# Analysis of Cauchy Sequence Convergence and Uniqueness in MR-Metric Spaces

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*Abstract:* - This paper delves into the exploration of MR-metric spaces, [1], which serve as an advanced extension of D-metric spaces, focusing on the behavior of MR-Cauchy and MR-convergent sequences. It identifies the criteria necessary for ensuring both the existence and uniqueness of fixed points for self-maps within MR-metric spaces that are complete and bounded. The findings introduce innovative fixed-point theorems under particular contraction conditions and provide examples to illustrate the characteristics and dynamics of sequences within these spaces.

*Key-Words:* - MR − metric space, M-Convergent, M-Cauchy, exists point theorems, self-mappings

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## 1 Introduction

Metric spaces play an important role in mathematical analysis and form the basis for understanding concepts such as sequence convergence, continuity, and fixed points. Over the years, extended frameworks such as D-metric spaces have been developed to address more complex relations and structures. Recently, the MR metric space[1], has been introduced as an advanced generalization, providing innovative tools to explore sequence dynamics and map behavior in generalized settings. These areas significantly extend the applicability of metric theories to new domains and promote opportunities for deeper theoretical investigation. This research focuses on the fundamental properties and practical implications of MR metric spaces, with particular attention to the necessary conditions for convergence of sequences and self-mapping to fixed points. By proposing new definitions and establishing refined conditions, the study extends classical results in both metric and fixed point theories. Through detailed examples and theoretical proofs, the paper demonstrates the peculiar properties of fixed points under specific contraction criteria and emphasizes the relevance of MR metric spaces to advance nonlinear analysis and mathematical research. For further information, we refer the reader [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25].

Definition 1 [1] Let $\mathcal{X}$ be a nonempty set and let $R > 1$ be a fixed real number. A mapping $M : \mathcal{X} \times \mathcal{X} \times \mathcal{X} \longrightarrow [0, \infty)$ is called an $MR$-metric if it fulfills the following axioms for every choice of $a, b, c \in \mathcal{X}$:

(M1) $M(a, b, c) \geq 0$.

(M2) $M(a, b, c) = 0$ if and only if $a = b = c$.

(M3) $M(a, b, c)$ is invariant under any permutation $p(a, b, c)$ of its arguments; i.e. $M(a, b, c) = M(p(a, b, c))$.

(M4) (Tetrahedral inequality) For an arbitrary element $\ell \in \mathcal{X}$,

$$M(a, b, c) \ \leq \ R \left[ M(a, b, \ell) + M(a, \ell, c) + M(\ell, b, c) \right].$$

If such an $M$ is defined on $\mathcal{X}$, the ordered pair $(\mathcal{X}, M)$ is said to be an $MR$-metric space.

Definition 2 Let $(\mathcal{Y}, M)$ be an $MR$-metric space and let $\{a_n\} \subseteq \mathcal{Y}$ be a sequence. We say that $\{a_n\}$ is $MR$-convergent if there exists a point $d \in \mathcal{Y}$ such that for every $\varepsilon > 0$ one can find an integer $N \in \mathbb{N}$ with the property

$$M(a_n, a_m, d) \ < \ \varepsilon \qquad \text{for all } m, n \geq N.$$

In this situation we write $a_n \underset{MR}{\longrightarrow} d$; the element $d$ is called the limit of the sequence and we say that $\{a_n\}$ $MR$-converges to $d$.

Definition 3 Let $(\mathcal{Y}, M)$ be an $MR$-metric space. A sequence $\{a_n\} \subseteq \mathcal{Y}$ is called $MR$-Cauchy if for every $\varepsilon > 0$ there exists an index $N \in \mathbb{N}$ such that

$$M(a_n, a_m, a_p) \ < \ \varepsilon \qquad \text{for all } m, n, p \geq N.$$

Definition 4 An $MR$-metric space $(\mathcal{X}, M)$ is termed bounded if there exists a positive constant $L > 0$ satisfying

$$M(a,b,c) \;\leq\; L \qquad \text{for every } a,b,c \in \mathcal{X}.$$

In such a case $L$ is referred to as an $MR$-bound for the space $(\mathcal{X}, M)$.

Definition 5 A subset $E$ of an $MR$-metric space $(\mathcal{X}, M)$ is referred to as $M$-bounded whenever one can find a positive number $L$ such that

$$M(a,b,c) \leq L$$

for all $a,b,c \in E$.

This paper builds on these fundamental results by extending the concepts to the realm of $MR$-metric spaces. We present new definitions and results related to convergence and Cauchy sequences in this context and investigate the properties of points existing in $MR$-metric spaces, providing new insights into their structure and potential applications.

## 2 Main Results

We now present a simple example of a complete $MR$-metric space with a convergent sequence, where the sequence converges at every point in the space.

Example 6 Let $X = \{2^{-n} : n \in \mathbb{N}\}$. Define $M : X \times X \times X \to \mathbb{R}^+$ by

$$M(a,b,c) = \begin{cases} 0, & \text{if } a=b=c, \\ \frac{1}{R}\,\min\{\max\{a,b\},\ \max\{b,c\}, & \\ \quad \max\{c,a\}\}, & \text{otherwise.} \end{cases} \tag{1}$$

Then $(X, M)$ is a complete $MR$-metric space. Moreover, the sequence $\{2^{-n}\}_{n=1}^{\infty}$ is $MR$-Cauchy and $MR$-converges to every $a \in X$.

Proof 7 (M1)–(M3): By (1), $M(a,b,c) \geq 0$, $M(a,b,c) = 0 \iff a = b = c$, and symmetry holds.

(M4) Tetrahedral inequality: Let $a,b,c \in X$. If $a = b = c$, (M4) holds. Assume $a \geq b \geq c$ (otherwise permute). Then $M(a,b,c) = b/R$. For any $\ell \in X$, if $\ell \geq b$ then $M(\ell,b,c) = b/R$; if $\ell < b$ then $M(a,b,\ell) = b/R$. Thus

$$M(a,b,c) = \frac{b}{R} \leq R\big[M(\ell,b,c) + M(a,\ell,c) + M(a,b,\ell)\big] = R \cdot \frac{b}{R} = b.$$

Completeness: Let $\{x_n\} = \{a_n\}$ be $MR$-Cauchy. Two cases:

Case 1: $\exists N,\ \forall n \geq N,\ x_n = a_N$. Then $x_n \to a_N$.

Case 2: $\{x_n\}$ not eventually constant. Assume $\lim_{n\to\infty} x_n \neq 0$ in $\mathbb{R}$. Then $\exists \varepsilon > 0$ and infinite indices with $x_n \geq \varepsilon$. Choose $i,j,k$ with $k > j > i > N$, $x_i, x_j \geq \varepsilon$, $x_k \neq x_j$. Then

$$M(x_i,x_j,x_k) = \frac{1}{R}\min\{\max\{x_i,x_j\}, \max\{x_j,x_k\}, \max\{x_k,x_i\}\} \geq \frac{\varepsilon}{R}.$$

contradicting the $MR$-Cauchy property. Hence $x_n \to 0$ in $\mathbb{R}$. For any $a \in X$ and $m,n \in \mathbb{N}$,

$$\begin{aligned} M(a,x_m,x_n) &\leq \frac{1}{R}\min\{\max\{a,x_m\}, \max\{x_m,x_n\}, \max\{x_n,a\}\} \\ &\leq \max\{x_m,x_n\} \to 0. \end{aligned}$$

Thus $x_n \xrightarrow[MR]{} a$ for every $a \in X$, proving completeness.

Theorem 8 Let $(\mathcal{X}, M)$ be a complete and bounded $MR$-metric space, and let $\phi : \mathcal{X} \to \mathcal{X}$. If $\exists\, \alpha \in [0,1)$ such that

$$M(\phi a, \phi b, \phi c) \leq \alpha\, M(a,b,c), \quad \forall a,b,c \in \mathcal{X}, \tag{2}$$

then $\phi$ admits a unique fixed point in $\mathcal{X}$.

Remark 9 The necessity of completeness and boundedness in Theorem 8 is demonstrated by Example 10, where all hypotheses except the existence conclusion hold, yet no fixed point exists.

Example 10 Let $(X, M)$ be as in Example 6. Define $\phi : X \to X$ by $\phi(a) = a/2$. Then $\forall a,b,c \in X$,

$$M(\phi a, \phi b, \phi c) = \frac{1}{2R} M(a,b,c). \tag{3}$$

However, $\phi$ has no fixed point in $X$, and for any $a \in X$, $\{\phi^n(a)\}$ $MR$-converges to every $x \in X$.

Proof 11 For $a = b = c$, $M(\phi a, \phi b, \phi c) = 0 = \frac{1}{2R} M(a,b,c)$. If $a \geq b \geq c$,

$$M(\phi a, \phi b, \phi c) = M\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right) = \frac{b}{2R} = \frac{1}{2R} M(a,b,c).$$

Hence (3) holds. Clearly $\phi(a) = a$ has no solution in $X$. Moreover, for any $a \in X$, $\phi^n(a) = 2^{-n}a$, and by Example 6, $\{2^{-n}a\}$ converges to every $x \in X$.

Theorem 12 Let $(\mathcal{X}, M)$ be a complete and bounded $MR$-metric space and $\phi : \mathcal{X} \to \mathcal{X}$ satisfy (2). Then for each $a \in \mathcal{X}$, $\{\phi^n(a)\}$ is $MR$-Cauchy. If $\exists a_0 \in \mathcal{X}$ such that $\{\phi^n(a_0)\}$ has only finitely many limit points, then $\phi$ possesses a unique fixed point.

Example 13 Let $X = \{0\} \cup \{1/n : n \in \mathbb{N}\}$. Define $M : X \times X \times X \to \mathbb{R}^+$ by

$$M(a, b, c) = \frac{1}{R}\big[|a - b| + |b - c| + |c - a|\big]. \quad (4)$$

Then $(X, M)$ is a complete, bounded $MR$-metric space where every convergent sequence has a unique limit. Define $\phi(a) = \frac{1}{2R}a$; then $\phi$ satisfies (2) with $\alpha = 1/(2R)$ and has unique fixed point 0.

Proposition 14 Let $(\mathcal{X}, M)$ be an $MR$-metric space and assume $M$ is sequentially continuous in one variable. Then every $MR$-convergent sequence has a unique limit.

Proof 15 Let $x_n \to p$ and $x_n \to q$. By sequential continuity in one variable,

$$\lim_{n\to\infty} M(x_n, p, p) = M(p, p, p) = 0,$$
$$\lim_{n\to\infty} M(x_n, p, p) = M(q, p, p).$$

Hence $M(q, p, p) = 0$, so $q = p$.

Theorem 16 Let $(\mathcal{X}, M)$ be an $MR$-metric space, $\phi : \mathcal{X} \to \mathcal{X}$, and $a_0 \in \mathcal{X}$ such that $O_\phi(a_0)$ is bounded and every $MR$-Cauchy sequence converges. Suppose $\exists \alpha \in [0, 1)$ with

$$M(\phi a, \phi b, \phi c) \le \frac{\alpha}{R} \max\big\{M(a, b, c),\ M(a, \phi a, c)\big\}, \quad \forall a, b, c \in \overline{O_\phi(a_0)}. \quad (5)$$

Then $\phi$ admits a unique fixed point in $\mathcal{X}$.

Remark 17 Theorems 18 and 21 extend and refine the ideas in Theorem 16.

Theorem 18 Let $(\mathcal{X}, M)$ be an $MR$-metric space and $\phi : \mathcal{X} \to \mathcal{X}$. Let $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ be non-decreasing with $\phi(t) < t$ for $t > 0$. Suppose $\exists a_0 \in \mathcal{X}$ with

$$\sup\{M(a_0, \phi^i a_0, \phi^j a_0) : i, j \in \mathbb{N} \cup \{0\}\} < \infty,$$

and $\forall a, b, c \in \overline{O_\phi(a_0)}$,

$$M(\phi a, \phi b, \phi c) \le \frac{1}{R}\, \phi(\max\{M(a, u, v) : u, v \in \{a, \phi a, b, c\}\}). \quad (6)$$

Then $\{\phi^n(a_0)\}$ is $MR$-Cauchy. If it converges to $p \in \mathcal{X}$ and $M$ is sequentially continuous in any two variables at $p$, then $p$ is the unique fixed point of $\phi$ in $\overline{O_\phi(a_0)}$.

Proof 19 Set $x_n = \phi^n a_0$. For $r, s \in \mathbb{N} \cup \{0\}$ and $n \in \mathbb{N}$,

$$\begin{aligned} M(x_n, \phi^n a_r, \phi^n a_s) &\le \frac{1}{R}\phi\Big(\max\{M(x_{n-1}, \phi^{n-1}a_p, \phi^{n-1}a_q) : p, q \in \{0, 1, r, s\}\}\Big) \\ &\le \phi^2\Big(\max\{M(x_{n-2}, \phi^{n-2}a_i, \phi^{n-2}a_j) : i, j \in \{0, 1, r, s\}\}\Big) \\ &\le \cdots \le \phi^n(\max\{M(a_0, a_i, a_j) : i, j \in \{0, 1, r, s\}\}). \end{aligned}$$

Let $L = \sup\{M(a_0, \phi^i a_0, \phi^j a_0)\} < \infty$. Then $M(x_n, \phi^n a_r, \phi^n a_s) \le \phi^n(L) \to 0$, so $\{x_n\}$ is $MR$-Cauchy.

If $x_n \to p$ and $M$ is sequentially continuous,

$$M(x_n, x_n, \phi p) \le \frac{1}{R}\phi(\max\{M(x_{n-1}, u, v) : u, v \in \{x_{n-1}, x_n, p\}\}) \to 0,$$

so $M(p, p, \phi p) = 0$ and $\phi p = p$. Uniqueness follows from (6) as in the proof of Theorem 21.

Remark 20 For $a, b, c \in \mathcal{X}$, define $\alpha(a, b, c) = \max\{M(a, u, v) : u, v \in \{a, \phi a, b, c\}\}$. By symmetry of $M$, (6) is equivalent to

$$M(\phi a, \phi b, \phi c) \le \phi(\min\{\alpha(a, b, c), \alpha(b, c, a), \alpha(c, a, b)\}). \quad (7)$$

Theorem 21 Let $(\mathcal{X}, M)$ be an $MR$-metric space, $\phi : \mathcal{X} \to \mathcal{X}$, and $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ non-decreasing with $\phi(t) < t$ for $t > 0$. Assume $\exists a_0 \in \mathcal{X}$ such that $O_\phi(a_0)$ is $MR$-bounded and $\forall a, b, c \in \overline{O_\phi(a_0)}$,

$$M(\phi a, \phi b, \phi c) \le \frac{1}{R}\, \phi(\max\{M(u, v, w) : u, v, w \in \{a, \phi a, b, \phi b, c, \phi c\}\}). \quad (8)$$

Then $\{\phi^n(a_0)\}$ is $MR$-Cauchy. If $\phi^n(a_0) \to p$ and $M$ is sequentially continuous in any two variables, then $p$ is the unique fixed point of $\phi$ in $\overline{O_\phi(a_0)}$.

Proof 22 Set $x_n = \phi^n a_0$ and $\beta_n = \sup\{M(x_i, x_j, x_k) : i, j, k \ge n\}$. Then $\beta_0 < \infty$. For $i, j, k \ge n$,

$$\begin{aligned} M(x_i, x_j, x_k) &= M(\phi x_{i-1}, \phi x_{j-1}, \phi x_{k-1}) \\ &\le \frac{1}{R}\phi(\max\{M(u, v, w) : u, v, w \in \{x_{i-1}, x_{j-1}, x_{k-1}, x_k\}\}) \\ &\le \phi(\beta_{n-1}). \end{aligned}$$

Thus $\beta_n \le \phi(\beta_{n-1}) \le \phi^n(\beta_0) \to 0$, so $\{x_n\}$ is $MR$-Cauchy.

If $x_n \to p$ and $M$ is sequentially continuous,

$$M(x_n, x_n, \phi p) \leq \phi(\max\{M(u,v,w) : u,v,w \in \{x_{n-1}, x_n, p, \phi p\}\}) \to \phi(\max\{M(p,p,\phi p), M(p,\phi p,\phi p)\}),$$

$$M(p,\phi p,\phi p) \to \phi(\max\{M(p,p,\phi p), M(p,\phi p,\phi p)\}).$$

Hence $\max\{M(p,p,\phi p), M(p,\phi p,\phi p)\} \leq \phi(\max\{M(p,p,\phi p), M(p,\phi p,\phi p)\})$, forcing $\max = 0$, so $\phi p = p$. Uniqueness follows similarly.

Remark 23 In Example 13, $M$ is sequentially continuous in all variables, verifying Theorems 18 and 21.

Definition 24 A map $\phi : \mathcal{X} \to \mathcal{X}$ is sequentially continuous at $p \in \mathcal{X}$ if $x_n \to p$ implies $\phi x_n \to \phi p$. It is sequentially continuous on $\mathcal{X}$ if this holds at every point.

Theorem 25 Let $(\mathcal{X}, M)$ be an $MR$-metric space, $\phi : \mathcal{X} \to \mathcal{X}$, and $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ non-decreasing with $\phi^n(t) \to 0$ for all $t > 0$. Suppose $\exists a_0 \in \mathcal{X}$ with $O_\phi(a_0)$ $MR$-bounded and $\forall a,b,c \in O_\phi(a_0)$,

$$M(\phi a, \phi b, \phi c) \leq \phi(\sup\{M(u,v,w) : u,v,w \in O_\phi(a) \cup O_\phi(b) \cup O_\phi(c)\}). \tag{9}$$

Then $\{\phi^n(a_0)\}$ is $MR$-Cauchy. If it converges to a unique $p \in \mathcal{X}$ and $\phi$ is sequentially continuous at $p$, then $\phi p = p \in \overline{O_\phi(a_0)}$.

Proof 26 Define $x_n$ and $\beta_n$ as in Theorem 21. For $i,j,k \geq n$,

$$\begin{aligned} M(x_i, x_j, x_k) &\leq \phi(\sup\{M(u,v,w) : \\ &\qquad u,v,w \in O_\phi(x_{i-1}) \cup O_\phi(x_{j-1}) \\ &\qquad \cup O_\phi(x_{k-1})\}) \\ &\leq \phi(\beta_{n-1}). \end{aligned}$$

Hence $\beta_n \leq \phi(\beta_{n-1}) \leq \phi^n(\beta_0) \to 0$, so $\{x_n\}$ is $MR$-Cauchy. If $x_n \to p$ uniquely and $\phi$ is sequentially continuous at $p$, then $\phi x_n \to \phi p$, but $\phi x_n = x_{n+1} \to p$, so $\phi p = p$.

Remark 27 If $\phi$ is continuous at 0 with $\phi(0) = 0$ and $M(\phi a, \phi b, \phi c) \leq \phi(M(a,b,c))$, then $\phi$ is sequentially continuous.

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