Further Results and Application on Bipolar Fuzzy Soft Environments under Interval-Valued Influence

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Abstract: - The rapidly evolving nature of uncertainty issues requires researchers to continuously develop current mathematical tools. For example, employee selection techniques in government institutions are subject to rigorous evaluation by human resources department committees, during which positive and negative impressions are formed when interviewing candidates applying for these jobs. Therefore, in order to cover this impression, we will present in this work a mathematical model called bipolar fuzzy soft, associated with the influence of interval form. This model study translates positive and negative thinking into an interval degree of ambiguity ranging between [0,1] for the positive side and an interval degree ranging between [-1,0] for the negative side. On BIV-FSS, we define and study basic set operations, including the extended union, intersection of two BIV-FSSs, and the complement operation of BIV-FSS. Some properties and numerical examples that illustrate. Finally, we will demonstrate the practical side of this work by presenting a new algorithm that relies on the tools proposed in this work to solve one of the decision-making problems in the administrative aspect (employee selection).

Key-Words: - Bipolar fuzzy set, Bipolar fuzzy soft set, Interval valued fuzzy set, Interval valued bipolar fuzzy set, Decision-making.

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1 Introduction

In [1] the theory of fuzzy set (FS) is proposed as a mathematical extension of crisp set in order to handle uncertainty in the various real applications. The traditional FS is represented by a well-single functioning membership function (single belonging function) whose starting point is the universal set and whose rest is the closed interval [0,1]. The dynamics of coupling the elements of the comprehensive set with the single membership function has attracted considerable interest from users, especially those who employ these mathematical tools in the decision-making process, i.e., choosing the optimal option from among several presented options. For example, if the comprehensive set contains three cars and one of them is to be chosen, using the single membership function, each of these cars can be assigned a value belonging to the closed interval [0, 1]. Accordingly, the choice will be for the car that obtained a score close to 1.In search of more flexibility, [2] applied the concept of interval-valued FSs (IV-FSs) when he changed the form of value from single value to interval value, looking for more flexibility to handle uncertainty in the various real applications. The issues that FS dealt with were dealt with by IVFS, who proved to be superior, flexible, and more adaptable in putting data on daily life problems.

At present, IV-FSs have attracted wide attention from many researchers, and they have made some achievements. [3] developed some applications on IV-FS information. [4] developed some common measures on IV-FSs and used these measures in handling DM problems. The soft set (SS) [5] was merged with FS, [6]. [7] founded hybrid models called IV-FSSs when they merged IV-FSs and SSs. By looking to SS structures, it has a clear representation of the elements of the universal set. The SS has many extensions, [8], [9], [10], [11], [12], [13], [14]. However, the integration of SSs is not limited to the FSs; rather, it extends to other extensions of the FSs, [15] defined normal groups on Q-neutrosophic soft sets as a new algebraic approach of FSs . [16]. modified a new hybrid of the Fermatean neutrosophic soft set. [17] introduced the concept of an effective fuzzy soft set, and he showed its operation.

In any case, the nature of thinking of the decision maker is characterized by two sides: positive and negative. That is, for every situation, the human mind takes two sides: positive and negative. To illustrate this idea, [18] suggests the idea of bipolar fuzzy sets (BFS) where every belonging function has two positive and negative sides, called positive belonging function and the negative belonging function, and both of them have a stable location that differs from the other, meaning that the short period that we mentioned previously expands to [-1,1]. Now there are many contributions based on this idea, including bipolar interval -valued fuzzy set (BIVFS) [19] when we change the positive and negative constraints from single to interval form, and other works, [20], [21], [22], [23], [24].

Therefore, in order to cover this impression, we will present in this work a mathematical model called bipolar fuzzy soft, associated with the influence of interval form. This model study translates positive and negative thinking into an interval degree of ambiguity ranging between [0,1] for the positive side and an interval degree ranging between [-1,0] for the negative side. On BIV-FSS, we define and study basic set operations, including the extended union, intersection of two BIV-FSSs, the complement operation of BIV-FSS. Some properties and numerical examples that illustrate. Finally, we will demonstrate the practical side of this work by presenting a new algorithm that relies on the tools proposed in this work to solve one of the decision-making problems in the administrative aspect (employee selection).

2 Preliminaries

Definition 2.1 [1] The following framework

$$\Gamma_{FS} = \{(\dot{\tau}, \mathfrak{B}(\dot{\tau})), \forall \dot{\tau} \in X\}$$

on non – empty universe set X define as a fuzzy set (FS) where the component $\mathfrak{B}(\dot{\tau})$ give as a true membership function from X to $[0,1]$.

Definition 2.2 [2] The following framework

$$\Gamma_{IVFS} = \{(\dot{\tau}, \mathfrak{B}(\dot{\tau})), \forall \dot{\tau} \in X\}$$

on non – empty universe set X define as an interval valued-fuzzy set (IV-FS) where the component $\mathfrak{B}(\dot{\tau}) = [\mathfrak{B}^L(\dot{\tau}), \mathfrak{B}^U(\dot{\tau})]$ give as a true lower membership (TLM) and a true upper membership (TUM) functions from X to $[0,1]$.

Definition 2.3 [2] Let $\Gamma_{1,IVFS} = \{(\dot{\tau}, \mathfrak{B}_1(\dot{\tau})), \forall \dot{\tau} \in X\}$ where $\mathfrak{B}_1(\dot{\tau}) = [\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_1^U(\dot{\tau})]$and $\Gamma_{2,IVFS} = \{(\dot{\tau}, \mathfrak{B}_2(\dot{\tau})), \forall \dot{\tau} \in X\}$ where $\mathfrak{B}_2(\dot{\tau}) = [\mathfrak{B}_2^L(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})]$ then

Then the following properties are done,

i. Complement:

The $(\Gamma_{1,FS})^C = \{(\dot{\tau}, \mathfrak{B}_1^C(\dot{\tau})), \forall \dot{\tau} \in X\}$
$= \{(\dot{\tau}, [1 - \mathfrak{B}_1^U(\dot{\tau}), 1 - \mathfrak{B}_1^L(\dot{\tau})]), \forall \dot{\tau} \in X\}$.

ii. Union:

The
$\Gamma_{1,FS} \cup \Gamma_{2,FS} = \{max[\mathfrak{B}_1(\dot{\tau}), \mathfrak{B}_2(\dot{\tau})],, \forall \dot{\tau} \in X\}$
$= \{max[\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_2^L(\dot{\tau})], max[\mathfrak{B}_1^U(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})], \forall \dot{\tau} \in X\}$.

iii. Intersection:

The
$\Gamma_{1,FS} \cap \Gamma_{2,FS} = \{min[\mathfrak{B}_1(\dot{\tau}), \mathfrak{B}_2(\dot{\tau})],, \forall \dot{\tau} \in X\}$.
$= \{min[\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_2^L(\dot{\tau})], min[\mathfrak{B}_1^U(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})], \forall \dot{\tau} \in X\}$.

iv. Subset:
The $\Gamma_{1,FS} \subseteq \Gamma_{2,FS}$ if $\mathfrak{B}_1(\dot{\tau}) \leq \mathfrak{B}_2(\dot{\tau})$
such that
$\mathfrak{B}_1^L(\dot{\tau}) \leq \mathfrak{B}_2^L(\dot{\tau})$ and $\mathfrak{B}_1^U(\dot{\tau}) \leq \mathfrak{B}_2^U(\dot{\tau})$

v. Equals:
The $\Gamma_{1,FS} = \Gamma_{2,FS}$ if $\mathfrak{B}_1(\dot{\tau}) = \mathfrak{B}_2(\dot{\tau})$.
such that
$\mathfrak{B}_1^L(\dot{\tau}) = \mathfrak{B}_2^L(\dot{\tau})$ and $\mathfrak{B}_1^U(\dot{\tau}) = \mathfrak{B}_2^U(\dot{\tau})$.

Definition 2.4 [2] Let $\Gamma_{1,IVFS} = \{(\dot{\tau}, \mathfrak{B}_1(\dot{\tau})), \forall\dot{\tau} \in X\}$ where $\mathfrak{B}_1(\dot{\tau}) = [\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_1^U(\dot{\tau})]$ and $\Gamma_{2,IVFS} = \{(\dot{\tau}, \mathfrak{B}_2(\dot{\tau})), \forall\dot{\tau} \in X\}$ where $\mathfrak{B}_2(\dot{\tau}) = [\mathfrak{B}_2^L(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})]$ then

- **i.** The $\Gamma_{1,IVFS} \subseteq \Gamma_{2,IVFS}$ if $\mathfrak{B}_1^L(\dot{\tau}) \leq \mathfrak{B}_2^L(\dot{\tau})$ and $\mathfrak{B}_1^U(\dot{\tau}) \leq \mathfrak{B}_2^U(\dot{\tau})$.
- **ii.** The $\Gamma_{1,IVFS} = \Gamma_{2,IVFS}$ if $\mathfrak{B}_1^L(\dot{\tau}) = \mathfrak{B}_2^L(\dot{\tau})$ and $\mathfrak{B}_1^U(\dot{\tau}) = \mathfrak{B}_2^U(\dot{\tau})$.
- **iii.** $(\Gamma_{1,IVFS})^C = \left\{\left(\dot{\tau}, \mathfrak{B}_1^C(\dot{\tau})\right), \forall\dot{\tau} \in X\right\}$=$\{(\dot{\tau}, [1 - \mathfrak{B}_1^U(\dot{\tau}), 1 - \mathfrak{B}_1^L(\dot{\tau})]), \forall\dot{\tau} \in X\}$.
- **iv.** $\Gamma_{1,IVFS} \cup \Gamma_{2,IVFS} = \{max[\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_2^L(\dot{\tau})], max[\mathfrak{B}_1^U(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})], \forall\dot{\tau} \in X\}$.
- **v.** $\Gamma_{1,IVFS} \cap \Gamma_{2,IVFS} = \{min[\mathfrak{B}_1^L(\dot{\tau}), \mathfrak{B}_2^L(\dot{\tau})], min[\mathfrak{B}_1^U(\dot{\tau}), \mathfrak{B}_2^U(\dot{\tau})], \forall\dot{\tau} \in X\}$.

Definition 2.5 [2] The following framework
$$\Gamma_{BFS} = \{(\dot{\tau}, \mathfrak{B}^+(\dot{\tau}), \mathfrak{B}^-(\dot{\tau})), \forall\dot{\tau} \in X\}$$
on non – empty universe set X define as a bipolar fuzzy set (BFS) where the components $\mathfrak{B}^+(\dot{\tau}), \mathfrak{B}^-(\dot{\tau})$ give as a positive true membership (PTM) function from X to $[0,1]$ and negative true membership (NTM) from function from X to $[-1,0]$.

Definition 2.6 [11]
Let $\Gamma_{1,BFS} = \{(\dot{\tau}, \mathfrak{B}_1^+(\dot{\tau}), \mathfrak{B}_1^-(\dot{\tau})), \forall\dot{\tau} \in X\}$ and $\Gamma_{2,BFS} = \{(\dot{\tau}, \mathfrak{B}_2^+(\dot{\tau}), \mathfrak{B}_2^-(\dot{\tau})), \forall\dot{\tau} \in X\}$ then

- **i.** The $\Gamma_{1,BFS} \subseteq \Gamma_{2,BFS}$ if $\mathfrak{B}_1^+(\dot{\tau}) \leq \mathfrak{B}_2^+(\dot{\tau})$ and $\mathfrak{B}_1^-(\dot{\tau}) \leq \mathfrak{B}_2^-(\dot{\tau})$.
- **ii.** The $\Gamma_{1,BFS} = \Gamma_{2,BFS}$ if $\mathfrak{B}_1^+(\dot{\tau}) = \mathfrak{B}_2^+(\dot{\tau})$ and $\mathfrak{B}_1^-(\dot{\tau}) = \mathfrak{B}_2^-(\dot{\tau})$.
- **iii.** $(\Gamma_{1,BFS})^C = \left\{\left(\dot{\tau}, \mathfrak{B}_1^C(\dot{\tau})\right), \forall\dot{\tau} \in X\right\}$=$\{(\dot{\tau}, [1 - \mathfrak{B}_1^+(\dot{\tau}), 1 + \mathfrak{B}_1^L(\dot{\tau})]), \forall\dot{\tau} \in X\}$.
- **iv.** $\Gamma_{1,BFS} \cup \Gamma_{2,BFS} = \{max[\mathfrak{B}_1^+(\dot{\tau}), \mathfrak{B}_2^+(\dot{\tau})], max[\mathfrak{B}_1^-(\dot{\tau}), \mathfrak{B}_2^-(\dot{\tau})], \forall\dot{\tau} \in X\}$.
- **v.** $\Gamma_{1,BFS} \cap \Gamma_{2,BFS} = \{min[\mathfrak{B}_1^+(\dot{\tau}), \mathfrak{B}_2^+(\dot{\tau})], min[\mathfrak{B}_1^-(\dot{\tau}), \mathfrak{B}_2^-(\dot{\tau})], \forall\dot{\tau} \in X\}$.

Definition 2.7 [20] The following order pair $(\Gamma, A_{i=1,2,\dots,n})$is called SS where $\Gamma: A_i \longrightarrow P(X)$ such that $A_i \subseteq$ Yis given as the set of parameters and $P(X)$is the power set of X.

Definition 2.8 [23] The following framework
$$(\Gamma_{FSS}, A_i) = \{(\dot{\tau}, \mathfrak{B}^+(\dot{\tau})(\sigma), \mathfrak{B}^-(\dot{\tau})(\sigma)), \forall\dot{\tau} \in X \; and \; \sigma \in A_i \subseteq \mathrm{Y}\}$$
is called BFSSs on nonempty soft universal (X, Y). Where $\mathfrak{B}^+(\dot{\tau})(\sigma)$ is soft-$PTMF$ and $\mathfrak{B}^-(\dot{\tau})(\sigma)$ $soft - NTMF$.

3 The Mathematical Structure of Bipolar Interval Fuzzy Soft Sets (BIV-FSSs)

In this part, we propose the new framework definition of our concept BIV-FSS with fundamental operations like empty BIV-FSS, absolute BIV-FSS, subset BIV-FSS, and equality between two BIV-FSS. Also, to clarify our model more, we will give some numerical examples.
Now, we will provide the main definition of our propose model BIV-FSS.

Definition 3.1 For X be non – empty universe set and Y be a parametrized set contains all parametrized that represent X element's. Then, the
$$\Gamma_{A_i} = \{(\dot{\tau}, \mathfrak{B}^+(\dot{\tau})(\sigma), \mathfrak{B}^-(\dot{\tau})(\sigma)), \forall\tau \in \dot{X} \; and \; \sigma\in A_i \subseteq \mathrm{Y}\}$$
$$= \{(\dot{\tau}, [\mathfrak{B}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}^{+,U}(\dot{\tau})(\sigma)], [\mathfrak{B}^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}^{-,U}(\dot{\tau})(\sigma)]) | \forall\dot{\tau} \in X\}.$$
Where
$\mathfrak{B}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}^{+,U}(\dot{\tau})(\sigma): (X, A_i \subseteq \mathrm{Y}) \to [0,1]$,
$\mathfrak{B}^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}^{-,U}(\dot{\tau})(\sigma): (X, A_i \subseteq \mathrm{Y}) \to [-1,0]$.
Here the true function $\mathfrak{B}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}^{+,U}(\dot{\tau})(\sigma)$ are denotes to lower and upper soft-$PTMF$ and the true function $\mathfrak{B}^{-,L}(\dot{\tau})(\sigma),, \mathfrak{B}^{-,U}(\dot{\tau})(\sigma)$ are denotes to lower and upper soft-$NTMF$.

Definition 3.2 Let
$$\Gamma = \left\{\left(\dot{\tau}, \mathfrak{B}^{1^+}(\dot{\tau})(\sigma), \mathfrak{B}^{1^-}(\dot{\tau})(\sigma)\right), \forall\dot{\tau} \in X \text{ and } \sigma \in Y\right\}$$
be a BIV-FSSs on nonempty soft universal (X, Y). Then we defined the complement operation on Γ as the following
$$\Gamma^C = \{(\dot{\tau}, \mathfrak{B}^{+c}(\dot{\tau})(\sigma), \mathfrak{B}^{-c}(\dot{\tau})(\sigma)), \forall\dot{\tau} \in X \text{ and } \sigma \in Y\} .$$

where

$\mathfrak{B}^{+c}(\dot{\tau})(\sigma) = \mathfrak{B}_1{}^{+,L,c}(\dot{\tau})(\sigma) = 1 - \mathfrak{B}_1{}^{+,U}(\dot{\tau})(\sigma)$,
$\mathfrak{B}^{+c}(\dot{\tau})(\sigma) = \mathfrak{B}_1{}^{+,U,c}(\dot{\tau})(\sigma) = 1 - \mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma)$,

qnd
$\mathfrak{B}^{-c}(\dot{\tau})(\sigma) = \mathfrak{B}_1{}^{-,L,c}(\dot{\tau})(\sigma) = 1 + \mathfrak{B}_1{}^{-,U}(\dot{\tau})(\sigma)$,
$\mathfrak{B}^{-c}(\dot{\tau})(\sigma) = \mathfrak{B}_1{}^{-,U,c}(\dot{\tau})(\sigma) = 1 + \mathfrak{B}^{1-,L}(\dot{\tau})(\sigma)$.
Here Γ^C denotes as BIV-FSS-complement.

Example 3.1.
The part $\Gamma_1(\sigma_1) = \{\dot{\tau}_1, \langle[0.1,0.4], [-0.5, -0.3]\rangle\}$ is BIV-FSSs on nonempty soft universal (X, Y). Then the $\Gamma_1^C(\sigma_2) = \{\dot{\tau}_1, \langle[0.6,0.9], [-0.7, -0.5]\rangle\}$ is a complement of $\Gamma_1(\sigma_1)$.

Proposition 3.1 If $\Gamma_{BIV-FSSs} = \{(\dot{\tau}, \mathfrak{B}^+(\dot{\tau})(\sigma), \mathfrak{B}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ be a BIV-FSSs on nonempty soft universal (X, Y). Then $(\Gamma^C)^C = \Gamma$.

Proof: This proposition can be proven using the techniques in the **Definition 3.2.**

We have
$\Gamma_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}^+(\dot{\tau})(\sigma), \mathfrak{B}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$
and

$$\Gamma^c_{BIV-FSS} = \left\{\left(\dot{\tau}, \left(\mathfrak{B}^+(\dot{\tau})(\sigma)\right)^C, \left(\mathfrak{B}^-(\dot{\tau})(\sigma)\right)^C\right), \forall \dot{\tau} \in X\right\}$$

$$= \left\{\begin{pmatrix} \dot{\tau}, ([\mathfrak{B}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}^{+,U}(\dot{\tau})(\sigma)])^C, \\ ([\mathfrak{B}^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}^{-,U}(\dot{\tau})])^C \end{pmatrix}, \forall \dot{\tau} \in X\right\}$$

$$= \left\{\begin{pmatrix} \dot{\tau}, \left[\left(\mathfrak{B}^{+,L}(\dot{\tau})\right)^C, \left(\mathfrak{B}^{+,U}(\dot{\tau})\right)^C\right], \\ \left[\left(\mathfrak{B}^{-,L}(\dot{\tau})\right)^C, \left(\mathfrak{B}^{-,U}(\dot{\tau})\right)^C\right] \end{pmatrix}, \forall \dot{\tau} \in X\right\}$$

$$= \left\{\begin{pmatrix} \dot{\tau}, \left[\left(1 - \mathfrak{B}^{+,U}(\dot{\tau})\right), \left(1 - \mathfrak{B}^{+,L}(\dot{\tau})\right)\right], \\ \left[\left(1 + \mathfrak{B}^{-,U}(\dot{\tau})\right), \left(1 + \mathfrak{B}^{-,L}(\dot{\tau})\right)\right] \end{pmatrix}, \forall \dot{\tau} \in X\right\}$$

Thus

$$(\Gamma^c_{BIV-FS})^c = \left\{\begin{pmatrix} \dot{\tau}, \left[\left(1 - \mathfrak{B}^{+,U}(\dot{\tau})\right)^c, \left(1 - \mathfrak{B}^{+,U}(\dot{\tau})\right)^c\right], \\ \left[\left(1 + \mathfrak{B}^{-,U}(\dot{\tau})\right)^c, \left(+\mathfrak{B}^{-,U}(\dot{\tau})\right)^c\right] \end{pmatrix}, \forall \dot{\tau} \in X\right\}.$$

$$= \left\{\begin{pmatrix} \dot{\tau}, \left[\left(1 - \left(\mathfrak{B}^{+,U}(\dot{\tau})\right)^C\right), \left(1 - \left(\mathfrak{B}^{+,L}(\dot{\tau})\right)^C\right)\right], \\ \left[\left(1 - \left(\mathfrak{B}^{-,U}(\dot{\tau})\right)^C\right), \left(1 - \left(\mathfrak{B}^{-,L}(\dot{\tau})\right)^C\right)\right] \end{pmatrix}, \forall \dot{\tau} \in X \right\}$$

$$= \left\{\begin{pmatrix} \dot{\tau}, \left[\left(1 - \left(1 - \mathfrak{B}^{+,L}(\dot{\tau})\right)\right), \left(1 - \left(1 - \mathfrak{B}^{+,U}(\dot{\tau})\right)\right)\right], \\ \left[\left(1 - \left(1 + \mathfrak{B}^{-,L}(\dot{\tau})\right)\right), \left(1 - \left(1 + \mathfrak{B}^{-,U}(\dot{\tau})\right)\right)\right] \end{pmatrix}, \forall \dot{\tau} \in X \right\}$$

$$= \{(\dot{\tau}, [\mathfrak{B}^{+,L}(\dot{\tau}), \mathfrak{B}^{+,U}(\dot{\tau})], [\mathfrak{B}^{-,L}(\dot{\tau}), \mathfrak{B}^{-,U}(\dot{\tau})]), \forall \dot{\tau} \in X\}$$

$$= \left\{\left(\dot{\tau}, \mathfrak{B}^+(\dot{\tau}), \mathfrak{B}^-(\dot{\tau})\right), \forall \dot{\tau} \in X\right\} = \Gamma_{BIV-FS}.$$

Example 3.2. Consider the parts given in an **example 3.1** such that

$$(\Gamma_1, A_1) = \left\{ \Gamma_1(\sigma_1) \begin{Bmatrix} \dot{\tau}_1, \langle[0.1,0.4], [-0.5, -0.3]\rangle \\ \dot{\tau}_2, \langle[0.6,0.8], [-0.4, -0.2]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7], [-0.2, -0.9]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5], [-0.1, -0.1]\rangle \end{Bmatrix} \right\}$$

Then

$$(\Gamma_1, A_1)^C = \left\{ \Gamma_1(\sigma_1) \begin{Bmatrix} \dot{\tau}_1, \langle[0.6,0.9], [-0.7, -0.5]\rangle \\ \dot{\tau}_2, \langle[0.2,0.4], [-0.8, -0.6]\rangle \\ \dot{\tau}_3, \langle[0.3,0.6], [-0.1, -0.8]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5], [-0.9, -0.9]\rangle \end{Bmatrix} \right\}$$

Definition 3.3. Let $\Gamma_1 = \{(\dot{\tau}, \mathfrak{B}_1{}^+(\dot{\tau})(\sigma), \mathfrak{B}_1{}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X \text{ and } \sigma \in Y\}$ and $\Gamma_2 = \{(\dot{\tau}, \mathfrak{B}_2{}^+(\dot{\tau})(\sigma), \mathfrak{B}_2{}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X \text{ and } \sigma \in Y\}$ be two BIV-FSSs on nonempty soft universal (X, Y). Then the union between both of them denotes as$\Gamma_1 \cup \Gamma_2 = \Gamma_3$ and given as following formal.

$$\Gamma_3 = \begin{Bmatrix} \{(\dot{\tau}, max\{\mathfrak{B}_1{}^+(\dot{\tau})(\sigma), \mathfrak{B}_2{}^+(\dot{\tau})(\sigma))\} \\ = max[\mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{+,L}(\dot{\tau})(\sigma)], \\ max[\mathfrak{B}_1{}^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{+,U}(\dot{\tau})(\sigma)]. \\ \{(\dot{\tau}, min\{\mathfrak{B}_1{}^-(\dot{\tau})(\sigma), \mathfrak{B}_2{}^-(\dot{\tau})(\sigma))\} \\ = min[\mathfrak{B}_1{}^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,L}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_1{}^{-,U}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,U}(\dot{\tau})(\sigma)]. \end{Bmatrix}$$

This operation denotes as : $\Gamma_1 \cup \Gamma_2$.

Definition 3.4
Let $\Gamma_1 = \{(\dot{\tau}, \mathfrak{B}_1{}^+(\dot{\tau})(\sigma), \mathfrak{B}_1{}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X \text{ and } \sigma \in Y\}$ and $\Gamma_2 = \{(\dot{\tau}, \mathfrak{B}_2{}^+(\dot{\tau})(\sigma), \mathfrak{B}_2{}^-(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X \text{ and } \sigma \in Y\}$ be two BIV-FSSs on nonempty soft universal (X, Y). Then the intersection between both of them denotes as $\Gamma_1 \cap \Gamma_2 = \Gamma_3$ and given as following formal:

$$\Gamma_3 = \begin{Bmatrix} \{(\dot{\tau}, min\{\mathfrak{B}_1{}^+(\dot{\tau})(\sigma), \mathfrak{B}_2{}^+(\dot{\tau})(\sigma))\} \\ = min[\mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{+,L}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_1{}^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{+,U}(\dot{\tau})(\sigma)]. \\ \{(\dot{\tau}, max\{\mathfrak{B}_1{}^-(\dot{\tau})(\sigma), \mathfrak{B}_2{}^-(\dot{\tau})(\sigma))\} \\ = max[\mathfrak{B}_1{}^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,L}(\dot{\tau})(\sigma)], \\ max[\mathfrak{B}_1{}^{-,U}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,U}(\dot{\tau})(\sigma)]. \end{Bmatrix}$$

This operation denotes as : $\Gamma_1 \cap \Gamma_2$.

Example 3.3 Consider the parts given in an **example 3.1** such that

$$(\Gamma_1, A_1) = \left\{ \begin{array}{l} \Gamma_1(\sigma_1)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.1,0.4],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.6,0.8],[-0.4,-0.2]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.2,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \\ \Gamma_1(\sigma_2)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.3,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.2,0.5],[-0.6,-0.5]\rangle \end{array}\right\} \\ \Gamma_1(\sigma_3)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.1,0.4],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.6,0.8],[-0.4,-0.2]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.6]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \end{array} \right\}$$

and let $A_2 \subseteq \mathrm{Y} = \{\sigma_1, \sigma_2\}$ such that

$$(\Gamma_2, A_2) = \left\{ \begin{array}{l} \Gamma_2(\sigma_1)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.3,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.2,0.5],[-0.6,-0.5]\rangle \end{array}\right\} \\ \Gamma_2(\sigma_2)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.3,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.2,0.5],[-0.6,-0.5]\rangle \end{array}\right\} \end{array} \right\}$$

Then

$$\Gamma_1 \cup \Gamma_2 = \left\{ \begin{array}{l} \Gamma(\sigma_1)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.2,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \\ \Gamma(\sigma_2)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.3,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.2,0.5],[-0.6,-0.5]\rangle \end{array}\right\} \\ \Gamma(\sigma_3)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.1,0.4],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.6,0.8],[-0.4,-0.2]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.6]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \end{array} \right\}$$

And

$$\Gamma_1 \cap \Gamma_2 = \left\{ \begin{array}{l} \Gamma(\sigma_1)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.2,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \\ \Gamma(\sigma_2)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.3,0.7],[-0.3,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.8,0.9],[-0.7,-0.1]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.9]\rangle \\ \dot{\tau}_4, \langle[0.2,0.5],[-0.6,-0.5]\rangle \end{array}\right\} \\ \Gamma(\sigma_3)\left\{\begin{array}{l} \dot{\tau}_1, \langle[0.1,0.4],[-0.5,-0.3]\rangle \\ \dot{\tau}_2, \langle[0.6,0.8],[-0.4,-0.2]\rangle \\ \dot{\tau}_3, \langle[0.4,0.7],[-0.9,-0.6]\rangle \\ \dot{\tau}_4, \langle[0.5,0.5],[-0.1,-0.1]\rangle \end{array}\right\} \end{array} \right\}$$

Proposition 3.2

Let $\Gamma^1_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}^+_1(\dot{\tau})(\sigma), \mathfrak{B}^-_1(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$,$\Gamma^2_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}^+_2(\dot{\tau})(\sigma), \mathfrak{B}^-_2(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ and $\Gamma^3_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}^+_3(\dot{\tau})(\sigma), \mathfrak{B}^-_3(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ be three BIV-FSSs on nonempty soft universal (X, Y). Then the following tip is considered convincing:

i. $\Gamma^1_{BIV-FSS} \cup \Gamma^1_{\Phi} = \Gamma^1_{\Phi} \cup \Gamma^1_{BIV-FSS} = \Gamma^1_{BIV-FSS}$.

ii. $\Gamma^1_{BIV-FSS} \cap \Gamma^1_{\Phi} = \Gamma^1_{\Phi} \cap \Gamma^1_{BIV-FS} = \Gamma^1_{\Phi}$.

iii. $\Gamma^1_{BIV-FSS} \cup \Gamma^1_{\mathrm{X}} = \Gamma^1_{\mathrm{X}} \cup \Gamma^1_{BIV-FSS} = \Gamma^1_{\mathrm{X}}$.

iv. $\Gamma^1_{BIV-FS} \cap \Gamma^1_{\mathrm{X}} = \Gamma^1_{\mathrm{X}} \cap \Gamma^1_{BIV-FS} = \Gamma^1_{BIV-FS}$.

Proof (i): This proof depends on the definition of union who clearing above such that, take

$$\Gamma^1_{BIV-FS} \cup \Gamma^1_{\Phi} = \left\{ \begin{pmatrix} \dot{\tau}, \max[\mathfrak{B}^+_1(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{+}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_1{}^{-}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{-}(\dot{\tau})(\sigma)] \end{pmatrix}, \forall \dot{\tau} \in X \right\}$$

$$= \left\{ \left(\tau, \left\{ \begin{array}{l} max[\mathfrak{B}^{+,L}_1(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{+,L}(\dot{\tau})(\sigma)], \\ max[\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma), \mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma)] \end{array} \right\}, \left\{ \begin{array}{l} min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{-,L}(\dot{\tau})(\sigma)\right], \\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{-,U}(\dot{\tau})(\sigma)\right] \end{array} \right\} \right) \right\} \quad \textbf{(1)}$$

$$= \left\{ \left(\dot{\tau}, \left\{ \begin{array}{l} max\left[\mathfrak{B}^{1^{+,L}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{+,L}(\dot{\tau})(\sigma)\right], \\ max\left[\mathfrak{B}^{1^{+,U}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{+,U}(\dot{\tau})(\sigma)\right] \end{array} \right\}, \left\{ \begin{array}{l} min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{-,L}(\dot{\tau})(\sigma)\right], \\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma), \mathfrak{B}_{\Phi}{}^{-,U}(\dot{\tau})(\sigma)\right] \end{array} \right\} \right) \right\} \quad \textbf{(2)}$$

From **(1)** and **(2)** and through the previous definitions, we guarantee that the result will be $\Gamma^1_{BIV-FS} = \{(\dot{\tau}, \mathfrak{B}_1{}^{+}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X \text{ } and \text{ } \sigma \in Y\}$.

Proof (ii): This proof depends on the definition of union who clearing above such that ,take

$$\Gamma^1_{BIV-FS}\cap\Gamma^1_{\Phi}$$
$$=\left\{\begin{pmatrix}\dot{\tau},\min[\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^+(\dot{\tau})(\sigma)],\\ max[\mathfrak{B}_1{}^-(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^-(\dot{\tau})(\sigma)]\end{pmatrix},\forall\dot{\tau}\in X\right\}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}min[\mathfrak{B}^{+,L}_1(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{+,L}(\dot{\tau})(\sigma)],\\ min[\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma),\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma)]\end{Bmatrix},\begin{Bmatrix}max\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ max\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(3)}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max\left[\mathfrak{B}^{1^{+,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{+,L}(\dot{\tau})(\sigma)\right],\\ max\left[\mathfrak{B}^{1^{+,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{+,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(4)}$$

From **(3)** and **(4)** and through the previous definitions, we guarantee that the result will be

$\Gamma^1_{\Phi}=\{(\dot{\tau},\mathfrak{B}^+_{\Phi}(\dot{\tau})(\sigma),\mathfrak{B}^-_{\Phi}(\dot{\tau})(\sigma)),\forall\dot{\tau}\in X\ and\ \sigma\in Y\}$.

Proof (iii): This proof depends on the definition of union who clearing above such that, take

$$\Gamma^1_{BIV-FS}\cup\Gamma^1_{X}$$
$$=\left\{\begin{pmatrix}\dot{\tau},\max[\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^+(\dot{\tau})(\sigma)],\\ min[\mathfrak{B}_1{}^-(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^-(\dot{\tau})(\sigma)]\end{pmatrix},\forall\dot{\tau}\in X\right\}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max[\mathfrak{B}^{+,L}_1(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,L}(\dot{\tau})(\sigma)],\\ max[\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma),\mathfrak{B}^{+,U}_X(\dot{\tau})(\sigma)]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(5)}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max\left[\mathfrak{B}^{1^{+,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,L}(\dot{\tau})(\sigma)\right],\\ max\left[\mathfrak{B}^{1^{+,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(6)}$$

From **(5)** and **(6)** and through the previous definitions, we guarantee that the result will be

$\Gamma^1_{X}=\{(\dot{\tau},\mathfrak{B}^+_{X}(\dot{\tau})(\sigma),\mathfrak{B}^-_{X}(\dot{\tau})(\sigma)),\forall\dot{\tau}\in X\ and\ \sigma\in Y\}$.

Proof (vi): This proof depends on the definition of union who clearing above such that ,take

$$\Gamma^1_{BIV-FS}\cap\Gamma^1_{X}$$
$$=\left\{\begin{pmatrix}\dot{\tau},\min[\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^+(\dot{\tau})(\sigma)],\\ max[\mathfrak{B}_1{}^-(\dot{\tau})(\sigma),\mathfrak{B}_{\Phi}{}^-(\dot{\tau})(\sigma)]\end{pmatrix},\forall\dot{\tau}\in X\right\}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}min[\mathfrak{B}^{+,L}_1(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,L}(\dot{\tau})(\sigma)],\\ min[\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma),\mathfrak{B}^{+,U}_X(\dot{\tau})(\sigma)]\end{Bmatrix},\begin{Bmatrix}max\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ max\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(7)}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max\left[\mathfrak{B}^{1^{+,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,L}(\dot{\tau})(\sigma)\right],\\ max\left[\mathfrak{B}^{1^{+,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{+,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_{X}{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(8)}$$

From **(7)** and **(8)** and through the previous definitions, we guarantee that the result will be

$=\{(\dot{\tau},\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}^-_1(\dot{\tau})(\sigma)),\forall\dot{\tau}\in X\ and\ \sigma\in Y\}$
$=\Gamma^1_{BIV-FS}$

Proposition 3.3
Let $\Gamma^1_{BIV-FS}=\{(\dot{\tau},\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}^-_1(\dot{\tau})(\sigma)),\forall\dot{\tau}\in X\}$ and $\Gamma^2_{BIV-FSS}=\{(\dot{\tau},\mathfrak{B}^+_2(\dot{\tau})(\sigma),\mathfrak{B}^-_2(\dot{\tau})(\sigma)),\forall\dot{\tau}\in X\}$ be two BIV-FSSs on nonempty soft universal (X,Y). Then the following tip is considered convincing:
i. $(\Gamma^1_{BIV-FS}\cup\Gamma^2_{BIV-FS})=(\Gamma^2_{BIV-FS}\cup\Gamma^1_{BIV-FS})$.
ii. $(\Gamma^1_{BIV-FS}\cap\Gamma^2_{BIV-FS})=(\Gamma^2_{BIV-FS}\cap\Gamma^1_{BIV-FS})$.

Note: These points fulfil the commutative law.

Proof (i): This proof depends on the definition of union who clearing above such that, take

$$\Gamma^1_{BIV-FS}\cup\Gamma^2_{BIV-FS}$$
$$=\left\{\begin{pmatrix}\dot{\tau},\max[\mathfrak{B}^+_1(\dot{\tau})(\sigma),\mathfrak{B}_2{}^+(\dot{\tau})(\sigma)],\\ min[\mathfrak{B}_1{}^-(\dot{\tau})(\sigma),\mathfrak{B}_2{}^-(\dot{\tau})(\sigma)]\end{pmatrix},\forall\dot{\tau}\in X\right\}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max[\mathfrak{B}^{+,L}_1(\dot{\tau})(\sigma),\mathfrak{B}_2{}^{+,L}(\dot{\tau})(\sigma)],\\ max[\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma),\mathfrak{B}^{+,U}_2(\dot{\tau})(\sigma)]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_2{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_2{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(9)}$$
$$=\left\{\left(\dot{\tau},\begin{Bmatrix}max[\mathfrak{B}^{+,L}_2(\dot{\tau})(\sigma),\mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma)],\\ max[\mathfrak{B}^{+,U}_2(\dot{\tau})(\sigma),\mathfrak{B}^{+,U}_1(\dot{\tau})(\sigma)]\end{Bmatrix},\begin{Bmatrix}min\left[\mathfrak{B}^{2^{-,L}}(\dot{\tau})(\sigma),\mathfrak{B}_1{}^{-,L}(\dot{\tau})(\sigma)\right],\\ min\left[\mathfrak{B}^{2^{-,U}}(\dot{\tau})(\sigma),\mathfrak{B}_1{}^{-,U}(\dot{\tau})(\sigma)\right]\end{Bmatrix}\right)\right\}\quad\textbf{(10)}$$

From **(9) and (10)** and through the previous definitions, we guarantee that the result will be

$$= \left\{ \begin{pmatrix} \dot{\tau}, \max[\mathfrak{B}_2^{+}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{+}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_2{}^{-}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{-}(\dot{\tau})(\sigma)] \end{pmatrix}, \forall \dot{\tau} \in X \right\}$$
$$= \Gamma^2_{BIV-FS} \cup \Gamma^1_{BIV-FS}.$$

Proof (ii): This proof depends on the definition of union who clearing above such that, take $\Gamma^1_{BIV-FS} \cap \Gamma^2_{BIV-FS} =$

$$= \left\{ \begin{pmatrix} \dot{\tau}, \min[\mathfrak{B}_1^{+}(\dot{\tau})(\sigma), \mathfrak{B}_\Phi{}^{+}(\dot{\tau})(\sigma)], \\ max[\mathfrak{B}_1{}^{-}(\dot{\tau})(\sigma), \mathfrak{B}_\Phi{}^{-}(\dot{\tau})(\sigma)] \end{pmatrix}, \forall \dot{\tau} \in X \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} min[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{+,L}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma)] \end{Bmatrix}, \begin{Bmatrix} max\left[\mathfrak{B}^{1^{-,L}}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,L}(\dot{\tau})(\sigma)\right], \\ max\left[\mathfrak{B}^{1^{-,U}}(\dot{\tau})(\sigma), \mathfrak{B}_2{}^{-,U}(\dot{\tau})(\sigma)\right] \end{Bmatrix} \right) \right\} \quad \textbf{(11)}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} min[\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma)] \end{Bmatrix}, \begin{Bmatrix} max\left[\mathfrak{B}^{2^{-,L}}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{-,L}(\dot{\tau})(\sigma)\right], \\ max\left[\mathfrak{B}^{2^{-,U}}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{-,U}(\dot{\tau})(\sigma)\right] \end{Bmatrix} \right) \right\} \quad \textbf{(12)}$$

From (**11) and (12)** and through the previous definitions, we guarantee that the result will be

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} min[\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1{}^{+,L}(\dot{\tau})(\sigma)], \\ min[\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma)] \end{Bmatrix}, \right.$$
$$= \Gamma^2_{BIV-FS} \cap \Gamma^1_{BIV-FS}.$$

Proposition 3.4

Let $\Gamma^1_{BIV-FS} = \{(\dot{\tau}, \mathfrak{B}_1^{+}(\dot{\tau})(\sigma), \mathfrak{B}_1^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$,$\Gamma^2_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}_2^{+}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ and $\Gamma^3_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}_3^{+}(\dot{\tau})(\sigma), \mathfrak{B}_3^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ be three BIV-FSSs on nonempty soft universal (X, Y). Then the following tip is considered convincing:

i. $\Gamma^1_{BIV-FS} \cup (\Gamma^2_{BIV-FS} \cup \Gamma^3_{BIV-FS}) = (\Gamma^1_{BIV-FS} \cup \Gamma^2_{BIV-FS}) \cup \Gamma^3_{BIV-FS}$.

ii. $\Gamma^1_{BIV-FS} \cap (\Gamma^2_{BIV-FS} \cap \Gamma^3_{BIV-FS}) = (\Gamma^1_{BIV-FS} \cap \Gamma^2_{BIV-FS}) \cap \Gamma^3_{BIV-FS}$.

Note: These points satisfy the distribution law.

Proof (i). This proof depends on the definition of union **Definition 3.3**.
who clearing above such that, take $\Gamma^1_{BIV-FS} \cup (\Gamma^2_{BIV-FS} \cup \Gamma^3_{BIV-FS})$

$$= \left\{ \begin{pmatrix} \dot{\tau}, \mathfrak{B}_1^{+}(\dot{\tau})(\sigma), max\{\mathfrak{B}_2^{+}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+}(\dot{\tau})(\sigma)\}, \\ \mathfrak{B}_1^{-}(\dot{\tau})(\sigma), min\{\mathfrak{B}_2^{-}(\dot{\tau})(\sigma), \mathfrak{B}_3^{-}(\dot{\tau})(\sigma)\} \end{pmatrix} \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} [\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+U}(\dot{\tau})(\sigma)], \\ max\begin{bmatrix} max\{\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)\}, \\ max\{\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)\} \end{bmatrix} \end{Bmatrix}, \begin{Bmatrix} [\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+U}(\dot{\tau})(\sigma)], \\ min\begin{bmatrix} min\{\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)\}, \\ min\{\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)\} \end{bmatrix} \end{Bmatrix} \right) \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} \max[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)] \\ max[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)] \end{Bmatrix}, \begin{Bmatrix} \min[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)] \\ min[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)] \end{Bmatrix} \right) \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} \max[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma), \\ \max[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma), \end{Bmatrix}, \begin{Bmatrix} \min[\mathfrak{B}_1^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-,L}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{-,L}(\dot{\tau})(\sigma), \\ \min[\mathfrak{B}_1^{-,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-,U}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{-,U}(\dot{\tau})(\sigma) \end{Bmatrix} \right) \right\}$$

$$= \{(\dot{\tau}, max\{\mathfrak{B}_1^{+}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+}(\dot{\tau})(\sigma)\}, \mathfrak{B}_3^{+}(\dot{\tau})(\sigma), min\{\mathfrak{B}_1^{-}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-}(\dot{\tau})(\sigma)\}, \mathfrak{B}_3^{-}(\dot{\tau})(\sigma))\}$$
$$= (\Gamma^1_{BIV-FS} \cup \Gamma^2_{BIV-FS}) \cup \Gamma^3_{BIV-FS}.$$

Proof (ii). This proof depends on the definition of intersection **Definition 3.4**.
who clearing above such that, take $\Gamma^1_{BIV-FS} \cap (\Gamma^2_{BIV-FS} \cap \Gamma^3_{BIV-FS})$

$$= \left\{ \begin{pmatrix} \dot{\tau}, \mathfrak{B}_1^{+}(\dot{\tau})(\sigma), min\{\mathfrak{B}_2^{+}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+}(\dot{\tau})(\sigma)\}, \\ \mathfrak{B}_1^{-}(\dot{\tau})(\sigma), max\{\mathfrak{B}_2^{-}(\dot{\tau})(\sigma), \mathfrak{B}_3^{-}(\dot{\tau})(\sigma)\} \end{pmatrix} \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} [\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+U}(\dot{\tau})(\sigma)], \\ min\begin{bmatrix} min\{\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)\}, \\ min\{\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)\} \end{bmatrix} \end{Bmatrix}, \begin{Bmatrix} [\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_1^{+U}(\dot{\tau})(\sigma)], \\ max\begin{bmatrix} max\{\mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)\}, \\ max\{\mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)\} \end{bmatrix} \end{Bmatrix} \right) \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} \min[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)] \\ min[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma)] \end{Bmatrix}, \begin{Bmatrix} \max[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)] \\ max[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma)] \end{Bmatrix} \right) \right\}$$

$$= \left\{ \left(\dot{\tau}, \begin{Bmatrix} \min[\mathfrak{B}_1^{+,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,L}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{+,L}(\dot{\tau})(\sigma), \\ \min[\mathfrak{B}_1^{+,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{+,U}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{+,U}(\dot{\tau})(\sigma), \end{Bmatrix}, \begin{Bmatrix} \max[\mathfrak{B}_1^{-,L}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-,L}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{-,L}(\dot{\tau})(\sigma), \\ \max[\mathfrak{B}_1^{-,U}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-,U}(\dot{\tau})(\sigma)], \mathfrak{B}_3^{-,U}(\dot{\tau})(\sigma) \end{Bmatrix} \right) \right\}$$
$$= (\Gamma^1_{BIV-FS} \cap \Gamma^2_{BIV-FS}) \cap \Gamma^3_{BIV-FS}.$$

Proposition 3.5

Let $\Gamma^1_{BIV-FS} = \{(\dot{\tau}, \mathfrak{B}_1^{+}(\dot{\tau})(\sigma), \mathfrak{B}_1^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ and $\Gamma^2_{BIV-FSS} = \{(\dot{\tau}, \mathfrak{B}_2^{+}(\dot{\tau})(\sigma), \mathfrak{B}_2^{-}(\dot{\tau})(\sigma)), \forall \dot{\tau} \in X\}$ be two BIV-FSSs on nonempty soft universal (X, Y). Then the following tip is considered as **DE Morgan laws:**

i. $(\Gamma^1_{BIV-FS} \cup \Gamma^2_{BIV-FSS})^C = (\Gamma^1_{BIV-FS})^C \cap (\Gamma^2_{BIV-FS})^C$

ii. $(\Gamma^1_{BIV-FS} \cap \Gamma^2_{BIV-FSS})^C = (\Gamma^1_{BIV-FS})^C \cup (\Gamma^2_{BIV-FS})^C$.

Proof. The proof of this **Propositions** is clear and in the same order as the proof for Propositions **3.2, 3.3,** and **3.4**, as it depends on.

4 Application of BIV-FSSs in Decision-Making

The human resources department (HRD) plays an important role in government and private sector companies, specifically in the mechanism of selecting competent employees according to strict standards imposed by the company. This department contains experts who have full knowledge of discovering and evaluating qualifications. In this context, all of these experts have positive and negative characteristics when interviewing each applicant for these jobs. In this section, and based on the above, we will organize a hypothetical problem based on hypothetical data that can be extracted from the opinions of experts in a private sector company. One company announced the availability of four distinct jobs, prompting numerous individuals to apply for them. Here, the company decided to hire two experts from the human resources department, each of whom interviewed a sample of applicants to select the best two candidates for the announced jobs. Now, let $X = \{\dot{\tau}_1, \dot{\tau}_2, \dot{\tau}_3, \dot{\tau}_4, \dot{\tau}_5, \dot{\tau}_6, \dot{\tau}_7, \dot{\tau}_8, \dot{\tau}_9, \dot{\tau}_{10}\}$ consider a non-empty universe set, which represents ten individuals who are interested in applying for these jobs and $A \subseteq Y = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ consider a non-empty parametrized set contains all parametrized that represent X element's where $\sigma_1 = Educational\ Qualifications$, $\sigma_2 = work\ experience\ years$, $\sigma_3 = Soft\ Skills$ and $\sigma_4 = Cultural\ Fit$. To streamline the interview process of applicants by experts. The experts decided that each expert would interview five applicants and choose one. Accordingly, each expert interviewed five applicants, and each expert had a positive and negative impression of each applicant according to the standards approved by the company. This impression can be organised and analysed using our proposed concept as follows.

$$(\Gamma, A_1)_1 = \left\{ \begin{array}{l}
\Gamma(\sigma_1) = \left\{ \begin{array}{l}
\dot{\tau}_1, \langle [0.1,0.4], [-0.5,-0.3] \rangle \\
\dot{\tau}_2, \langle [0.6,0.8], [-0.4,-0.2] \rangle \\
\dot{\tau}_3, \langle [0.4,0.7], [-0.2,-0.9] \rangle \\
\dot{\tau}_4, \langle [0.5,0.5], [-0.1,-0.1] \rangle \\
\dot{\tau}_5, \langle [0.5,0.5], [-0.1,-0.1] \rangle \\
\dot{\tau}_6, \langle [0.1,0.4], [-0.9,-0.3] \rangle \\
\dot{\tau}_7, \langle [0.6,0.8], [-0.6,-0.5] \rangle \\
\dot{\tau}_8, \langle [0.4,0.7], [-0.7,-0.8] \rangle \\
\dot{\tau}_9, \langle [0.5,0.5], [-0.3,-0.1] \rangle \\
\dot{\tau}_{10}, \langle [0.5,0.5], [-0.3,-0.2] \rangle
\end{array} \right\} \\
\Gamma(\sigma_2) = \left\{ \begin{array}{l}
\dot{\tau}_1, \langle [0.3,0.7], [-0.3,-0.3] \rangle \\
\dot{\tau}_2, \langle [0.8,0.9], [-0.7,-0.1] \rangle \\
\dot{\tau}_3, \langle [0.4,0.7], [-0.9,-0.9] \rangle \\
\dot{\tau}_4, \langle [0.2,0.5], [-0.6,-0.5] \rangle \\
\dot{\tau}_5, \langle [0.2,0.5], [-0.6,-0.5] \rangle \\
\dot{\tau}_6, \langle [0.1,0.4], [-0.9,-0.3] \rangle \\
\dot{\tau}_7, \langle [0.6,0.8], [-0.6,-0.5] \rangle \\
\dot{\tau}_8, \langle [0.4,0.7], [-0.7,-0.8] \rangle \\
\dot{\tau}_9, \langle [0.5,0.5], [-0.3,-0.1] \rangle \\
\dot{\tau}_{10}, \langle [0.5,0.5], [-0.3,-0.2] \rangle
\end{array} \right\} \\
\Gamma(\sigma_3) = \left\{ \begin{array}{l}
\dot{\tau}_1, \langle [0.1,0.4], [-0.5,-0.3] \rangle \\
\dot{\tau}_2, \langle [0.6,0.8], [-0.4,-0.2] \rangle \\
\dot{\tau}_3, \langle [0.4,0.7], [-0.9,-0.6] \rangle \\
\dot{\tau}_4, \langle [0.5,0.5], [-0.1,-0.1] \rangle \\
\dot{\tau}_5, \langle [0.7,0.42], [-0.7,-0.4] \rangle \\
\dot{\tau}_6, \langle [0.1,0.4], [-0.9,-0.3] \rangle \\
\dot{\tau}_7, \langle [0.6,0.8], [-0.6,-0.5] \rangle \\
\dot{\tau}_8, \langle [0.4,0.7], [-0.7,-0.8] \rangle \\
\dot{\tau}_9, \langle [0.5,0.5], [-0.3,-0.1] \rangle \\
\dot{\tau}_{10}, \langle [0.5,0.5], [-0.3,-0.2] \rangle
\end{array} \right\} \\
\Gamma(\sigma_4) = \left\{ \begin{array}{l}
\dot{\tau}_1, \langle [0.3,0.4], [-0.5,-0.3] \rangle \\
\dot{\tau}_2, \langle [0.5,0.7], [-0.7,-0.5] \rangle \\
\dot{\tau}_3, \langle [0.2,0.6], [-0.2,-0.9] \rangle \\
\dot{\tau}_4, \langle [0.4,0.9], [-0.5,-0.1] \rangle \\
\dot{\tau}_5, \langle [0.4,0.9], [-0.5,-0.1] \rangle \\
\dot{\tau}_6, \langle [0.1,0.4], [-0.9,-0.3] \rangle \\
\dot{\tau}_7, \langle [0.6,0.8], [-0.6,-0.5] \rangle \\
\dot{\tau}_8, \langle [0.4,0.7], [-0.7,-0.8] \rangle \\
\dot{\tau}_9, \langle [0.5,0.5], [-0.3,-0.1] \rangle \\
\dot{\tau}_{10}, \langle [0.5,0.5], [-0.3,-0.2] \rangle
\end{array} \right\}
\end{array} \right\}$$

Thus the two BIV-FSSs $(\Gamma, A_1)_1$ and $(\Gamma, A_1)_2$ refer to expert opinions regarding the extent of experts' satisfaction with the applicant's performance during the interview. For example the term $(\Gamma(\sigma_1)(\dot{\tau}_1), \langle [0.1,0.4], [-0.5,-0.3] \rangle)$ refer to that the first expert's positive perception of the applicant $(\dot{\tau}_1)$ ranges between $\langle [0.1,0.4] \rangle$ and the first expert's negative perception of the applicant

($\dot{\tau}_1$) ranges between $\langle[-0.5, -0.3]\rangle$. Now we organize a multi-step algorithm 1 that analyzes positive and negative impressions and generates a score that experts can rely on to make the right decision.

Algorithm 1

Step 1. Fetch information on BIV-FSS evaluations. The information calm from the one expert's discussions on evaluation is given in$(\Gamma, A_1)_1$.

Step 2. Convert BIV-FSS to into single value using the following formula:

$$\boldsymbol{M_i^+} = \frac{\frac{\sum_{i=1}^{k} \mathfrak{B}_i^{+,L}(\dot{\tau})(\sigma)}{k} + \frac{\sum_{i=1}^{k} \mathfrak{B}_i^{+,U}(\dot{\tau})(\sigma)}{k}}{\mathbf{2}}$$

$$\boldsymbol{M_i^-} = \frac{\frac{\sum_{i=1}^{k} \mathfrak{B}_i^{-,L}(\dot{\tau})(\sigma)}{k} + \frac{\sum_{i=1}^{k} \mathfrak{B}_i^{-,U}(\dot{\tau})(\sigma)}{k}}{\mathbf{2}}$$

Step 3. Obtaining the degree of the positive function and the negative function, both individually.

Step 4. Obtaining a comparison table for a positive and negative function.

Step 5. Calculate the final score by subtracting the negative score function from the positive score function.

Now we start implementing the algorithm given above in the form of sequential tables as shown below:

Step 1 is shown above in the object $(\Gamma, A_1)_1$ as in Table 1 and Table 2 in Appendix.

Accordingly, based on the results obtained in Table 2, it can be said that the committee formed in the Human Resources Department has recommended that those qualified to fill the positions are: $\dot{\tau}_2, \dot{\tau}_3, \dot{\tau}_8$, and $\dot{\tau}_{10}$.

In addition to Table 1 and Table 2 (Appendix), we also present Figure 1 (Appendix) that shows the variance between the results obtained from Algorithm 1.

5 Conclusion

This work aims to introduce a new algebraic mathematical concept called BIV-FSS, which deals with daily life situations that have both positive and negative characteristics. That is, the mechanism of this concept is to follow the logic of human thinking, which works to analyze any daily life situation into a positive and a negative characteristic. This concept has the ability to provide an accurate description of any daily life situation by judging it from both positive and negative aspects. Accordingly, we presented this thesis in the form of three main chapters, where the first chapter provides a comprehensive summary of the definitions and basic properties that we relied on to build this concept. As for the second chapter, we presented in detail the basic definition of our proposed concept. After that, we presented the basic properties of this concept, explaining each property with a numerical example to clarify the mechanism of work. This concept is derived from set theory, and accordingly, we defined the basic operations on this concept, which are as follows: union, intersection, and complement, in addition to other operations. We supported all definitions of these operations with illustrative numerical examples. As future studies of these tools, we recommend their application to showed the mechanism for employing our proposed concept in solving multi-criteria decision-making problems by using fixed-step algorithms. Also, as future studies, these tools can be applied with a number of current research works such as [25], [26], [27], [28], [29], [30], [31], [32].

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Contribution of Individual Authors to the Creation of a Scientific Article (Ghostwriting Policy)

- ALI JARADAT, FAISAL AL-SHARQI, and ABDULLRAHMAN A. AL-MAQBALI prepared, created and presented this work, specifically writing the initial draft.
- ABDULLAH ALSOBOH and FAISAL AL-SHARQI: Supervision and leadership responsibility for the implementing research activities.
- YOUSEF AL-QUDAH is responsible for project management and coordination for planning and implementing the research activity.

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APPENDIX

Table 1. Values obtained from M_i^+ and M_i^-

Candidates	M_i^+	M_i^-	(M_i^+, M_i^-)
$\dot{\tau}_1$	$0,338$	-0.375	$(0,338,-0,375)$
$\dot{\tau}_2$	$0,321$	$-0,234$	$(0,321,-0,234)$
$\dot{\tau}_3$	$0,453$	$-0,264$	$(0,453,-0,264)$
$\dot{\tau}_4$	$0,224$	$-0,354$	$(0,224,-0,354)$
$\dot{\tau}_5$	$0,361$	$-0,563$	$(0,361,-0,563)$
$\dot{\tau}_6$	$0,215$	$-0,276$	$(0,215,-0,276)$
$\dot{\tau}_7$	$0,324$	$-0,432$	$(0,324,-0,432)$
$\dot{\tau}_8$	$0,446$	$-0,306$	$(0,446,-0,306)$
$\dot{\tau}_9$	$0,317$	$-0,405$	$(0,317,-0,405)$
$\dot{\tau}_{10}$	$0,602$	$-0,274$	$(0,602,-0,274)$

Source: created by the authors

Table 2. Comparison table for a positive and negative function

Candidates	(M_i^+, M_i^-)	Comparison Value	Result
$\dot{\tau}_1$	$(0,338,-0,375)$	$(5,3)=4$	NO
$\dot{\tau}_2$	$(0,321,-0,234)$	$(3,9)=6$	YES
$\dot{\tau}_3$	$(0,453,-0,264)$	$(8,8)=8$	YES
$\dot{\tau}_4$	$(0,224,-0,354)$	$(1,4)=2,5$	NO
$\dot{\tau}_5$	$(0,361,-0,563)$	$(6,0)=3$	NO
$\dot{\tau}_6$	$(0,215,-0,276)$	$(0,6)=3$	NO
$\dot{\tau}_7$	$(0,324,-0,432)$	$(4,1)=2,5$	NO
$\dot{\tau}_8$	$(0,446,-0,306)$	$(7,5)=6$	YES
$\dot{\tau}_9$	$(0,317,-0,405)$	$(2,2)=2$	NO
$\dot{\tau}_{10}$	$(0,602,-0,274)$	$(9,7)=8$	YES

Source: created by the authors

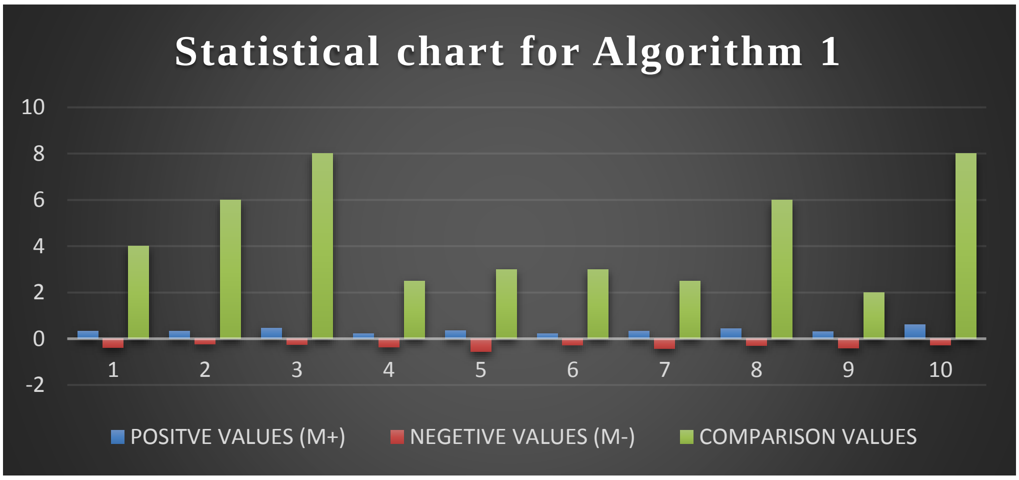

Fig. 1: Shows the variance between the results obtained from Algorithm 1
Source: created by the authors