initial mass of the spacecraft, assumed to start its motion in a circular orbit with an altitude of 200 km (LEO), is 2133.30 kg. The engine is supposed to have a specific impulse I_s = 1550 s, a specific energy ε = 0.06, a thrust level of T = 2.70 mN at AU. It is important to note that these characteristics of the low thrust engine are based in solar energy. The magnitude of the force depends on the solar radiation received, that depends on the distance between the spacecraft and the Sun. So, the initial conditions can specify the magnitude of the force at a specific distance, chosen to be at the Sun-Earth distance. The results obtained by this type of propulsion system will be later compared with the ones obtained by a propulsion system that have a fixed value for the magnitude of the force. For the calculations shown below, the time zero corresponds to the date initial launch.

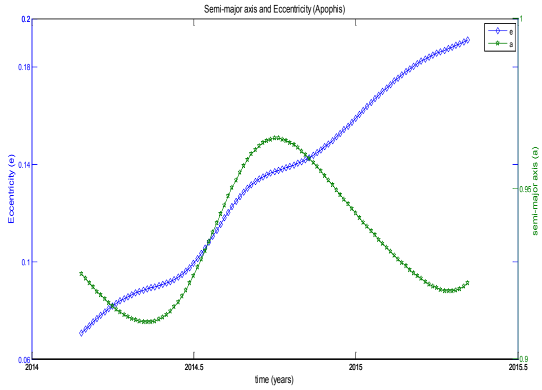

Figure 1 - Evolution of semi-major axis (AU) and eccentricity (admensional) for the transfer trajectory of the spacecraft from the Earth to the asteroid Apophis using the PPS1350.

Figure 1 shows the evolution of the semi-major axis (in astronomical units) and the eccentricity (admensional) of the transfer trajectory for the spacecraft going from the Earth to the asteroid Apophis. It is visible that the eccentricity has a monotonically increases, while the semi-major axis show oscillations. Those adjustments in the orbit of the spacecraft are made in order to cause an encounter between the spacecraft and the comet.

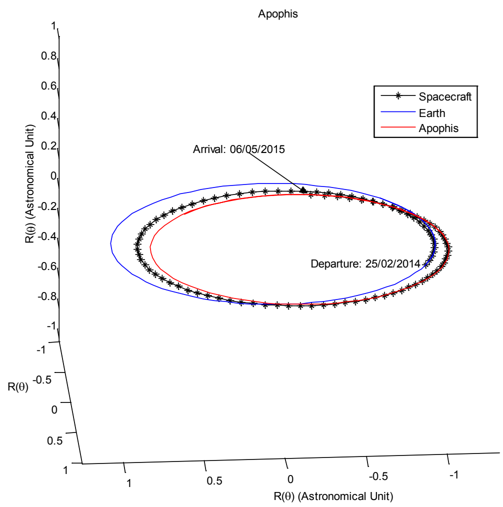

Figure 2 - Transfer trajectory of the spacecraft from the Earth to the asteroid Apophis using the engine PPS1350. Departure: 25/02/2014, arrival: 06/05/2015, duration of the maneuver: 435 days.

Figure 2 shows the transfer trajectory in the heliocentric system. The departure date for this maneuver is 25 of February, 2014, with an arrival date of 06 of May, 2015. It means that the duration of the maneuver is 435 days.

The switching function (S_f) is the final value of a quantity (S) that specifies the regions where the thrust is turned on ($S_f > 0$, red circle) and the regions where it is turned off, which represents a coast arc ($S_f < 0$, blue circle). Figure 3 shows these results. Note that, for this particular solution, the engine stays switched on for the entire trajectory. Other solutions may have a different behavior and may alternate burning arcs with coasting arcs, similar to what happens in the next simulation.

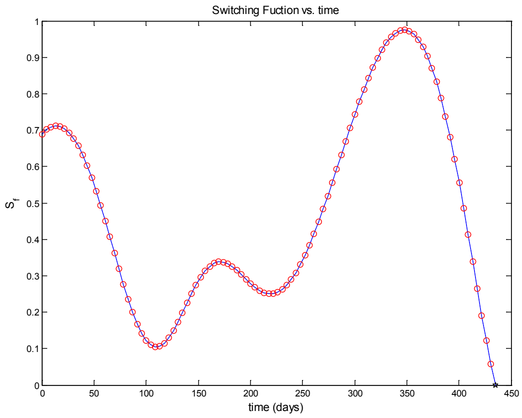

Figure 3 - The Switching Function for the Trajectory to the asteroid Apophis.

Figure 4 shows the evolution of the energy and the Hamiltonian of the system. The duration of the mission (ΔT) is approximately 435 days and the consumption of fuel (Δm) is 500.41 kg, delivering a total mass of 1632.89 kg in the asteroid.

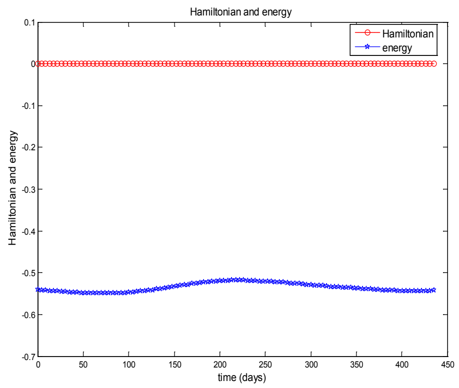

Figure 4 - The Hamitonian and the energy of the transfer trajectory.

Those results were obtained by using the optimization procedure to find optimal trajectories, in terms of the maximization of the spacecraft final mass (so, the minimization of fuel consumption). It is also possible to consider some other constraints, like the duration of the travel time or the inclusion of regions where it is not allowed to turn the engine on, etc. So, these trajectories depend on the mission objectives. It is also possible to reduce the travel time with some more spend of propellant.

5.2 Mission Apophis: Phall 1 - UNB

Now we perform similar simulations with the goal of testing a different propulsion system, the so called Phall 1. It is a system under development by the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), in Brazil. It uses a plasma propulsion system based on Stationary Plasma Thrusters (SPT). This project uses permanent magnets to generate the magnetic field, so reducing the electric consumption. It delivers a constant force to the spacecraft, opposite to what happens to propulsion systems based on solar energy, as shown in the previous section. The characteristics of this propulsion system are:

1. the initial mass of the spacecraft, that is assumed to start its motion with an altitude of 200 km in a circular LEO, is 2133.30 kg;
2. the specific impulse of the engine is I_s = 1607 s and the specific energy is ε = 0.06;
3. T = 2.126mN is the level of the magnitude of the thruster.

Figure 5 shows the trajectory of the spacecraft, marking the points where it leaves the Earth and arrives at the asteroid. The mission duration time (ΔT) is approximately 325 days and the fuel consumption (Δm) is 604.26, delivering a total mass of 1529.04 kg in the asteroid.

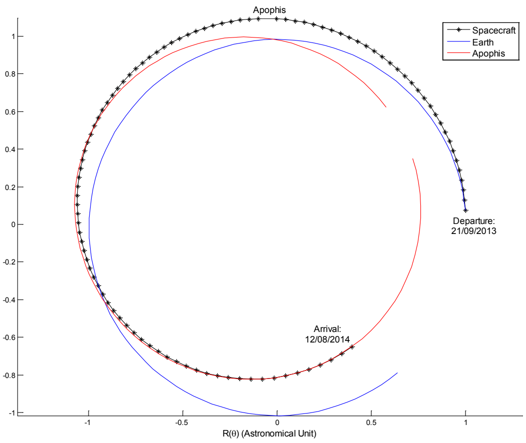

Figure 5 - Transfer trajectory of the spacecraft from the Earth to the asteroid Apophis using the thruster Phall 1 (UNB). Departure: 21/09/2013, arrival: 12/08/2014, duration of the maneuver: 325.69 days.

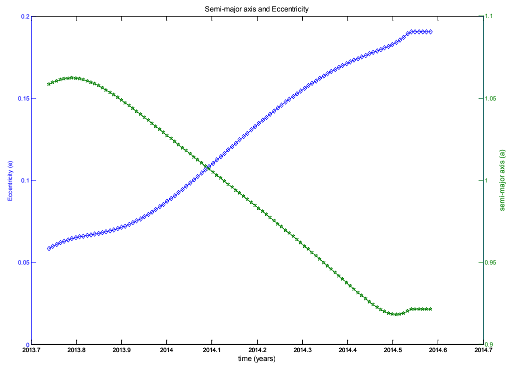

Figure 6 – The semi-major axis (AU) and eccentricity (admensional) for the transfer trajectory of the spacecraft from the Earth to the asteroid Apophis, with Phall 1 (UNB).

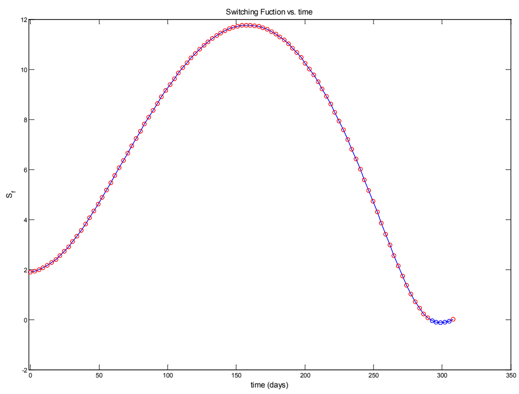

Figure 7 - The Switching Function for the transfer using the Phall 1 propulsion system.

Figure 7 shows the Switching Function. It is clear that there is an alternation between the propulsion arcs and the arcs without propulsion at the end of the transfer (time near 300 days). This fact is obtained from the switching function (S_f) as a solution of the optimization procedure. The duration of the transfer is about 325 days.

Trajectories obtained by using the Phall 1 propulsion system had been analyzed combined with gravity assisted maneuvers and it was verified that this is an interesting approach that reduces the amount of fuel consumed. This concept makes possible the use of launch vehicles that are under development, like the VLS-2, under development in Brazil, which can inject in LEO (Low Earth Orbit) a satellite medium sized. This satellite can later use the solar electric propulsion (SEP) to place it in any desired orbit, maximizing the use of fuel by combining the propulsion system with maneuvers that use assistance of the gravity [7] .

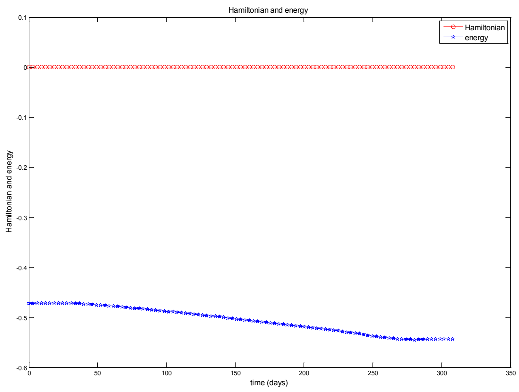

Figure 8 - The Hamitonian and the energy of the transfer trajectory when using the Phall 1.

The performance parameters of Phall 1 are competitive with known electromagnet Hall thrusters found on the literature [6].

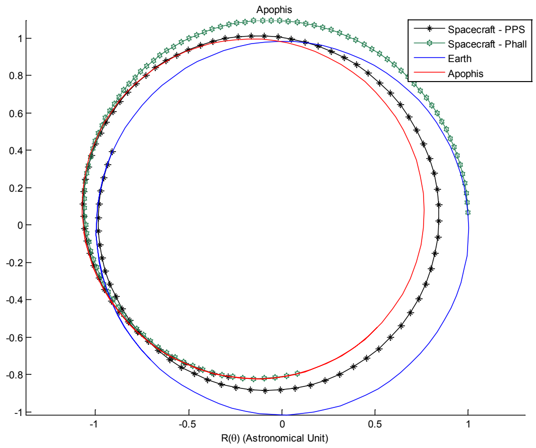

Figure 9 - The comparison of the trajectories when using the two different propulsion systems.

From the simulations, it is observed that the results for both propulsion systems are similar. They also indicates that Phall 1 is better than PPS1350 (Table 2 and Figure 9) in terms of time, but worst in terms of fuel consumption. Those differences come from the fact that the force is constant for the Phall 1. So, although the magnitude of the force of the PPS-1350 has an initial magnitude (when near Earth) larger, it decreases when the spacecraft goes to more distant places. This is why, in terms of average, Phall 1 has a larger magnitude for the propulsion system and this fact explains why it can make the maneuver faster, but expending more fuel.

Table 2 – The comparison between the travel time and fuel consumption for both propulsion systems, Phall 1 and PPS-1350

	PPS1350	***Phall 1***
ΔT(days)	435	325.69
Final mass in the asteroid (kg)	1632.89	1529.04
Fuel consumed	500.41	604.26

The literature shows results for similar missions (Direct, EGA, EMGA) for asteroids 200TC70 and 1989UQ (see references [6], [7] and [9]). The same parameters are analyzed, such as: semi-major axis (a), eccentricity (e) switching function (Sf), Hamiltonian, energy of the orbit, the initial mass, the consumption of propellant (Δm) (parameter optimization), ΔV electric (ΔVel) and the important dates for the mission (departure, arrival and flyby).

5. Conclusion

Analysis like the ones made in the present paper can help the plans for a mission to the asteroid Apophis, which is an asteroid that may collide with the Earth in the future. It means that a mission to this asteroid will be required to destroy or to change the orbit of this asteroid.

The results showed here gives an idea of how much fuel and how much time is required to accomplish this task for two different types of propulsions. It also shows that the Optimal Control technique applied here works well in this type of problem.

Regarding the comparison of the propulsion systems, this set of simulations shows that the propulsion system under development at the Brasilia University has some advantage over the more established PPS-1350, in terms of transfer time, and some disadvantage in terms of fuel consumption required by the maneuvers.

Of course more studies have to be made, but the information available here can be a good starting point for more detailed maneuvers, considering different types of propulsion systems, as well as more complex dynamics that takes into account the gravity of other bodies, like the Sun, or any other forced not modeled here.

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