The complete b-chromatic sum of the Mycielskian of paths

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Abstract: A b-coloring of a graph G is a proper vertex-coloring that for each class of color i, there is a vertex whose neighbors are of all colors but i. The b-chromatic number of G is the largest positive integer κ where a b-coloring of κ colors exists. The b-chromatic sum of G is the minimum sum of the colors of all vertices of G over all possible b-colorings that give the b-chromatic number. For the Mycielskian of path $\mu(P_n)$, the b-chromatic sum $\varphi'(\mu(P_n))$ is known for all n except when $10 \le n \le 15$. In this work, we give the value of $\varphi'(\mu(P_n))$ when $10 \le n \le 15$. This completes the list of the b-chromatic sum of $\mu(P_n)$.

Key-Words: b-coloring, b-domination, b-chromatic number, b-chromatic sum, The Minimum Sum, Mycielskian, path

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1 Introduction

For a proper vertex-coloring c with κ colors of $G = (V(G), E(G))$, we let the color class C_i be the set of the vertices of color i for $i \in \{1, \ldots, \kappa\}$. A vertex $w \in V(G)$ is a *b-dominating vertex* or a *b-vertex* if its neighbors contains at least one representative of every color class other than its own. If there exists a b-vertex in every color class, then c is called a *b-coloring*. The *b-chromatic number*, [1], of a graph G is the largest positive integer κ where G can have a b-coloring $c : V(G) \to \{1, \ldots, \kappa\}$, denoted by $\varphi(G)$. Among the b-coloring c giving $\varphi(G)$, if $\sum_{v \in V(G)} c(v)$ is the smallest, then such sum is the *b-chromatic sum*, [2], of G, denoted by $\varphi'(G)$. Some results on the b-chromatic number appear in [3], [4], [5], [6]. The b-chromatic sum of several types of graphs have been explored, such as wheel graph, cycle and complete graph, [2], [7].

For a graph $G = (V(G), E(G))$ where $V(G) = \{v_1, v_2, \ldots, v_n\}$, the *Mycielskian of graph* $\mu(G)$ of G is obtained by adding $n+1$ vertices to the graph G so that $V(\mu(G)) = \{u\} \cup U \cup V$ where $U = \{u_1, \ldots, u_n\}$, $V = V(G)$ and $E(\mu(G)) = \{u_i v_j : j = i-1 \text{ or } j = i+1 \text{ for } 1 \le i \le n\} \cup \{uu_i : 1 \le i \le n\} \cup E(G)$. Let P_n be an n-vertex path. An example of $\mu(P_n)$ is shown in Figure 1.

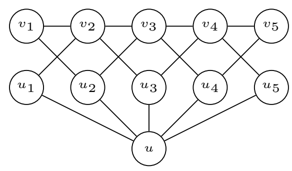

Fig. 1: $\mu(P_5)$. Source: created by the authors.

In [8], the values of $\varphi'(\mu(P_n))$ for $n \le 6$ and $n = 8$ were given, and upper bounds in other cases were also given. Later, the exact values of $\varphi'(\mu(P_n))$ when $n = 7, 9$ and $n \ge 16$ were given in [9]. They also reduced the upper bound given in [8]. From such results, it remains to determine the exact value of $\varphi'(\mu(P_n))$ for $10 \le n \le 15$. In this work, we show that $\varphi'(\mu(P_{11})) = 51$ and $\varphi'(\mu(P_n)) = 3n + 19$ if $n = 10$ or $12 \le n \le 15$. This completes the list of the exact values of $\varphi'(\mu(P_n))$ for all n.

Mycielskian graphs may be used in multiprocessor task scheduling problem, [10]. The applications of the b-coloring appear in the postal mail sorting system, [11], and clustering healthcare data, [12]. An application of a chromatic sum or the Minimum Color Sum (MCS), which is the lowest sum of the colors of all vertices among the proper colorings, appeared in resource allocation, [13]. The vertex set used in the resource allocation problems is the set of processors. The adjacency between two vertices corresponds to two processors that cannot operate simultaneously. The chromatic sum helps reducing the average response time of the processors. For the b-chromatic sum, it can also be applied in the same manner in the problem that requires the b-coloring instead of the proper coloring.

2 Preliminary results

In the preliminary-result section, we provide known results on $\varphi(\mu(P_n))$ and $\varphi'(\mu(P_n))$.

The following theorem gives the exact value of

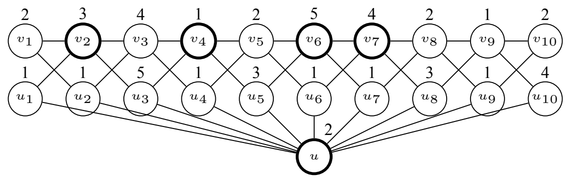

Fig. 2: A b -coloring with the sum of $3n+19$ when $n=10$. Source: [9].

$\varphi(\mu(P_n))$ for all $n \geq 2$.

Theorem 2.1 ([14]). *Let $n \geq 2$. Then*

$$\varphi(\mu(P_n)) = \begin{cases} 3, & \text{if } 2 \leq n \leq 4, \\ 4, & \text{if } 5 \leq n \leq 7, \\ 5, & \text{if } n \geq 8. \end{cases}$$

The following theorem gives $\varphi'(\mu(P_n))$ when $1 \leq n \leq 9$ or $n \geq 16$.

Theorem 2.2. *([8], [9]) Let $n \geq 1$. Then*

$$\varphi'(\mu(P_n)) = \begin{cases} 3n+1, & n=1, \\ 3n+2, & n=3, \\ 3n+3, & n=2,4, \\ 3n+6, & n=5,6,7, \\ 3n+20, & n=8, \\ 3n+19, & n=9 \text{ or } n \geq 16. \end{cases}$$

Theorem 2.3 shows that $\varphi'(\mu(P_n))$ is either $3n+18$ or $3n+19$ when $10 \leq n \leq 15$. For $n \geq 10$, the b-colorings giving the sum of $3n+19$ were presented in [9], as a part of the proof of Theorem 2.3. They constructed b-colorings in the cases of $n = 10, 11$ individually and then extended the construction to $n \geq 12$. An example of the b-coloring with the sum of $3n+19$ when $n=10$, [9], is presented in Figure 2. We later show in Theorem 3.3 that $\varphi'(\mu(P_n)) = 3n+18$ only if $n = 11$.

Theorem 2.3 ([9]). *For $10 \leq n \leq 15$, we have $3n+18 \leq \varphi'(\mu(P_n)) \leq 3n+19$ and a b-coloring with the sum of $3n+19$ exists for each of such n.*

The subsequent lemmas play a crucial role in establishing the main theorem.

Lemma 2.4 ([9]). *Let $n \geq 8$ and c_n be a b-coloring that gives the $\varphi'(\mu(P_n))$. The following conditions are satisfied:*

1. *the b-vertices are contained in $V(P_n) \cup \{u\}$;*
2. *each color appears at least twice in $V(P_n) \cup U_n$;*
3. *$|c_n(V(P_n))| = 5$;*
4. *$|C_{c_n(u)}| \leq \lceil \frac{n}{2} \rceil + 1$.*

The following Lemma shows that the cardinalities of the classes of colors 3,4 and 5 are fixed and the color of u must be 2. The cardinalities of C_4 and C_5 appear in the term of n.

Lemma 2.5 ([9]). *For $10 \leq n \leq 15$, if $\varphi'(\mu(P_n))$ is $3n+18$, then $|C_1| = n-1, |C_2| = n-6, |C_3| = 3, |C_4| = 3$, $|C_5| = 2$ and $c_n(u) = 2$.*

3 Main Results

In this section, we show that $\varphi'(\mu(P_n)) = 3n+18$ when $n = 11$ and $\varphi'(\mu(P_n)) = 3n+19$ when $n \in \{10, 12, 13, 14, 15\}$. Theorem 2.3 stated that $\varphi'(\mu(P_n))$ is either $3n+18$ or $3n+19$ when $10 \leq n \leq 15$. Lemma 3.1 and Lemma 3.2 will provide necessary conditions for $\varphi'(\mu(P_n)) = 3n+18$. These conditions will lead to an example of a b-coloring whose colors are summed to $51 = 3n+18$ for $n = 11$, as shown in Figure 3. Hence, $\varphi'(\mu(P_n)) = 3n+18$ for $n = 11$. On the other hand, for $n \in \{10, 12, 13, 14, 15\}$, the necessary conditions lead to a contradiction in Theorem 3.3. Hence, $\varphi'(\mu(P_n)) = 3n+19$.

Lemma 3.1. *For $10 \leq n \leq 15$, let c_n be a b-coloring which gives $\varphi'(\mu(P_n)) = 3n+18$. Then, there exists $\ell \in \{1, \dots, n-7\}$ where $c_n(v_j)$ and $c_n(u_j)$ for $\ell \leq j \leq \ell+7$ are as shown in Table 1, up to the left and right reflection and the switch between color 3 and color 4. Furthermore, there are unique b-vertices of colors 1, 3, 4 and 5.*

Proof. By Lemma 2.5, we have $|C_1| = n-1$, $|C_2| = n-6$, $|C_3| = 3$, $|C_4| = 3$, $|C_5| = 2$ and $c_n(u) = 2$. From Lemma 2.4, the b-vertices must be in $V(P_n) \cup \{u\}$.

Let $C_5 = \{v_t, x\}$ where v_t is a b-vertex of color 5 and $x \in \{v_s, u_s\}$. Since $c_n(u) = 2$, we have $C_2 \cap U_n = \emptyset$. One of the two neighbors v_{t-1} and v_{t+1} of v_t must has color 2. We may assume $c_n(v_{t+1}) = 2$. The b-vertices of colors $1, 3, 4$ must be in $V(P_n)$. Since each of them must be adjacent to either v_t or x, we have that $\{v_{t-1}, v_{s-1}, v_{s+1}\}$ are the set of unique b-vertices of colors $1, 3$ and 4.

We will show that $c_n(v_{t-1}) \neq 1$. Suppose to the contrary that $c_n(v_{t-1}) = 1$. Since the numbers of the vertices with colors 3 and 4 are the same, we may assume that $c_n(v_{s-1}) = 3$ and $c_n(v_{s+1}) = 4$. The colors of $v_{t-2}, v_{t-1}, v_t, v_{t+1}$ are $2, 1, 5, 2$, respectively. Since v_{t-1} and v_t have no common neighbor, we need two vertices of color 3 and two vertices of color 4 as neighbors of v_{t-1} and v_t in U_n. We have $C_3 \cup C_4 = \{v_{s-1}, v_{s+1}, u_{t-2}, u_{t-1}, u_t, u_{t+1}\}$ and there is no other vertex of color 3 or 4. Now, we consider the

Table 1: Two possibilities for the colorings of the vertices v_i and u_i for $\ell \le i \le \ell+7$ where $\{d_1, d_1'\} = \{4,5\}$ and $\{d_2, d_2'\} = \{1,3\}$. The circles indicate b-vertices. Source: created by the authors.

i	ℓ	$\ell+1$	$(=s)$ $\ell+2$	$\ell+3$	$\ell+4$	$\ell+5$	$(=t)$ $\ell+6$	$\ell+7$
$c_n(v_i)$	2	3	d_1	1	2	4	5	2
$c_n(u_i)$	1		d_1'		3	d_2	1	d_2'

i	ℓ	$\ell+1$	$(=t)$ $\ell+2$	$\ell+3$	$\ell+4$	$(=s)$ $\ell+5$	$\ell+6$	$\ell+7$
$c_n(v_i)$	2	4	5	2	1	d_1	3	2
$c_n(u_i)$	d_2	1	d_2'	3		d_1'		1

Table 2: The b-vertices of colors $1, 3, 4$ and 5 where $\{d_1, d_2\} = \{3, 4\}$. Source: created by the authors.

$c_n(v_{s-1})$	$c_n(v_{s+1})$	$c_n(v_{t-1})$	$c_n(v_t)$
d_1	1	d_2	5
1	d_1	d_2	5

neighbors of the b-vertices v_{s-1} and v_{s+1}. Since $N(\{v_{s-1}, v_{s+1}\})$ and $\{u_{t-2}, u_{t-1}, u_t, u_{t+1}\}$ have at most one common member, it is not possible for both v_{s-1} and v_{s+1} to have neighbors of colors 4 and 3, respectively. Thus v_{s-1} and v_{s+1} cannot both be b-vertices, a contradiction. Hence $c_n(v_{t-1}) \neq 1$. Therefore $c_n(v_{t-1}) \in \{3,4\}$ and $1 \in \{c_n(v_{s-1}), c_n(v_{s+1})\}$ as in Table 2.

Next, if x is another b-vertex of color 5, then $x = v_s$, but v_s has no neighbor of color 2, a contradiction. So v_t is the unique b-vertex of color 5. Thus, $\{v_{s-1}, v_{s+1}, v_{t-1}, v_t\}$ are the set of the unique b-vertices of colors $1, 3, 4$ and 5.

Now, the colorings of such b-vertices of $\mu(P_n)$ for $10 \le n \le 15$ can be described in Table 2 (up to the left and right reflection). We note that $|C_3| = |C_4| = 3$ and $\{d_1, d_2\} = \{3,4\}$. Up to the switch between color 3 and color 4, we will proceed under the case when $d_1 = 3$ and $d_2 = 4$. The other case when $d_1 = 4$ and $d_2 = 3$ can be done in the same manner. We then have that $c_n(v_{t-1}) = 4$, $\{c_n(u_{t-2}), c_n(u_t)\} = \{c_n(u_{t-1}), c_n(u_{t+1})\} = \{1,3\}$ and $c_n(v_{t-2}) = c_n(v_{t+1}) = 2$. The vertices v_{s-1} and v_{s+1} are not adjacent to v_t. So, either $c_n(v_s) = 5$ or $c_n(u_s) = 5$. Since $c_n(u) = 2$, we have $c_n(v_{s-2}) = c_n(v_{s+2}) = 2$ or $c_n(v_s) = 2$. The colors of the vertices are shown in Table 3.

Since $|C_3| = 3$, either $s+2 = t-2$ or $s-2 = t+1$. Thus, there exists $\ell \in \{1, \dots, n-7\}$ such that $\{\ell, \dots, \ell+7\} = \{s-2, \dots, s+2, t-1, \dots, t+1\}$ with the possible colors shown in Table 1. □

Next, we will determine the colors of the vertices v_i and u_i where $i \notin \{\ell, \ell+1, \dots, \ell+7\}$. The following lemma shows that, for the vertices v_i, they

Table 3: Four possibilities for the coloring of the neighbors of v_{s-1} and v_{s+1}, and the colorings of the neighbors of v_{t-1} and v_t, where $\{d_1, d_1'\} = \{4,5\}$, $\{d_2, d_2'\} = \{1,4\}$, $\{d_3, d_3'\} = \{3,4\}$ and $\{d_4, d_4'\} = \{d_5, d_5'\} = \{1,3\}$. The circles indicate b-vertices. Source: created by the authors.

i	$t-2$	$t-1$	t	$t+1$
$c_n(v_i)$	2	4	5	2
$c_n(u_i)$	d_4	d_5	d_4'	d_5'

i	$s-2$	$s-1$	s	$s+1$	$s+2$
$c_n(v_i)$	2	3	d_1	1	2
$c_n(u_i)$	1		d_1'		3

i	$s-2$	$s-1$	s	$s+1$	$s+2$
$c_n(v_i)$	2	1	d_1	3	2
$c_n(u_i)$	3		d_1'		1

i	$s-2$	$s-1$	s	$s+1$	$s+2$
$c_n(v_i)$	d_2	3	2	1	d_3
$c_n(u_i)$	d_2'		5		d_3'

i	$s-2$	$s-1$	s	$s+1$	$s+2$
$c_n(v_i)$	d_3	1	2	3	d_2
$c_n(u_i)$	d_3'		5		d_2'

are of color 2 except only exactly two of them that have color 1. On the other hand, for the vertices u_i, all of them have color 1 except only one of them that has color 4.

Let ℓ be the index obtained from Lemma 3.1, we define $A = \{u_i, v_i : \ell \le i \le \ell+7\}$ and $B = V(\mu(P_n)) \setminus (A \cup \{u\})$.

Lemma 3.2. *For $10 \le n \le 15$, if $\varphi'(\mu(P_n)) = 3n+18$, then the followings are true:*

1. *exactly two vertices in $B \cap V(P_n)$ have color 1 and the rest have color 2;*
2. *for other vertices in U_n which have not been colored in Table 1, exactly one of them has color 4 and the rest have color 1.*

Proof. Let c_n be a b-coloring of $\mu(P_n)$ with the color sum of $3n+18$. By Lemma 2.5, the cardinalities of the color classes are $|C_1| = n-1$, $|C_2| = n-6$, $|C_3| = 3$, $|C_4| = 3$, $|C_5| = 2$. It also gives $c(u) = 2$. By Lemma 3.1, there exists $\ell \in \{1, \dots, n-7\}$ where the colors of the vertices in $A = \{v_i, u_i : \ell \le i \le \ell+7\}$ are corresponding to Table 1. There exist $s, t \in \{\ell, \dots, \ell+7\}$ that $\{c_n(u_s), c_n(v_s)\} = \{4,5\}$ and $c_n(v_{t-1}) = 4, c_n(v_t) = 5$ and $c_n(u_t) \in \{1,3\}$. To complete remaining of the proof, it suffices to show that $|C_1 \cap V(B) \cap V(P_n)| = 2$ and there is $m \neq s$ such that $c_n(u_m) = 4$.

If u is a b-vertex, then $c_n(v_s) = 4$ and $c_n(u_s) = 5$. Since u must have a neighbor of color 4, there exists $m \neq s$ where $c_n(u_m) = 4$. Since $|C_3 \cap U_n| = 2$ and the remaining vertices of color 3, 4 and 5 are already assigned, we have $|(C_3 \cup C_4 \cup C_5) \cap U_n| = 4$. Since $|C_2 \cap U_n| = 0$, we have $|C_1 \cap U_n| = n - 4$.

Now, we suppose that u is not a b-vertex. Thus, there exists a b-vertex of color 2 in P_n. Since such b-vertex must have a neighbor of color 5, it follows that v_{t+1} is the b-vertex of color 2. For the upper coloring in Table 1, the vertex v_{t+1} is not adjacent to a vertex of color 3, a contradiction. For the lower coloring in Table 1, we have $c_n(u_t) = 3$ and $c_n(u_{t+2}) = 4$. Thus, there exists $m = t + 2 \neq s$ where $c_n(u_m) = 4$. We note that, in this case, $c_n(u_{t+3}) = 4$ and $c_n(v_{t+3}) = 5$. We have now completely assigned the colors 3, 4 and 5. Thus $|(C_3 \cup C_4 \cup C_5) \cap U_n| = 4$. Hence $|C_1 \cap U_n| = n - 4$.

Since $|C_1| = n - 1$, we have $|C_1 \cap V(B) \cap V(P_n)| = (n - 1) - (n - 4) - 1 = 2$. Now, the proof is completed. $\square$

From the upper bound in Theorem 2.3 and the result in Lemma 3.2, we can complete the main theorem.

Theorem 3.3. *For* $10 \leq n \leq 15$*, we have*

$$\varphi'(\mu(P_n)) = \begin{cases} 3n + 18, & n = 11, \\ 3n + 19, & n \neq 11. \end{cases}$$

Proof. Let $n \in \{10, 11, \ldots, 15\}$. Suppose that $\varphi'(\mu(P_n)) = 3n + 18$. By Lemma 3.2, exactly two vertices in $B \cap V(P_n)$ have color 1 and the rest have color 2. In addition, for other vertices in U_n which have not been colored in Table 1, exactly one of them has color 4 and the rest have color 1.

Consider the upper coloring in Table 1. (The case of the lower coloring can be treated in the same manner.) Since $v_\ell = 2$ and $u_\ell = 1$, we cannot assign either color 1 or 2 to $v_{\ell-1}$. Hence, $\ell = 1$ and $c_n(v_9), c_n(v_{10}), c_n(v_{11}), \ldots$ are $1, 2, 1, \ldots$, respectively. Since exactly two vertices in $\{v_9, v_{10}, \ldots, v_n\}$ have color 1 and the rest have color 2, we have $11 \leq n \leq 12$. For $n = 12$, we have $c_n(u_{10}) = 4 = c_n(u_{12})$, a contradiction. Hence $n = 11$ and we have a coloring in Figure 3 with the color sum of 51.

From Theorem 2.3, we have $\varphi'(\mu(P_n)) = 3n+18$ for $n = 11$ and $\varphi'(\mu(P_n)) = 3n+19$ otherwise. Now, the proof is completed. $\square$

From Theorem 2.2 and Theorem 3.3, we have the values of $\varphi'(\mu(P_n))$ for all positive integer n as shown in Table 4.

Analysis of the necessary condition for $\varphi'(\mu(P_n)) = 3n + 18$ leads to a contradiction when the value is lower than the b-chromatic sum. A similar analysis may possibly be further used to analyze the lower bound of the b-chromatic sum of other graphs.

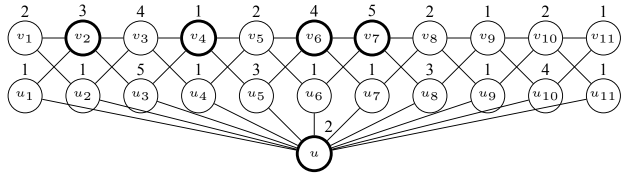

Fig. 3: A b -coloring with $\varphi'(\mu(P_n)) = 3n + 18$ when $n = 11$. Source: created by the authors.

Table 4: The exact values of $\varphi'(\mu(P_n))$ for all positive integer n. Source: created by the authors.

n	$\varphi'(\mu(P_n))$	reference
1	$3n + 1$	[8]
3	$3n + 2$	[8]
$2, 4$	$3n + 3$	[8]
$5, 6, 7$	$3n + 6$	[8], [9]
8	$3n + 20$	[8]
$9, 10$	$3n + 19$	[9], and our result
11	$3n + 18$	our result
$n \geq 12$	$3n + 19$	[9], and our result

4 Conclusion

In this work, the b-chromatic sum of the Mycielskian of paths is studied. The study, [8], previously gave the b-chromatic sum of $\mu(P_n)$ where $n \leq 6$ and $n = 8$, while, [9], later determined the values for $n = 7, 9$ and $n \geq 16$. Therefore, our results complete the characterization of the b-chromatic sum of all the Mycielskian of paths of all length as appeared in Table 4.

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflicts of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.