Distinguishing profile deviations from a part's deformation using the maximum normed residual test

ALI AIDIBE, ANTOINE S. TAHAN, GAD N. ABENHAIM Mechanical Engineering Department École de technologie supérieure 1100, rue Notre-Dame Ouest Montréal (Québec) H3C1K3 CANADA ali.aidibe.1@ens.etsmtl.ca

Abstract: - Non-rigid parts, in *free-state*, may have a considerable different shape than their nominal model due to dimensional and geometric variations of manufacturing process, gravity loads and residual stress induced distortion. Therefore, sorting profile deviation from a part's deformation by comparing the part's nominal shape to its scanned *free-state* shape is a challenging task. This task is a key step in the Iterative Displacement Inspection (IDI) algorithm used for the inspection of non-rigid parts without the use of costly specialized fixtures. This paper proposes the use of the statistical maximum normed residual test to improve the aforementioned identification task. Thirty two simulated manufactured parts are studied to show that the proposed method reduces the type I and II identification error of the IDI method.

Key-Words: - Rigid registration, non-rigid registration, quality control, tolerancing, inspection, metrology, non-rigid parts, deformation, Geometric dimensioning and tolerancing (GD&T).

1 Introduction

One of the important tasks that have to be taken into consideration in the industry is the inspection of manufactured parts. At the end of the manufacturing process we must verify if the produced part respects the functional requirements under a given tolerance. The problem of the dimensional and geometric variations (GD&T) on mechanical components has been studied by many researchers in the case of rigid parts. Despite those research there still no viable solutions in the case of non-rigid parts. Nonrigid parts, in *free- state*, may have a different form than their CAD model due to inherent variation of manufacturing process, gravity loads, residual stress induced distortion. and/or assembly force. Specifically, the inspection of such parts poses difficulties and has significant costs industries because they need specialized fixtures. Therefore Automatic inspection becomes essential.

This paper proposes a method enabling the distinction between the geometrical defects due to error in the manufacturing process and the deformations due to the flexibility of the parts in the case of thin shells during the inspection process. The distinction allows for the detection of profile variations without the need of conformation jig.

Fig.1 illustrates an example of a conformation jig used in the inspection of an automotive body part.

Extending the work of Abenhaim and Tahan [1] on the inspection of non-rigid part, this paper focuses on the identification module of the iterative displacement inspection (IDI) method proposed by the latter with the following assumptions:

- The part to inspect is a quasi-constant thin shell.
- In a *free-state*, the manufactured parts elastic deformation is greater than the tolerances required profile.
- The defects are not distributed all over the part. In other words, they are localized.

• Inspection is limited to the defects in the surface profile as defined by ASME Y14.5-2009.



Fig.1 Inspection of non-rigid part using a jig -Source: Volvo, PREVOST Car

Firstly a short background research is presented. Afterward, the general methodology is exposed in order to focus on the implementation of statistical maximum normed residual test in the IDI algorithm. Finally, the implementation and evaluation of the proposed approach is tested on many case studies representing typical parts in the transport industry.

2 Background

A part is considered 'non-rigid' if the typical value of the deformation resulting by applying a force of 15-20 lbf/linear foot is more than 10% of its assigned profile tolerance. The value of force is dictated by what is commonly used in a manual assembly line. With this definition, many types of parts in aeronautics and automotive industry can be grouped. For example, the wallboard (Skin), pieces of thin-walled structure (spar, ribs, etc...) and components for the interior finish of such planes shown in Fig.2. The quality control of such parts requires a special approach. As mentioned previously, jigs and fixtures are needed to constrain dedicated and follow the component during the inspection. Therefore the aim of our research is to identify the magnitude and location of defects induced by the inherent variation manufacturing processes, from a cloud of points collected in a condition without specialized fixtures (Fig.3).



Fig.2 Examples of non-rigid parts in the aerospace industry

Abenhaim and Tahan [1] developed IDI algorithm. The IDI allows for the surface profile inspection of a non-rigid part without the need of a specialized jig. The method works through a comparison of two sets of points, one from the mesh of the CAD model and one from the scanned manufactured part, despite the significant difference in their respective geometries. The method outlined operates by iteratively deforming smoothly the CAD mesh until it matches the scanned part without profile deviation or measurement noise. This matching process is made possible with the introduction of the identification method, which enables the effects of profile deviations to be distinguished from the deformations due to the positioning of the part and its flexibility. This work focuses on improving the

IDI identification techniques to distinguish between the defaults and the deformations.



Fig.3 The concept of the inspection of non-rigid parts: (a) CAD, (b) Free State (with deformations and defects) (c) CAD deformed (d) Profile deviations

The state of the art in machine vision inspection research and technology has been presented recently by Malamas et al. [21]. They classified the contemporary applications in the industry according to their measured parameters (i.e. dimensions, surface, assembly and operation) and to their degrees of freedom. After the removal of manufacturing forces, flexible part could be subjected to significant distortion. This free-state variation is principally due to weight and flexibility of the part and the release of internal stresses resulting from fabrication. The inspection of freeform surfaces belonging to non-rigid parts has been presented by Ascione and Polini [4]. In their proposed a fixture work, they assembly methodology that enables both to simulate the mating part interface and to locate the part in coordinate measuring machines working volume. Then, they used a method for the evaluation of the actual surface with respect to its nominal model based on their Euclidean distance. Finally, a method based on a finite element analysis was proposed to evaluate the effects of the measuring force, induced by the touch probe on the inspected surface, on the measurement results. For the alignment of deformable parts that do not require any fixtures, Weckenmann et al. [25] as well as Jaramillo et al. [13] proposed an approach based on a finite element method to obtain a physical deformation of the original CAD model, and radial basis functions to approximate this deformation faster and in realtime, opening the door to on-line inspection of deformable parts. Li and Gu [26] provided a comprehensive literature review of methodologies, techniques and various processes of inspections of

19

parts with free-form surfaces. They discussed the profile verification techniques for free-form surface inspection with and without datums. The inspection of free-from surfaces includes two major processes: (1) the localization of measurement data to design coordinate system based on the datum reference information or a number of extracted surface features: and (2) the further localization based on the surface characteristics so that the deviation of the measured surface from the design model is minimized. Caulier [7] proposed a general free-form stripe image interpretation approach on the basis of a four step procedure: (i) comparison of different feature-based image content description techniques, (ii) determination of optimal feature sub-groups, (iii) fusion of the most appropriate ones, and (iv) selection of the optimal features. She applies this technique to a broader range of surface geometries and types, i.e. to free-form rough and free-form specular shapes. Caulier and Bourennane [6] proposed a general free-form surface inspection approach relying on the projection of a structured light pattern and the interpretation of the generated stripe structures by means of Fourier-based features. Lin et al. [20] explored automated visual inspection of surface defects in a light-emitting diode (LED) wavelet-based principal chip by applying component analysis (WPCA) and Hotelling statistic (WHS) approaches to integrate the multiple wavelet characteristics. The principal component analysis of WPCA and the Hotelling control limit of WHS individually judge the existence of defects. Cristea [8] presents aspects of the design for an intelligent modular inspection system. This system consists of grouping the parts based on the relation between dimensional inspection process characteristics and modular design of all inspection equipments with a high universality and flexibility degree.



Fig.4 Automated 3D shape inspection (Background)

In our proposed method, the defects are identified as outliers of the Euclidian distance by an iterative method. Hawkins [11] define an outlier as an observation that deviates so much from other observations as to arouse suspicions that it was generated by a different mechanism.

Fagarasan [9] provides a comparison between different methods of fault detection and some examples of the fault detection and identification procedure for industrial processes. Aggarwal and Yu [2] developed a method for outlier detection especially suited to very high dimensional data sets by using the evolutionary search technique. Angiulli and Fassetti [3] proposed a method for detecting distance-based outliers in data streams under the sliding window model. The novel notion of onetime outlier query is introduced in order to detect anomalies in the current window at arbitrary pointsin-time. Breunig et al. [5] assigned to each object a degree of being an outlier. This degree is called the local outlier factor (LOF) of an object (Identifying density-based local outliers). It is a local in that the degree depends on how isolated the object is with respect to the surrounding neighborhood. Hsiao et al. [12] developed an efficient algorithm which converts outlier problem to pattern and relative deviation degree (RDD) problem. They present a new mechanism to distinguish outliers from the remainder in univariate dataset. Knorr et al. [14] proposed finding strongest and weak outliers and their corresponding structural intensional knowledge. In 2001, they proposed a robust space transformation called the Donoho-Stahel estimator to support operations such as nearest neighbor search, distance-based clustering and outlier detection [15]. Koufakou and Georgiopoulos [16] presented a fast distributed outlier detection algorithm for mixed attribute datasets that deals with sparse high-dimensional data. The algorithm called outlier detection for mixed attribute datasets (ODMAD) identifies outliers based on the categorical attributes first, and then focuses on subsets of data in the continuous space by utilizing information about these subsets from the categorical attribute space. Jan et al. [17] presented an outlier detection framework that is closely related to statistical non parametric density estimation methods with a variable kernel to yield a robust local density estimation. Outliers are then detected by comparing the local density of each point to the local density of its neighbors. Li and Kitagawa [18] took an Example-Based approach based on the notion of the distance based (DB) Outliers and examine behaviors of projections of the outlier examples in high dimensional datasets. To address the problem with the curse of dimensionality, they employed a Subspace-Based method to bring down the dimensionality of detected spaces. Thus, they proposed a method whose central ideas are making the best of users' examples to omit boring predefined parameters. They did so by detecting an optimal subspace where these examples perform more abnormal behaviors than in others, and picking out outliers having similar characteristics to examples. Limas et al. [19] proposed a method of outlier detection and data cleaning for both normal and non-normal multivariate data sets. This method named the PAELLA algorithm is based on an iterated local fit without a priori metric assumptions. They proposed a new approach supported by finite mixture clustering which provides good results with large data sets. Because the relationship between the samples and the extreme values in a data set is so dependent upon the distributional properties of the data set in question. Mingxi and Jermaine [22] considered the problem of estimating the extreme values in a data set by looking at a small number of samples from it by devising a Bayesian framework that uses previously observed queries to make a statistically rigorous guess as to the type of query that is currently under consideration. Sarker and Kitagawa [24] used the definition of the distance based outlier detection proposed by Knorr [14] and proposed a distributed algorithm for detecting outliers for shared nothing distributed systems. The algorithm finds top n outliers in its rank based on the distance of a point to its kth nearest neighbor. Rehtn et al. [23] presented a method to estimate the noise distance in noise clustering based on the preservation of the hyper volume of the feature space. The main purpose of noise clustering is to reduce the influence of outliers on the regular clusters. Zhang Ji et al. [27] proposed a technique named Stream projected outlier detector (SPOT), equipped with incrementally updatable data synapses, to deal with the problem of projected outlier detection in high-dimensional data streams.

3 Methodology

As mentioned previously, in our proposed method, the defects are identified as outliers of the Euclidian distance by an iterative method. To illustrate this idea, Fig.5 presents an example of the outliers identified as defects by the IDI algorithm after 150 iterations. This identification has been performed by using a threshold defined by the user. In order to eliminate the user interaction, this research proposes the use of the statistical maximum normed residual test [10].

By the definition, the maximum normed residual test (also known as the Grubbs test) consists of:

- Detecting outliers in univariate data.
- Assuming data comes from normal distribution.
- Detecting one outlier at a time, removing the outlier, and repeating.
- H_0 : There is no outlier in data, H_A : There is at least one outlier



Fig.5 Outlier detection - D_{si} : Projection of the distance $(s_i - c_i)$ on the normal n_{si}

The test is based on the difference of the mean of the sample and the most extreme data considering the standard deviation s as shown in equation 1.

$$G = \frac{\max|\mathbf{x} - \bar{\mathbf{x}}|}{s} \tag{1}$$

Reject H_0 if

$$G > \frac{N-1}{\sqrt{N}} \sqrt{\frac{t_{(\overline{\alpha}, N-2)}^2}{N-2+t_{(\overline{\alpha}, N-2)}^2}} = G_{critical} \qquad (2)$$

With $t_{(\bar{\alpha}/_{2N},N-2)}^2$: Critical value of the *t*-distribution with (N-2) degrees of freedom and a significance level of $\bar{\alpha}/_{2N}$ used to compute the confidence level. In our case, we use an alpha $(\bar{\alpha})$ of 0.05 that indicates a 95 percent confidence level.

In this work, the maximum normed residual test is implemented in the identification module of IDI (Fig.6) as described in the section 4.6 of Abenhaim *et al.* [1] paper.



Fig.6 Grubbs implemented in IDI, Kmax is the maximum iteration

4 Results

In order to evaluate the performance of the new implementation, three case studies shape representing typical non-rigid parts for transport industry are studied (Fig.7). All the parts are in aluminum gauge 14 (0.7213 mm) with a Young modulus of 7×10^{10} N/m² and a density of 2700 kg/m^3 . Table 1 represents the parameters for the algorithm used in the case studies. In order to make comparison between original the the IDI identification method and the proposed herein, type I error (α) and the type II error (β) are used thereafter. Type I error (α) is the error of rejecting a "correct" null hypothesis (H_0), and type II error (β) is the error of not rejecting a "false" null hypothesis (H_0) . In other words, α is rejecting a default when it should not have been rejected and β is failing to reject a default when it should have been rejected (β is a false detection). Assuming that the density is uniform throughout the part, the performance of the proposed method and of the IDI's identification method can be compared using a point-base metric instead of surface-base metric. In the ideal case the 2 types of errors must be equal to zero.

Analyzing Figures 8, 9,10 and 11 and Table 2, one can notice that the implementation of the maximum normed residual test has remarkably reduced the number of points with type I errors (α) and with type II errors (β) in the case of quasi constant surface (case a) compared to their original IDI identification method.

In the case of omega shape (case b) and freeform surface (case c), this implementation has slightly reduced the number of points with type I errors (α) and has remarkably reduced (only in case b) the number of points with type II errors (β) compared to their original IDI identification method.



Fig.7 Descriptions of the case studies (a) quasiconstant surface (b) Omega shape Ω (c) freeform surface – [1]

| Table 1 Parameters for the algorithm used in the | e |
|--------------------------------------------------|---|
| case studies - [1] | |

| Simulation configuration | | | | | | |
|--------------------------|--------------------------|------------------------------------|------------------|--|--|--|
| Case Study | F1 | F2 | K _{max} | | | |
| а | 2N force on point A | 3N force on point A and B | 150 | | | |
| b | 10 N force on point C | 5N force on point C and D | 500 | | | |
| с | 2N force on point E | 10 mm displacement of curve F-E | 300 | | | |



Fig.8 Case a-F1 (a) V1 (b) V2 (c) V3 (d) V4 (e) V5 (f) detailed view of (e) / The red circles (O) represent the defaults detected and the blue squares (\Box) represent the imposed defaults



Fig.9 Case a-F2 (a) V1 (b) V2 (c) V3 (d) V4 (e) V5 (f) detailed view of (e) / The red circles (O) represent the defaults detected and the blue squares (D) represent the imposed defaults



Fig.10 Case b-F1 Default detection: (a) V1- IDI (b) V1- Grubbs (c) V2 - IDI (d) V2 - Grubbs V2 (e) V3 - IDI (f) V3 - Grubbs / The red circles (O) represent the defaults detected and the blue squares (\Box) represent the imposed defaults



| Fig.11 Case c-F1 (a) IDI - V5 (b) Grubbs - V5 |
|----------------------------------------------------|
| (c) Detailed view of (a) and (b) / The red circles |
| (O) represent the defaults detected and the blue |
| squares (\Box) represent the imposed defaults |

Table 2 Type I and II errors

| Case Studies | | | IDI | | GRUBBS | |
|-----------------|-----|----------------------------------|---------------------------|----------------------|---------------------------|-------------------------|
| | | Imposed profile deviations | Type I error (α) | Type II error (β) | Type I error (α) | Type II error (β) |
| a-F1 | V0* | 0 | 0 | 0 | 0 | 0 |
| | V1 | 27 | 10 | 20 | 2 | 0 |
| | V2 | 56 | 36 | 13 | 24 | 0 |
| | V3 | 67 | 49 | 7 | 33 | 0 |
| | V4 | 96 | 76 | 10 | 46 | 0 |
| | V5 | 138 | 111 | 6 | 56 | 0 |

| a-F2 | V0* | 0 | 0 | 0 | 0 | 0 | |
|-------------|-----|-----|-----|----|-----|---|--|
| | V1 | 27 | 10 | 20 | 10 | 6 | |
| | V2 | 56 | 40 | 15 | 31 | 6 | |
| | V3 | 67 | 48 | 5 | 38 | 1 | |
| | V4 | 96 | 71 | 10 | 59 | 0 | |
| | V5 | 138 | 111 | 6 | 72 | 0 | |
| | V0* | 0 | 0 | 0 | 0 | 0 | |
| Ь F1 | V1 | 76 | 15 | 44 | 31 | 0 | |
| D-F1 | V2 | 73 | 24 | 44 | 30 | 0 | |
| | V3 | 112 | 51 | 76 | 48 | 0 | |
| b-F2 | V0* | 0 | 0 | 0 | 0 | 0 | |
| | V1 | 76 | 28 | 25 | 21 | 0 | |
| | V2 | 73 | 31 | 22 | 44 | 0 | |
| | V3 | 112 | 62 | 61 | 60 | 0 | |
| | V0* | 0 | 0 | 0 | 0 | 0 | |
| | V1 | 52 | 48 | 0 | 43 | 1 | |
| e-F1 | V2 | 62 | 55 | 0 | 50 | 1 | |
| C-11 | V3 | 103 | 90 | 0 | 80 | 3 | |
| | V4 | 110 | 103 | 0 | 92 | 1 | |
| | V5 | 211 | 195 | 0 | 171 | 1 | |
| c-F2 | V0* | 0 | 0 | 0 | 0 | 0 | |
| | V1 | 52 | 48 | 2 | 40 | 7 | |
| | V2 | 62 | 54 | 1 | 49 | 1 | |
| | V3 | 103 | 82 | 1 | 73 | 6 | |
| | V4 | 110 | 94 | 1 | 87 | 2 | |
| | V5 | 211 | 184 | 0 | 171 | 2 | |

***V0** tests are performed to ensure that the method does not induce a bias. In other words, no defects should be detected if there aren't any imposed defects in the simulation part.

Another improvement compared to the original IDI identification module is that the identification threshold is not estimated by a trial and error process. Herein, the maximum normed residual test uses a constant parameter $\overline{\alpha} = 0.05$ that corresponds to $\pm \sigma \approx 95\%$ of the set.

5 Conclusion

In this paper, the problematic of the inspection of the non-rigid parts without specialized fixtures is presented. The review of the literature covering the major aspects of the problem shows that the inspection of non-rigid parts is still a real problem for transport industry. Dealing with this problematic, this paper presents implementing the maximum normed residual test in the IDI identification module followed by three case Compared with the original IDI studies. identification module, the results show that the proposed method reduces the type I and type II errors. In addition, in contrast with the IDI's identification method, the proposed method does not need a user-specified threshold based on a trial and error process. Future research is underway to validate the methodology.

The authors would like to thank the National Sciences and Engineering Research Council (NSERC) for their support and financial contribution.

References:

- G. N. Abenhaim, A. S. Tahan, A. Desrochers et al., A Novel Approach for the Inspection of Flexible Parts Without the Use of Special Fixtures, *Journal of Manufacturing Science* and Engineering ASME, Vol.133,2 2011, pp. 1-11.
- [2] C. C. Aggarwal, S Yu, Philip, An effective and efficient algorithm for high- dimensional outlier detection, *VLDB Journal*, Vol. 14, No. 2, 2005, pp. 211-21.
- [3] F. Angiulli, Fasseti, F., Distance-based outlier queries in data streams: the novel task and algorithms, *Data Mining and Knowledge Discovery*, Vol. 20, No. 2, 2010, pp. 290-324.
- [4] R. Ascione, and W. Polini, Measurement of nonrigid freeform surfaces by coordinate measuring machine, *International Journal of Advanced Manufacturing Technology*, Vol. 51, No. 9-12, 2010, pp. 1055-1067.
- [5] M. M. Breunig, Kriegel, H., Ng, R., Sander, J., LOF: identifying density- based local outliers, *SIGMOD Record*, Vol. 29, No. 2, 2000, pp. 93-104.
- [6] Y. Caulier, Fourier-based inspection of freeform reflective surfaces, Advanced Concept for Intelligent Vision Systems Proceedings, Vol. 5259, 2008, pp. 125-136.

- Y. Caulier, Inspection of complex surfaces by means of structured light patterns, *Optics Express*, Vol. 18, No. 7, 2010, pp. 6642-6660.
- [8] L. Cristea, and M. Manescu, Intelligent modular design of automatic dimensional inspection systems, WSEAS Transactions on Applied and Theoretical Mechanics, Vol. 3, No. 7, 2008, pp. 739-748.
- [9] I. Fagarasan, and S. S. ILiescu, Applications of fault detection methods to industrial processes, WSEAS Transactions on Systems, Vol. 7, No. 6, 2008, pp. 812-21.
- [10] F. E. Grubbs, Procedures for detecting outlying observations in samples, *Technometrics*, Vol. 11, No. 1,1969, pp. 1-21.
- [11] D. Hawkins, *Identification of Outliers*, Chapman and Hall: London, 1980.
- [12] C. A. Hsiao, Chen, H., Furuse, K., Ohbo, N., Figure and Ground: A Complete Approach to Outlier Detection, *IAENG Transactions on Engineering Technologies VOL 1*, Vol. 1089, 2009, pp. 70-81.
- [13] A. E. Jaramillo, P. Boulanger, and F. Prieto, On- line 3- D Inspection of Deformable Parts Using FEM Trained Radial Basis Functions, *IEEE 12th International Conference on Computer Vision Workshops, ICCV Workshops,* 2009, pp. 1733-1739.
- [14] E. M. Knorr, and R. T. Ng, Finding intensional knowledge of distance- based outliers, *Proceedings of the Twenty-Fifth International Conference on Very Large Data Bases*, 1999, pp. 211-222.
- [15] E. M. Knorr, R. T. Ng, and R. H. Zamar, Robust space transformations for distancebased operations, *Proceedings of the Seventh* ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2001, pp. 126-135.
- [16] A. Koufakou, and M. Georgiopoulos, A fast outlier detection strategy for distributed highdimensional data sets with mixed attributes, *DATA MINING AND KNOWLEDGE DISCOVERY*, Vol. 20, No. 2, 2010, pp. 259-289.
- [17] L. J. Latecki, A. Lazarevic, and D. Pokrajac, Outlier detection with kernel density functions, *Machine Learning and Data Mining in Pattern Recognition, Proceedings*, Vol. 4571 LNAI, 2007, pp. 61-75.
- [18] Y. Li, and H. Kitagawa, Example- based robust DB- outlier detection for high dimensional data, *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in*

Bioinformatics), Vol. 4947 LNCS, 2008, pp. 330-347.

- [19] M. C. Limas, J. Ordieres Meré, F. Martinez De Pison Ascacibar *et al.*, Outlier detection and data cleaning in multivariate non-normal samples: the PAELLA algorithm, *Data Mining and Knowledge Discovery*, Vol. 9, No. 2, 2004, pp. 171-87.
- [20] H.-D. Lin, C.-Y. Chung, and W.-T. Lin, Principal component analysis based on wavelet characteristics applied to automated surface defect inspection, WSEAS Transactions on Computers Research, Vol. 3, No. 4, 2008, pp. 193-202.
- [21] E. N. Malamas, E. G. M. Petrakisa, M. Zervakisa *et al.*, A survey on industrial vision systems, applications and tools, *Image and Vision Computing*, Vol. 21, No. 2, 2003, pp. 171-188.
- [22] W. Mingxi, and J. Chris, Guessing the extreme values in a data set: a Bayesian method and its applications, *VLDB Journal*, Vol. 18, No. 2, 2009, pp. 571-97.
- [23] F. Rehtn, F. Rehm, F. Klawonn *et al.*, A novel approach to noise clustering for outlier detection, *Soft Computing*, Vol. 11, No. 5, 2007, pp. 489-94.
- [24] B. K. Sarker, and H. Kitagawa, A distributed algorithm for outlier detection in a large database, *Lecture Notes in Computer Science*, Vol. 3433, 2005, pp. 300-309.
- [25] A. Weckenmann, J. Weickmann, Optical inspection of formed sheet metal parts applying fringe projection systems and virtual fixation, *Metrology and Scientific Instrumentation*, Vol. 13, 2006, pp. 321-334.
- [26] L. Yadong, and G. Peihua, Free-form surface inspection techniques state of the art review, *CAD Computer Aided Design*, Vol. 36, No. 13, 2004, pp. 1395-1417.
- [27] J. Zhang, Q. Liu, Q. Gao *et al.*, Detecting projected outliers in high-dimensional data streams, *Datatbase and Expert Systems Applications, Proceedings*, Vol. 5690, 2009, pp. 629-644.