A Probabilistic Theoric Framework for Uncertainty Cost Functions in Scheduling Optimization in Hybrid Renewable Microgrids

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Abstract: In this paper is developed a mathematical framework for uncertainty quantification in energy systems through Uncertainty Cost Functions (UCFs), in order to schedule the operation. We establish probabilistic models for solar photovoltaic generation, wind energy generation, and plug-in electric vehicles, deriving exact expressions for expected penalty costs using measure-theoretic probability. The main results include existence theorems for UCFs, closed-form solutions under specific distributional assumptions, and optimal scheduling policy.

Key-Words: Battery Energy Systems, Stochastic optimization, Renewable Energy Integration, Probabilistic Penalty Costs, Measure-theoretic Probability, Uncertainty Quantification, Operation Scheduling of Power Systems

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1 Introduction: Foundations of Uncertainty Cost Functions

The operation scheduling of power systems with high penetration of renewable energies (solar and wind) and electric vehicles must use uncertainty cost functions, [1], [2], [3], [4], [5]. They are useful to handle the intermittency of primary energy, [6], [7]. Here, we propose those functions.

Definition 1 (Uncertainty Cost Function). *Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For any scheduled power $W_s \in L^1(\Omega, \mathcal{F}, \mathbb{P})$ and available power $W_{av} \in L^1(\Omega, \mathcal{F}, \mathbb{P})$, the Uncertainty Cost Function is defined as equation 1:*

$$UCF(W_s) := \mathbb{E}[c_u(W_{av} - W_s)^+ + c_o(W_s - W_{av})^+] \tag{1}$$

where $(x)^+ := \max(x, 0)$, and c_u, c_o are underestimate/overestimate cost coefficients respectively.

Theorem 2 (Existence of UCF Solutions). *For any $\mathcal{F}$-measurable random variable W_{av} with $\mathbb{E}[|W_{av}|] < \infty$, and positive constants c_u, c_o, there exists a unique minimizer $W_s^* \in \mathbb{R}$ for the UCF, [8], [9], [10], [11].*

Proof. The UCF is convex as the sum of convex functions. By dominated convergence theorem, continuous differentiability follows from equation 2:

$$\frac{d}{dW_s}\mathbb{E}[UCF(W_s)] = c_u\mathbb{P}(W_{av} > W_s) - c_o\mathbb{P}(W_{av} < W_s) \tag{2}$$

Existence follows from coercivity: $\lim_{|W_s|\to\infty} UCF(W_s) = \infty$. □

1.1 Uncertainty Cost Framework

Definition 1 (Uncertainty Cost Framework). *Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space equipped with:*

- *Power injection process $W_{av} \in L^1(\Omega, \mathcal{F}, \mathbb{P})$*
- *Scheduled power parameter $W_s \in \mathbb{R}^+$*
- *Penalty operators $C_u(W_s, W_{av}) = c_u(W_{av} - W_s)^+$, $C_o(W_s, W_{av}) = c_o(W_s - W_{av})^+$*

The Uncertainty Cost Functional is the Radon-Nikodým derivative, as equation 3:

$$UCF(W_s) = \mathbb{E}^{\mathbb{P}}[C_u + C_o] = \int_\Omega [c_u(W_{av} - W_s)^+ + c_o(W_s - W_{av})^+]d\mathbb{P} \tag{3}$$

Theorem 3 (Existence and Uniqueness of Optimal Schedule). *For $\mathbb{P}$-integrable W_{av} with $\mathbb{E}[|W_{av}|] < \infty$ and $c_u, c_o > 0$, there exists a unique minimizer $W_s^* \in \mathbb{R}^+$ equation 4, [8],[11]:*

$$W_s^* = \arg\min_{W_s \geq 0} UCF(W_s) \tag{4}$$

Proof. Step 1: Convexity
The penalty operators C_u, C_o are convex in W_s as positive part functions. The expectation preserves convexity by linearity of integration.

Step 2: Coercivity (equations 5 and 6)
For $W_s \to \infty$:

$$UCF(W_s) \geq c_o\mathbb{E}[W_s - W_{av}] \geq c_o(W_s - \mathbb{E}[W_{av}]) \to \infty \tag{5}$$

For $W_s \to 0$:

$$UCF(W_s) \geq c_u\mathbb{E}[W_{av} - W_s] \geq c_u(\mathbb{E}[W_{av}] - W_s) \to c_u\mathbb{E}[W_{av}] \tag{6}$$

Step 3: Strict Convexity
The derivative using Dominated Convergence (equation 7):

$$\frac{d}{dW_s}UCF(W_s) = c_u\mathbb{P}(W_{av} > W_s) - c_o\mathbb{P}(W_{av} < W_s) \tag{7}$$

is strictly increasing as $\mathbb{P}(W_{av} > W_s)$ decreases monotonically. By Intermediate Value Theorem, unique W_s^* exists where derivative vanishes. □

2 Probabilistic Foundations for Scheduling in Microgrids

Lemma 1 (Change of Measure)**.** *Let $W = g(X)$ where X has PDF f_X. Then for any measurable h, [11], [12], [13] (equation 8):*

$$\mathbb{E}[h(W)] = \int h(g(x))f_X(x)dx = \int h(w)f_W(w)dw \tag{8}$$

where $f_W(w) = f_X(g^{-1}(w))|\frac{d}{dw}g^{-1}(w)|$ when g is monotonic.

2.1 Probabilistic Transformations

Lemma 2 (Measure-Theoretic Power Transformation)**.** *Given random variable $X \sim \mathbb{P}_X$ and Borel-measurable function $g : \mathbb{R} \to \mathbb{R}$, the transformed measure $\mathbb{P}_W = \mathbb{P}_X \circ g^{-1}$ satisfies equation 9,([11], [13]):*

$$\mathbb{E}^{\mathbb{P}_W}[h(W)] = \mathbb{E}^{\mathbb{P}_X}[h(g(X))] = \int_{\mathbb{R}} h(g(x))f_X(x)dx \tag{9}$$

For diffeomorphic g with g^{-1} differentiable (equation 10):

$$f_W(w) = f_X(g^{-1}(w))\left|\frac{d}{dw}g^{-1}(w)\right| \tag{10}$$

Corollary 4 (Log-Normal Irradiance Transformation)**.** *For solar irradiance $G \sim LogN(\lambda, \beta^2)$ and power conversion $W_{PV} = \phi(G)$, the photovoltaic power measure satisfies equation 11:*

$$f_{W_{PV}}(w) = \frac{1}{w\beta\sqrt{2\pi}}\exp\left(-\frac{(\ln w - \lambda)^2}{2\beta^2}\right)\mathbf{1}_{\{\phi^{-1}(w)\}} \tag{11}$$

where:

- *$f_{W_{PV}}(w)$: Probability density function (PDF) of the photovoltaic power W_{PV} evaluated at a specific power output w.*
- *w: A specific value of photovoltaic power (in Watts or per unit).*
- *λ: The mean of the natural logarithm of the solar irradiance G. It is one of the shape parameters of the Log-Normal distribution for G.*
- *β: The standard deviation of the natural logarithm of the solar irradiance G. Its square (β^2) is the variance of the natural logarithm of G, and it is the other shape parameter of the Log-Normal distribution.*
- *$\phi(G)$: The power conversion function that translates solar irradiance G into photovoltaic power W_{PV}. This function is piecewise defined, accounting for different irradiance regimes (e.g., non-linear behavior, cut-in and cut-out points).*
- *$\phi^{-1}(w)$: The inverse of the power conversion function, which takes a photovoltaic power output w and returns the corresponding solar irradiance G that would produce that power.*
- *$\mathbf{1}_{\{\phi^{-1}(w)\}}$: An indicator function that ensures the probability density is non-zero only for physically achievable power outputs. It evaluates to 1 if there exists a valid solar irradiance G such that $\phi(G) = w$, and 0 otherwise. This effectively defines the support of the PDF for W_{PV}.*

Corollary 5 (Rayleigh-Weibull Wind Power Transformation)**.** *For wind speed $v \sim Rayleigh(\sigma)$ with power conversion $W_w = \psi(v)$, the wind power measure satisfies equation 12:*

$$f_{W_w}(w) = \left(\frac{w-\kappa}{\rho^2\sigma^2}\right)\exp\left(-\frac{(w-\kappa)^2}{2\rho^2\sigma^2}\right) \mathbf{1}_{[v_i,v_r)}(\psi^{-1}(w)) + p_r\delta(w - W_r) \tag{12}$$

where:

- $\psi(v) = \rho v + \kappa$ *with* $\rho = \frac{W_r}{v_r - v_i}$, $\kappa = -\frac{W_r v_i}{v_r - v_i}$
- $p_r = \exp\left(-\frac{v_r^2}{2\sigma^2}\right) - \exp\left(-\frac{v_o^2}{2\sigma^2}\right)$
- $\mathbf{1}_{[v_i, v_r)}$ *indicates support in the linear conversion regime*
- $\delta(w - W_r)$ *represents Dirac measure at rated power*

2.2 Proof Methodology

Remark 6 (Change of Measure)**.** Step 1: *Apply Radon-Nikodym theorem to probability measure* $\mathbb{P}_W = \mathbb{P}_X \circ g^{-1}$ Step 2: *For monotonic* g*, use inverse function theorem to get the equation 13:*

$$dx = \left| \frac{d}{dw} g^{-1}(w) \right| dw \tag{13}$$

Step 3: *Substitute in expectation integral, equation 14:*

$$\mathbb{E}[h(W)] = \int h(g(x)) f_X(x) dx = \int h(w) f_X(g^{-1}(w)) \left| \frac{d}{dw} g^{-1}(w) \right| dw \tag{14}$$

Remark 7 (Measure-Theoretic Transformation)**.** Step 1: *Define pushforward measure* $\mathbb{P}_W(A) = \mathbb{P}_X(g^{-1}(A))$ Step 2: *For Borel* h*, apply change-of-variables formula, equation 15:*

$$\mathbb{E}^{\mathbb{P}_W}[h] = \int_\Omega h \circ g \, d\mathbb{P}_X = \int_{\mathbb{R}} h(g(x)) f_X(x) dx \tag{15}$$

Step 3: *For diffeomorphism* g*, use Jacobian transformation, equation 16:*

$$f_W(w) = f_X(g^{-1}(w)) \left| \det J_{g^{-1}}(w) \right| \tag{16}$$

Remark 8 (Solar Transformation)**.** Step 1: *Decompose* $\phi(G)$ *into piecewise regimes* Step 2: *For each regime* R_i*, compute equation 17:*

$$f_W^{R_i}(w) = f_G(\phi|_{R_i}^{-1}(w)) \left| \frac{d}{dw} \phi|_{R_i}^{-1}(w) \right| \tag{17}$$

Step 3: *Combine using indicator functions, equation 18:*

$$f_{W_{PV}}(w) = \sum_i f_W^{R_i}(w) \mathbf{1}_{\phi(R_i)}(w) \tag{18}$$

Remark 9 (Wind Transformation)**.** Step 1: *Handle discrete components first, equation 19:*

$$p_r = \mathbb{P}(v \in [v_r, v_o)) = \int_{v_r}^{v_o} \frac{v}{\sigma^2} e^{-v^2/2\sigma^2} dv \tag{19}$$

Step 2: *For linear regime* $v \in [v_i, v_r)$*, equation 20:*

$$f_W^{LR}(w) = \frac{d}{dw} F_v\left(\frac{w - \kappa}{\rho}\right) = \frac{v}{\sigma^2} \frac{dv}{dw}\Big|_{v = \frac{w-\kappa}{\rho}} \tag{20}$$

Step 3: *Combine measures using Dirac delta, equation 21:*

$$f_{W_w}(w) = f_W^{LR}(w) \mathbf{1}_{[0, W_r)}(w) + p_r \delta(w - W_r) \tag{21}$$

Remark 10. *The Dirac delta term* $\delta(w - W_r)$ *formally requires distributional analysis, but can be rigorously treated as equation 22:*

$$\int h(w) \delta(w - W_r) dw = h(W_r) \tag{22}$$

for test functions $h \in C_c^\infty(\mathbb{R})$*.*

3 Uncertainty Quantification in Hybrid Microgrids

3.1 Photovoltaic Uncertainty Quantification

Remark 11 (Solar Regime Classification)**.** *Define irradiance regimes:*

- *Sublinear Regime (SR):* $G \in (0, R_c]$ *with* $W_{PV}(G) = \frac{W_{PVr}}{G_r R_c} G^2$
- *Linear Regime (LR):* $G \in (R_c, \infty)$ *with* $W_{PV}(G) = \frac{W_{PVr}}{G_r} G$

Proposition 1 (Sublinear Regime Density)**.** *For* $W \in (0, W_{Rc})$*, the PV power density under SR is (equation 23):*

$$f_W^{SR}(w) = \frac{1}{2w\beta\sqrt{\pi}} \exp\left(-\frac{(\frac{1}{2}\ln\frac{wG_rR_c}{W_{PVr}} - \lambda)^2}{\beta^2}\right) \tag{23}$$

Proof. Apply Lemma 2.1 with $g(G) = \frac{W_{PVr}}{G_r R_c} G^2$, equation 24:

$$g^{-1}(w) = \sqrt{\frac{wG_rR_c}{W_{PVr}}}, \quad \frac{d}{dw} g^{-1}(w) = \frac{1}{2}\sqrt{\frac{G_rR_c}{W_{PVr}w}} \tag{24}$$

Substitute into Radon-Nikodým derivative and simplify. □

Proposition 2 (Linear Regime Density)**.** *For* $W \geq W_{Rc}$*, the PV power density under LR is (equation 25):*

$$f_W^{LR}(w) = \frac{1}{w\beta\sqrt{2\pi}} \exp\left(-\frac{(\ln\frac{wG_r}{W_{PVr}} - \lambda)^2}{2\beta^2}\right) \tag{25}$$

Theorem 12 (Photovoltaic UCF Decomposition). *The total uncertainty cost decomposes into regime-specific components, as shown in Equation* (26)*:*

$$UCF_{PV}(W_s) = UCF^{SR}(W_s) + UCF^{LR}(W_s) \tag{26}$$

where each component satisfies:

$$UCF^{SR}(W_s) = c_u \int_{W_s}^{W_{Rc}} (w - W_s) f_W^{SR}(w) dw \tag{27}$$

$$UCF^{LR}(W_s) = c_u \int_{W_{Rc}}^{\infty} (w - W_s) f_W^{LR}(w) dw + c_o \int_0^{W_s} (W_s - w) f_W^{SR\cup LR}(w) dw \tag{28}$$

The short-run component is defined in Equation (27)*, and the long-run component in Equation* (28)*.*

Theorem 13 (Closed-Form UCF Solution). *For $W_s \in (0, W_{Rc})$, the photovoltaic UCF admits a closed-form solution as shown in Equation* (29)*:*

$$\begin{aligned} UCF_{PV}(W_s) = \\ c_u W_{PVr} e^{2\lambda+2\beta^2} \Phi\left(\frac{\ln \frac{W_s G_r R_c}{W_{PVr}} - \lambda - 2\beta^2}{\beta}\right) \\ + c_o W_s \Phi\left(\frac{\ln \frac{W_s G_r}{W_{PVr}} - \lambda}{\beta}\right) \\ - c_o W_{PVr} e^{\lambda+\beta^2/2} \Phi\left(\frac{\ln \frac{W_s G_r}{W_{PVr}} - \lambda - \beta^2}{\beta}\right) \end{aligned} \tag{29}$$

3.2 Wind Energy Uncertainty Quantification

Remark 14 (Wind Regime Classification). *Define wind speed regimes for turbine operation:*

- *Dead Zone (DZ): $v \in [0, v_i) \cup [v_o, \infty)$ with $W_w(v) = 0$*
- *Linear Regime (LR): $v \in [v_i, v_r)$ with $W_w(v) = \rho v + \kappa$*
- *Saturation Regime (SR): $v \in [v_r, v_o)$ with $W_w(v) = W_r$*

where $\rho = \frac{W_r}{v_r - v_i}$ and $\kappa = -\frac{W_r v_i}{v_r - v_i}$.

Proposition 3 (Wind Power Measure Decomposition). *The wind power measure decomposes as presented in Equation* (30)*:*

$$\mathbb{P}_W = p_{DZ}\delta_0 + \mathbb{P}_{LR} + p_{SR}\delta_{W_r} \tag{30}$$

where the probabilities p_{DZ} and p_{SR} are defined by Equations (31) *and* (32) *respectively:*

$$p_{DZ} = 1 - e^{-v_i^2/2\sigma^2} + e^{-v_o^2/2\sigma^2} \tag{31}$$

$$p_{SR} = e^{-v_r^2/2\sigma^2} - e^{-v_o^2/2\sigma^2} \tag{32}$$

Theorem 15 (Linear Regime Density). *For $W \in (\kappa, W_r)$, the power density under the Linear Regime (LR) is given by Equation* (33)*:*

$$f_W^{LR}(w) = \frac{w - \kappa}{\rho^2 \sigma^2} \exp\left(-\frac{(w-\kappa)^2}{2\rho^2\sigma^2}\right) \tag{33}$$

Proof. Apply Lemma 2.1 with $g(v) = \rho v + \kappa$. The inverse function and its derivative are shown in Equation (34):

$$g^{-1}(w) = \frac{w - \kappa}{\rho}, \quad \frac{d}{dw} g^{-1}(w) = \frac{1}{\rho} \tag{34}$$

Substitute these into the Rayleigh density $f_v(v) = \frac{v}{\sigma^2} e^{-v^2/2\sigma^2}$. □

Theorem 16 (Wind UCF Decomposition). *The total wind uncertainty cost decomposes into components for the Linear Regime (LR), Saturation Regime (SR), and Dead Zone (DZ), as shown in Equation* (35)*:*

$$\begin{aligned} UCF_W(W_s) = UCF^{LR}(W_s) \\ + UCF^{SR}(W_s) \\ + UCF^{DZ}(W_s) \end{aligned} \tag{35}$$

with the individual components defined by Equations (36), (37), *and* (38) *respectively:*

$$UCF^{LR}(W_s) = c_u \int_{W_s}^{W_r} (w - W_s) f_W^{LR}(w) dw \tag{36}$$

$$UCF^{SR}(W_s) = c_u p_{SR} (W_r - W_s)^+ \tag{37}$$

$$UCF^{DZ}(W_s) = c_o \int_0^{W_s} (W_s - w) f_W^{DZ}(w) dw \tag{38}$$

Theorem 17 (Closed-Form Wind UCF). *The wind UCF admits the closed-form solution presented in Equation* (39)*:*

$$UCF_W(W_s) = c_u\rho\sigma\sqrt{\frac{\pi}{2}} \left[\Phi\left(\frac{W_r-\kappa}{\rho\sigma}\right) - \Phi\left(\frac{W_s-\kappa}{\rho\sigma}\right)\right] + c_u(W_r - W_s)p_{SR} + c_oW_sp_{DZ} - c_o\rho\sigma\sqrt{\frac{\pi}{2}}\Phi\left(\frac{W_s-\kappa}{\rho\sigma}\right) \tag{39}$$

where Φ is the standard normal CDF.

Proof. Decompose equation 40:

$$\int_{W_s}^{W_r}(w-W_s)f_W^{LR}(w)dw = \rho\sigma\sqrt{\frac{\pi}{2}}\left[\Phi\left(\frac{W_r-\kappa}{\rho\sigma}\right) - \Phi\left(\frac{W_s-\kappa}{\rho\sigma}\right)\right] - \sigma^2\left[e^{-(W_s-\kappa)^2/2\rho^2\sigma^2} - e^{-(W_r-\kappa)^2/2\rho^2\sigma^2}\right] \tag{40}$$

The Dirac measures contribute $p_{SR}(W_r - W_s)$ and $p_{DZ}W_s$. □

Corollary 18 (Optimal Wind Schedule)**.** *The optimal scheduled power W_s^* satisfies the condition given in Equation (41):*

$$c_u\left[1-\Phi\left(\frac{W_s^*-\kappa}{\rho\sigma}\right)\right] = c_o\left[\Phi\left(\frac{W_s^*-\kappa}{\rho\sigma}\right) + \frac{p_{DZ}}{f_W^{LR}(W_s^*)}\right] \tag{41}$$

Proof. Differentiating the UCF and setting it to zero yields Equation (42):

$$\frac{d}{dW_s}UCF_W(W_s) = c_u[1-F_W(W_s)] - c_oF_W(W_s) = 0 \tag{42}$$

where F_W is the cumulative distribution function. □

3.3 Electric Vehicle Uncertainty Quantification

Corollary 19 (Gaussian Battery Power Transformation)**.** *For PEV battery power $P_e \sim \mathcal{N}(\mu, \phi^2)$, the available power measure follows a Gaussian distribution as given by Equation (43):*

$$f_{P_e}(p) = \frac{1}{\phi\sqrt{2\pi}}\exp\left(-\frac{(p-\mu)^2}{2\phi^2}\right) \tag{43}$$

with the total uncertainty cost functional defined by Equation (44):

$$UCF_{PEV}(P_s) = c_u\mathbb{E}[(P_e - P_s)^+] + c_o\mathbb{E}[(P_s - P_e)^+] \tag{44}$$

Theorem 20 (PEV Underestimate Cost)**.** *The expected underestimate penalty cost, $\mathbb{E}[C_u(P_s)]$, can be expressed as shown in Equation (45):*

$$\mathbb{E}[C_u(P_s)] = c_u\left[\phi\varphi\left(\frac{P_s-\mu}{\phi}\right) + (\mu - P_s)\Phi\left(\frac{\mu-P_s}{\phi}\right)\right] \tag{45}$$

where φ is the standard normal PDF and Φ is its CDF.

Proof. Using the measure transformation lemma, the expectation integral can be evaluated as:

$$\mathbb{E}[(P_e - P_s)^+] = \int_{P_s}^{\infty}(p-P_s)\frac{1}{\phi\sqrt{2\pi}}e^{-(p-\mu)^2/2\phi^2}dp = \phi\varphi\left(\frac{P_s-\mu}{\phi}\right) + (\mu-P_s)\Phi\left(\frac{\mu-P_s}{\phi}\right)$$

This result is obtained via integration by parts and standard normal identities. □

Theorem 21 (PEV Overestimate Cost)**.** *The expected overestimate penalty cost, $\mathbb{E}[C_o(P_s)]$, satisfies Equation (46):*

$$\mathbb{E}[C_o(P_s)] = c_o\left[\phi\varphi\left(\frac{P_s-\mu}{\phi}\right) + (P_s-\mu)\Phi\left(\frac{P_s-\mu}{\phi}\right)\right] \tag{46}$$

Proof. Mirroring Theorem 5.1 (referring to the previous theorem, which was 5.1 in your original context) and utilizing the symmetry of the normal distribution, we consider the integral:

$$\mathbb{E}[(P_s - P_e)^+] = \int_{-\infty}^{P_s}(P_s - p)f_{P_e}(p)dp$$

This integral yields the dual expression through measure preservation. □

Theorem 22 (PEV UCF Closed-Form)**.** *The total uncertainty cost function for PEV, $UCF_{PEV}(P_s)$, combines as expressed in Equation (47):*

$$UCF_{PEV}(P_s) = (c_u + c_o)\phi\varphi\left(\frac{P_s-\mu}{\phi}\right) + c_u(\mu-P_s)\Phi\left(\frac{\mu-P_s}{\phi}\right) + c_o(P_s-\mu)\Phi\left(\frac{P_s-\mu}{\phi}\right) \tag{47}$$

Corollary 23 (Optimal PEV Schedule). *The optimal scheduled power P_s^* satisfies the condition stated in Equation (48):*

$$c_u \Phi\left(\frac{\mu - P_s^*}{\phi}\right) = c_o \Phi\left(\frac{P_s^* - \mu}{\phi}\right) \tag{48}$$

Proof. Differentiating $UCF_{PEV}(P_s)$ with respect to P_s and setting the result to zero gives:

$$\frac{d}{dP_s} UCF_{PEV}(P_s) = -c_u \Phi\left(\frac{\mu - P_s}{\phi}\right) + c_o \Phi\left(\frac{P_s - \mu}{\phi}\right) = 0$$

Using the identity $\Phi(-x) = 1 - \Phi(x)$ completes the proof and leads to Equation (48). □

Remark 24. *For symmetric costs ($c_u = c_o$), the optimal scheduled power is equal to the mean battery power: $P_s^* = \mu$.*

4 Main Results and Existence Optimal Scheduling Policy

4.1 Solar Photovoltaic Generation

Corollary 25 (Solar UCF). *For solar irradiance $G \sim$ Log-Normal(λ, β) with the piecewise power conversion function for $W_{PV}(G)$ defined in Equation (49):*

$$W_{PV}(G) = \begin{cases} \frac{W_{PVr} G^2}{G_r R_c} & 0 < G \leq R_c \\ \frac{W_{PVr} G}{G_r} & G > R_c \end{cases} \tag{49}$$

The expected components of the uncertainty cost function (UCF) are given by Equations (50) ($\mathbb{E}[C_u]$) and (51) ($\mathbb{E}[C_o]$):

$$\frac{c_u}{2}\left[(\mu - W_s)\, erf\left(\frac{\ln W_s - \lambda}{\beta\sqrt{2}}\right) + \beta e^{-(\ln W_s - \lambda)^2/2\beta^2}\right] \tag{50}$$

$$\frac{c_o}{2}\left[(W_s - \mu)\, erf\left(\frac{\ln W_s - \lambda}{\beta\sqrt{2}}\right) + \beta e^{-(\ln W_s - \lambda)^2/2\beta^2}\right] \tag{51}$$

4.2 Wind Energy Generation

Corollary 26 (Wind UCF). *For wind speed $v \sim$ Rayleigh(σ) with the power conversion function $W_w(v)$ defined piecewise in Equation (52):*

$$W_w(v) = \begin{cases} 0 & v \leq v_i \text{ or } v \geq v_o \\ \rho v + \kappa & v_i < v < v_r \\ W_r & v_r \leq v < v_o \end{cases} \tag{52}$$

The expected penalty costs, which constitute the wind UCF, satisfy Equation (53):

$$\mathbb{E}[UCF] = c_u \int_{W_s}^{W_r} \frac{w - \kappa}{\rho^2 \sigma^2} e^{-(w-\kappa)^2/2\rho^2\sigma^2} dw + c_o \left(\frac{W_s}{\sigma^2} e^{-W_s^2/2\sigma^2}\right) \tag{53}$$

4.3 Electric Vehicles

Corollary 27 (PEV UCF). *For PEV power $P_e \sim \mathcal{N}(\mu, \phi^2)$, the optimal scheduled power is given by the closed-form expression in Equation (54):*

$$W_s^* = \mu + \phi\sqrt{2}\, erf^{-1}\left(\frac{c_o - c_u}{c_o + c_u}\right) \tag{54}$$

5 Conclusion: Existence of Optimal Scheduling Policy

Definition 28 (Optimal Scheduling Problem). *Given an uncertainty cost function $UCF(W_s)$, the optimal scheduling problem is defined as finding the W_s^* that minimizes the expected cost, as formulated in Equation (55):*

$$W_s^* = \arg\min_{W_s \in \mathcal{W}} \mathbb{E}\left[c_u (W_{av} - W_s)^+ + c_o (W_s - W_{av})^+\right] \tag{55}$$

where $\mathcal{W}$ is the convex set of feasible schedules, satisfying the constraints presented in Equation (56):

$$\mathcal{W} = \{W_s \in L^2(\Omega, \mathcal{F}, \mathbb{P}) | W_{min} \leq W_s \leq W_{max} \text{ a.s.}\} \tag{56}$$

Theorem 29 (Existence and Uniqueness of Optimal Schedule). *For any square-integrable available power process $W_{av} \in L^2(\Omega, \mathcal{F}, \mathbb{P})$ with $\mathbb{E}[W_{av}^2] < \infty$, and strictly positive cost coefficients $c_u, c_o > 0$, there exists a unique optimal schedule $W_s^* \in \mathcal{W}$ solving the problem described in Equation (55).*

Proof. Step 1: Convexity Preservation
The mapping $W_s \mapsto UCF(W_s)$ remains convex over $\mathcal{W}$, as evidenced by its second derivative in Equation (57):

$$\nabla^2 UCF(W_s) = (c_u + c_o) f_{W_{av}}(W_s) \geq 0 \quad \text{a.e.} \tag{57}$$

Step 2: Weak Lower Semi-Continuity
For any weakly convergent sequence $W_s^n \rightharpoonup W_s$ in

L^2, the weak lower semi-continuity holds as per Equation (58):

$$\liminf_{n\to\infty} UCF(W_s^n) \geq UCF(W_s) \tag{58}$$

This is established by Fatou's lemma applied to the positive penalty functions.

Step 3: Coercivity
The radial unboundedness condition is satisfied, as indicated by Equation (59):

$$\lim_{\|W_s\|_{L^2}\to\infty} \frac{UCF(W_s)}{\|W_s\|_{L^2}} \geq \min(c_u, c_o) > 0 \tag{59}$$

By the Banach-Alaoglu theorem and strict convexity, there exists a unique minimizer in the weakly compact set $\mathcal{W}$. □

Corollary 30 (Optimal Scheduling Formula)**.** *The optimal schedule W_s^* satisfies the first-order condition expressed in Equation* (60)*:*

$$c_u \mathbb{P}(W_{av} > W_s^*) = c_o \mathbb{P}(W_{av} < W_s^*) \tag{60}$$

For a Gaussian $W_{av} \sim \mathcal{N}(\mu, \sigma^2)$, this leads to the closed-form solution presented in Equation (61)*:*

$$W_s^* = \mu + \sigma\sqrt{2}\,\mathrm{erf}^{-1}\left(\frac{c_o - c_u}{c_o + c_u}\right) \tag{61}$$

Remark 31. *This result generalizes to all distributions through their cumulative distribution functions, providing a universal scheduling rule modulated by cost ratios, as shown in Equation* (62)*:*

$$F_{W_{av}}(W_s^*) = \frac{c_o}{c_u + c_o} \tag{62}$$

Proposition 1 (Robustness to Distributional Uncertainty)**.** *For an ambiguously distributed W_{av} within a Wasserstein ball $\mathcal{B}_\epsilon(\mathbb{P}_0)$, the worst-case optimal schedule W_s^{rob} satisfies the inequality presented in Equation* (63)*:*

$$\sup_{\mathbb{P}\in\mathcal{B}_\epsilon(\mathbb{P}_0)} UCF(W_s^{rob}) \leq \inf_{W_s} \sup_{\mathbb{P}\in\mathcal{B}_\epsilon(\mathbb{P}_0)} UCF(W_s) + \epsilon(c_u + c_o) \tag{63}$$

Proof. Applying Kantorovich duality, the relationship between the Wasserstein distance and expectation is given by Equation (64):

$$\mathcal{W}_2(\mathbb{P}_0, \mathbb{P}) \leq \epsilon \implies \mathbb{E}_{\mathbb{P}}[f(W_s)] \leq \mathbb{E}_{\mathbb{P}_0}[f(W_s)] + \mathrm{Lip}(f)\epsilon \tag{64}$$

where $\mathrm{Lip}(UCF) \leq c_u + c_o$ by the Lipschitz continuity of the $(x)^+$ operator. □

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