The Further Studies of the c −credibility Measure in Fuzzy Environment

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Abstract: Actuarial assessments in insurance are inherently subjective due to various factors, including data limitations and the complexity of problems. Fuzzy mathematics and credibility theory provide robust frameworks to manage this subjectivity and uncertainty. Uncertain future events are closely linked with the credibility theory. This paper extends the properties of $c-$credibility measure. We generalized this measure in fuzzy environment and proved some properties of this measure. Those properties will provide more practical applications of the $c-$credibility measure of fuzzy events, especially in the context of insurance.

Key-Words: uncertainty, fuzzy measures, c-credibility measure, $c-$credibility in fuzzy environment.

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1 Introduction

Credibility theory is used to forecast an uncertain future event, mainly used in engineering, financial and actuarial mathematics. The basic credibility formula is Estimation = $z\cdot$ [Observation] + $(1 - z)\cdot$ [Prior information], where z represents the credibility weight assigned to empirical observations, and $1 - z$ denotes the complementary weight allocated to external data or historical knowledge. Rather than isolating a single "best" estimate, credibility theory synthesizes both sources through a weighted average, offering a balanced solution that harmonizes empirical evidence with contextual insights. The Bayesian credibility procedure is based on Bayes' theorem, which allows updating the probability of an event based on new information. In the insurance and actuarial context, the Bayesian procedure enables the determination of credible estimates of premiums, risks and reserves based on past data and new information.The Bayesian credibility procedure allows combining individual data with general market data to achieve more accurate estimates of risk and premiums.

An alternative version of the credibility theory for studying behave of fuzzy phenomena (CT-F), is formulated in [1]. Building on this, the authors of [2] established the credibility measure — a non-additive, self-dual measure derived as the arithmetic mean of possibility and necessity measures. Research paper [3] presents an overview of CT-F and discusses its implications, and also gives a note of the rationality of the choice of 0.5 in the CT-F formula. Further advancing the field, the authors in [4] generalized credibility measure, defined $c-$credibility measure, and aggregates possibility and necessity measures within fuzzy environments. In this study, we extend that work by broadening the theoretical scope of this measure in such contexts.

2 Preliminaries

This section revisits essential terminology and fundamental characteristics of operators defined over the interval $[0, 1]$ employed in subsequent analyses. For foundation details, we refer to the works [5] and [6]).

Definition 1. The non-increasing map $c : [0, 1] \to [0, 1]$ is defined as a **fuzzy complement**, if satisfies the boundary constraints: $c(0) = 1$ and $c(1) = 0$.

A function c is termed involutive.

Involutive fuzzy complement is a strictly increasing function.

Definition 2. The **equilibrium of fuzzy complement** is an element $\epsilon \in [0, 1]$, satisfying $c(\epsilon) = \epsilon$.

Every fuzzy complement possesses at most one equilibrium. Moreover, if c is continuous, it guarantees the existence of a unique equilibrium.

Definition 3. An **aggregation function** is a monotonically non-decreasing function $\psi_{[n]} : [0, 1]^n \to [0, 1]$, $n \in \mathbb{N}$ and satisfies boundary conditions: $\psi_{[n]}(0, ..., 0) = 0$, $\psi_{[n]}(1, ..., 1) = 1$ holds. $\psi_{[1]}(x) = i|_{[0,1]}(x) = x$, for $x \in [0, 1]$.

The function $\psi_{[n]}$ is idempotent if $\psi_{[n]}(x,...,x) = x$ for $x \in [0,1]$. $\psi_{[n]}$ is commutative for

$$\psi_{[n]}(x_1,...,x_n) = \psi_{[n]}(x_{p_1},...,x_{p_n})$$

and permutation $(p_1,...,p_n)$ of $\{1,...,n\}$.

The **dual aggregation function** ([4]) of an aggregation function $\psi_{[n]}$, shortly ψ, relative to a fuzzy complement c, is a function $\overline{\psi}_c$ (shortly $\overline{\psi}$) defined as

$$\overline{\psi}(x_1,...,x_n) = c(\psi(c(x_1),...,c(x_n))).$$

As established in [4] dual aggregation function itself constitutes an aggregation function.

For expanded discussions and related insights, we direct readers to [6], [7] and [8] as well as the associated references within these works.

Definition 4. Let X be an arbitrary non-empty set. A **fuzzy set** U with membership grades in $[0,1]$ is defined by its membership function $\mu_U : X \to [0,1]$. Equivalently, U can be expressed as the ordered pair (X, μ_U).

Fuzzy set operations are standard union, standard intersection and complement.

An important concept of the fuzzy sets is their α-cut. For the fuzzy subset A we define α-cut, $\alpha \in (0,1]$, as classical set

$$^{\alpha}U = \{u | \mu_U(u) \geq \alpha\}.$$

while for $\alpha = 0$, $^{\alpha}U = closure(\{u \in U \mid \mu_U(u) > 0\})$.

Besides, a fuzzy set W is said to be **normalized** if $\mu_U(u) = 1$, $u \in X$, while W is **convex** if its every α-cut is convex itself.

A fuzzy set S defined over $\mathbb{R}$ is called a **fuzzy number** if it is normalized and convex. The most famous example is triangular fuzzy number with the membership function

$$\mu(u) = \begin{cases} \frac{u-\tau}{m-\tau}, & \tau < u < m \\ \frac{v-u}{c-m}, & m < u < v \\ 0, & x \leq \tau \vee v \leq x \end{cases} \quad (1)$$

3 Fuzzy Measures

Definition 5. ([6]) X is a non-empty set and Υ is a non-empty collection of subsets of X, containing the empty set $\emptyset \in \Upsilon$. A function $\kappa : \Upsilon \to [0,\infty]$ is termed a **fuzzy measure- FM** (in the narrow sense) on Υ with following conditions:

$FM_1)$ $\kappa(\emptyset) = 0$,

$FM_2)$ $(\forall U, V \in \Upsilon)\ U \subset V \Rightarrow \kappa(U) \leq \kappa(V)$

$FM_3)$ $U_k \subset U_{k+1}$, $U_k \in \Upsilon$, $k \in \mathbb{N}$, $\bigcup\limits_{k=1}^{\infty} U_k \in \Upsilon \Rightarrow \kappa(\bigcup\limits_{k=1}^{\infty} U_k) = \lim\limits_{k\to\infty} \kappa(U_k)$ (continuity from below)

$FM_4)$ $U_k \supset U_{k+1}$, $U_k \in \Upsilon$, $k \in \mathbb{N}$, $\bigcap\limits_{k=1}^{\infty} U_k \in \Upsilon$ and there exists $k_0 \in \mathbb{N}$, $\kappa(U_{k_0}) < \infty \Rightarrow \kappa(\bigcap\limits_{k=1}^{\infty} U_k) = \lim\limits_{k\to\infty} \kappa(U_k)$, (continuity from above)

Furthermore, if $\kappa(X) = 1$, $X \in \Upsilon$, then κ is termed a regular fm. The structure (X, Υ, κ) is referred to as a FM space or semi-continuous FM space.

A FM κ is classified as lower or upper semi-continuous FM if it satisfies previous axioms 1-3 or $FM_1), FM_2)$ and $FM_4)$ respectively. If only one of conditions $FM_3)$ and $FM_4)$ is fulfilled, then κ is deemed a semi-continuous FM. If κ additionally fulfills $FM_1)$ and $FM_2)$, then κ is a FM in the broader sense.

Typically, Υ may be structured as different classes, semi-ring, etc. or $\mathcal{P}(X)$ [9].

FM‘s has been defined in [10]: Possibility function pos and Necessity function nec as dual.

The structure $(X, \mathcal{P}(X), pos)$ is termed as possibility space.

3.1 $c-$credibility Measure

Within the framework of his Uncertainty Theory, the researches proposed the credibility measure concept ([1], [11], [12], [13], [14]). Definition, some important properties and applications for the $c-$credibility measure can be find in [4], [15] and [16]. Also, $c-$credibility Extension Theorem is valid ([17]).

Consider an involutive fuzzy complement $c : [0,1] \to [0,1]$ with ϵ-equilibrium. The $c-$**credibility measure** on X (determined by c) is characterized by the set function $\mathsf{cr_m} : \mathcal{P}(X) \to [0,1]$ defined with:

$CR_1)$ $\mathsf{cr_m}(\emptyset) = 0$;

$CR_2)$ $(\forall U, V \in \mathcal{P}(X))\ \ U \subset V \Rightarrow \mathsf{cr_m}(U) \leq \mathsf{cr_m}(V)$;

$CR_3)$ $(\forall U \in \mathcal{P}(X))\ \ \mathsf{cr_m}(\overline{U}) = c(\mathsf{cr_m}(U))$;

$CR_4)$ $\mathsf{cr_m}(\bigcup\limits_{j\in\Lambda} U_j) = \sup\limits_{j\in\Lambda} \mathsf{cr_m}(U_j)$, $U_j \in \mathcal{P}(X)$, $j \in \Lambda$, $\sup\limits_{j\in\Lambda} \mathsf{cr_m}(U_j) < \epsilon$, where Λ is an arbitrary index set.

The structure $(X, \mathcal{P}(X), \mathsf{cr_m})$ is referred to as a $c-$**credibility space**.

Since the fuzzy complement c satisfies the involution property, the credibility measure adheres to the relation:

$$\mathsf{cr_m}(U) = c(\mathsf{cr_m}(\overline{U})). \tag{2}$$

Additionally, the measure fulfills the boundary conditions $\mathsf{cr_m}(X) = 1$ and $0 \leq \mathsf{cr_m}(U) \leq 1$.

Suppose the credibility measure for each singleton within the domain is predefined.

Theorem 1. Let X be a nonempty set and $\mathsf{cr_m} : \mathcal{P}(X) \to [0, 1]$ measure of credibility. Then so-called extension conditions credibility:

$$\sup_{u \in X} \mathsf{cr_m}(\{u\}) \geq \epsilon \tag{3}$$

$$\mathsf{cr_m}(\{u^*\}) \geq \epsilon \;\; \Rightarrow \;\; \sup_{u \neq u^*} \mathsf{cr_m}(\{u\}) = c(\mathsf{cr_m}(\{u^*\})). \tag{4}$$

Proof. If $\sup_{u \in X} \mathsf{cr_m}(\{u\}) < \epsilon$, then by use c and CR_4) we have $1 = \mathsf{cr_m}(X) = \mathsf{cr_m}(\bigcup_{u \in X} \{u\}) = \sup_{u \in X} \mathsf{cr_m}(\{u\}) < \epsilon$, and that is a contradiction, so it is $\sup_{u \in X} \mathsf{cr}\{u\} \geq \epsilon$.

Assume that $\mathsf{cr_m}(\{u^*\}) \geq \epsilon$, for some $u^* \in X$.

Applying CR_4), we get

$$c(\mathsf{cr_m}(\{u^*\})) = c(\mathsf{cr_m}(X \backslash \{u^*\})) = \mathsf{cr_m}(\bigcup_{u \in X \backslash \{u^*\}} \{u\})$$

$$= \sup_{u \in X \backslash \{u^*\}} \mathsf{cr_m}(\{u\}) = \sup_{u \neq u^*} \mathsf{cr_m}(\{u\}),$$

so the second formula is also correct. ☐

In this part we will introduce new measure called inf$-c-$credibility measure and some properties of this measure. Further, new properties of an integral based on $c-$credibility measure are introduced.

Let X be a nonempty set and Υ a nonempty class of subsets of X, such that $\emptyset, X \in \Upsilon$ and implication $U \in \Upsilon \Rightarrow \overline{U} \in \Upsilon$ holds.

With $\overline{U}$, we signed the complement of the set U.

If $\kappa : \Upsilon \to [0, 1]$ is a regular FM, then the map $\overline{\kappa}$ defined by

$$\overline{\kappa}(U) = 1 - \kappa(\overline{U}),$$

is called a **dual fuzzy measure** of FM κ.

Definition 6. Generalized dual set function of $\kappa : \Upsilon \to [0, 1]$, relative to a fuzzy complement c, is a set function $\overline{\kappa}_c$ (abbreviated $\overline{\kappa}$) given by:

$$\overline{\kappa}(U) = c(\kappa(\overline{U})).$$

When $\kappa : \Upsilon \to [0, 1]$ constitutes a regular FM (in the broader sense), the function $\overline{\kappa}$ is termed a **generalized dual measure** of κ.

For an involutive fuzzy complement c it is

$$\kappa(U) = c(\overline{\kappa}(\overline{U})).$$

It is easy to show that it is valid.

Generalized dual measure $\overline{\kappa}$ of regular FM (in the broader sense) $\kappa : \Upsilon \to [0, 1]$, with c, is a regular FM (in the broader sense).

Now we can defined new measure called inf$-c-$credibility measure.

Definition 7. $c : [0, 1] \to [0, 1]$ is an involutive fuzzy complement, with its ϵ. The **inf$-c-$credibility measure** on X is a set function $\mathsf{cr_m} : \mathcal{P}(X) \to [0, 1]$ that holds axioms CR_1), CR_2), CR_3) and

CR_5) $\mathsf{cr_m}(\bigcap_{j \in \Lambda} U_j) = \inf_{j \in \Lambda} \mathsf{cr_m}(U_j)$, for any sets $U_i \in \mathcal{P}(X)$, $i \in I$, for which it is $\inf_{j \in \Lambda} \mathsf{cr_m}(U_j) > \epsilon$, where Λ is an arbitrary.

$(X, \mathcal{P}(X), \mathsf{cr_m})$ structure we calling a **inf$-c-$credibility space**.

In the context of comparing measures, to avoid confusion, we marked $c-$credibility measure as sup$-c-$credibility measure, abbreviated with ${\mathsf{cr_m}}^s$, and inf$-c-$credibility measure with ${\mathsf{cr_m}}^i$.

As $c-$credibility $\mathsf{cr_m}$ is a FM in the broader sense, because of (2), $\mathsf{cr_m}$ is equal to its generalized dual measure $\overline{\mathsf{cr_m}}$, with the same involutive complement c.

Theorem 2. Generalized dual measure of sup$-c-$credibility measure ${\mathsf{cr_m}}^s$ (determined by c), with the same involutive fuzzy complement c, is a inf$-c-$credibility measure (determined by c).

Proof. Generalized dual measure of ${\mathsf{cr_m}}^s$ is $\overline{{\mathsf{cr_m}}^s}(A) = c({\mathsf{cr_m}}^s(\overline{U}))$.

For involutive c:

CR_1) $\overline{{\mathsf{cr_m}}^s}(\emptyset) = c({\mathsf{cr_m}}^s(\overline{\emptyset})) = c({\mathsf{cr_m}}^s(X)) = c(1) = 0$.

CR_2) $(\forall U, V \in \mathcal{P}(X))$ $U \subset V \Rightarrow \overline{U} \supset \overline{V} \Rightarrow {\mathsf{cr_m}}^s(\overline{U}) \geq {\mathsf{cr_m}}^s(\overline{V}) \Rightarrow \overline{{\mathsf{cr_m}}^s}(U) = c({\mathsf{cr_m}}^s(\overline{U})) \leq c({\mathsf{cr_m}}^s(\overline{V})) = \overline{{\mathsf{cr_m}}^s}(V)$.

CR_3) $(\forall U \in \mathcal{P}(X))$ $\overline{{\mathsf{cr_m}}^s}(\overline{U}) = c({\mathsf{cr_m}}^s((\overline{\overline{U}}))) = c({\mathsf{cr_m}}^s(U)) = {\mathsf{cr_m}}^s(\overline{U})$.

CR_5) $\overline{{\mathsf{cr_m}}^s}(\bigcap_{j \in \Lambda} U_j) = c({\mathsf{cr_m}}^s(\overline{\bigcap_{j \in \Lambda} U_j})) = c({\mathsf{cr_m}}^s(\bigcup_{j \in \Lambda} \overline{U_j})) = c(\sup_{j \in \Lambda} {\mathsf{cr_m}}^s(\overline{U_j})) = \inf_{j \in \Lambda} c({\mathsf{cr_m}}^s(\overline{U_j})) = \inf_{j \in \Lambda} \overline{{\mathsf{cr_m}}^s}(U_j)$, for any sets $U_j \in \mathcal{P}(X)$, $j \in \Lambda$, and $\inf_{j \in \Lambda} \overline{{\mathsf{cr_m}}^s}(U_j) > \epsilon$, where Λ is an arbitrary.

Notice, from $\epsilon < \inf_{j\in\Lambda} \overline{\mathsf{cr_m}}^s(U_j) = \inf_{j\in\Lambda} c(\mathsf{cr_m}^s(\overline{U_j})) = c(\sup_{j\in\Lambda} \mathsf{cr_m}^s(\overline{U_j}))$, it follows $\sup_{j\in\Lambda} \mathsf{cr_m}^s(\overline{U_j}) < \epsilon$. We used the well-known theorem:
If D is a bounded set of real numbers and $c : \mathbb{R} \to \mathbb{R}$ is a continuous strictly decreasing function, then $c(\sup D) = \inf c(D)$.

□

As $c-$credibility $\mathsf{cr_m}$ is a FM in the broader sense, because of (2), $\mathsf{cr_m}$ is equal to its generalized dual measure $\overline{\mathsf{cr_m}}$, with the same involutive fuzzy complement c.

4 Credibility of a Fuzzy Events

The concept of credibility within fuzzy environments has been established in [2] by defining it like arithmetic mean of pos and nec measures:

$$Cr(U) = \frac{1}{2}(pos(U) + nec(U)),$$

.

Here, the coefficient 0.5 is initially selected as a weighting factor; this choice will be extended to a broader framework in subsequent sections.

Consider a triangular fuzzy number X defined on $(X, \mathcal{P}(X), pos)$, characterized by (1).

For the fuzzy event $\{X \le w\}$ its possibility measure is defined as:

$$pos(\{X \le w\}) = \sup_{b\le w} \mu(b)$$

and for triangular fuzzy number that is

$$pos(\{X \le w\}) = \begin{cases} 0, & w \le \tau \\ \frac{w-\tau}{m-\tau}, & \tau < w < \upsilon \\ 1, & \upsilon \le w \end{cases}$$

Necessity of a fuzzy event $\{X \le w\}$ is defined as

$$nec(\{X \le w\}) = 1 - pos(\{X > w\}) = 1 - \sup_{b>w} \mu(b)$$
$$= \begin{cases} 0, & w \le m \\ \frac{w-m}{\upsilon-m}, & m < w < \upsilon \\ 1, & \upsilon \le w \end{cases}$$

In [4] authors gave the $c-$**credibility in a fuzzy environment** using aggregation function ψ

$$\mathsf{cr}_\psi(U) = \psi(pos(U), nec(U)), \tag{5}$$

and it is proven that the $c-$credibility in a fuzzy environment is a regular fuzzy measure in the broader sense.

Further, this measure (in relation to the aggregation function ψ) of a fuzzy events $\{X \le w\}$ is defined with (see [4]):

$$\mathsf{cr}_\psi(\{X \le w\}) = \psi(pos(\{X \le w\}), nec(\{X \le w\}))$$
$$= \begin{cases} 0, & w \le \tau \\ \psi(\frac{x-\tau}{m-\tau}, 0), & \tau < w \le m \\ \psi(1, \frac{w-m}{\upsilon-m}), & m < w < \upsilon \\ 1, & \upsilon \le w \end{cases}$$

Our results are presented below, generalized definition (5) and the suitable theorem.

Let $\psi_{[n]} : [0,1]^n \to [0,1]$, $n \in \mathbb{N}$, $n-$ary aggregation function. We denoted by ψ this function in the case when $n = 2$.

Definition 8. If κ and $\overline{\kappa}$ are fuzzy measure and appropriate generalized dual fuzzy measure, then

$$\mathsf{cr}_\psi(U) = \psi(\kappa(U), \overline{\kappa}(U)), \tag{6}$$

we call **generalized $c-$credibility measure in a fuzzy environment**.

Theorem 3. Let ψ be a continuous aggregation function. Generalized $c-$credibility measure in a fuzzy environment cr_ψ, defined by (6), is fuzzy measure.

Proof 1. $\kappa(U), \overline{\kappa}(U) \in [0,1]$, $U \in \Upsilon$ and $\psi : [0,1]^n \to [0,1]$ imply $\mathsf{cr}_\psi(U) \in [0,1]$.

FM_1) $\mathsf{cr}_\psi(\emptyset) = \psi(\kappa(\emptyset), \overline{\kappa}(\emptyset)) = \psi(0,0) = 0$.

FM_2) $U \subset V \Rightarrow \kappa(U) \le \kappa(V) \wedge \overline{\kappa}(U) \le \overline{\kappa}(V) \Rightarrow \mathsf{cr}_\psi(U) = \psi(\kappa(U), \overline{\kappa}(U)) \le \psi(\kappa(V), \overline{\kappa}(V)) = \mathsf{cr}_\psi(V)$.

FM_3) $U_k \uparrow$, $U_k \in \Upsilon$, $k \in \mathbb{N}$, $\bigcup_{k=1}^{\infty} U_k \in \Upsilon \Rightarrow \kappa(\bigcup_{k=1}^{\infty} U_k) = \lim_{k\to\infty} \kappa(U_k)$, $\overline{\kappa}(\bigcup_{k=1}^{\infty} U_k) = \lim_{k\to\infty} \overline{\kappa}(U_k)$:

$$\begin{aligned} \mathsf{cr}_\psi(\bigcup_{k=1}^{\infty} U_k) &= \psi(\kappa(\bigcup_{k=1}^{\infty} U_k), \overline{\kappa}(\bigcup_{k=1}^{\infty} U_k)) \\ &= \psi(\lim_{k\to\infty} \kappa(U_k), \lim_{k\to\infty} \overline{\kappa}(U_k)) \\ &= \lim_{k\to\infty} \psi\left(\kappa(U_k), \overline{\kappa}(U_k)\right) \\ &= \lim_{k\to\infty} \mathsf{cr}_\psi(U_k). \end{aligned}$$

which follows from the continuity of the aggregation function ψ and the known property:
If a function $\psi : X \to \mathbb{R}$, $X \subset \mathbb{R}^2$ is continuous and series $\{a_k\}$, $\{b_k\}$ converge to a, b, respectively, then $\lim_{k\to\infty} \psi(a_k, b_k) = \psi(a, b)$.

$FM_4)$ $U_k \downarrow,\ U_k \in \Upsilon,\ k \in \mathbb{N},$ $\bigcap_{k=1}^{\infty} U_k \in \Upsilon \Rightarrow \kappa(\bigcap_{k=1}^{\infty} U_k) = \lim_{k\to\infty} \kappa(U_k)$, $\overline{\kappa}(\bigcap_{k=1}^{\infty} U_k) = \lim_{k\to\infty} \overline{\kappa}(U_k)$, (the condition that k_0 exists so that $\kappa(U_{k_0}) < \infty$ is fulfilled because the measure κ is bounded):

$$\begin{aligned}
\mathsf{cr}_\psi(\bigcap_{k=1}^{\infty} U_k) &= h(\kappa(\bigcap_{k=1}^{\infty} U_k), \overline{\kappa}(\bigcap_{k=1}^{\infty} U_k)) \\
&= \psi(\lim_{k\to\infty} \kappa(U_k), \lim_{k\to\infty} \overline{\kappa}(U_k)) \\
&= \lim_{k\to\infty} \psi(\kappa(U_k), \overline{\kappa}(U_k)) \\
&= \lim_{k\to\infty} \mathsf{cr}_\psi(U_k).
\end{aligned}$$

If κ is regular measure, then cr_ψ is regular, because:
$\mathsf{cr}_\psi(X) = \psi(\kappa(X), \overline{\kappa}(X)) = \psi(1,1) = 1.$ □

Consider the dual function $\overline{\psi}$ derived from an aggregation function ψ, relative to the fuzzy complement c. Then

$$\begin{aligned}
\mathsf{cr}_\psi(\overline{U}) &= \psi(\kappa(\overline{U}), \overline{\kappa}(\overline{U})) \\
&= c\Big(c\big(\psi(c(c(\kappa(\overline{U}))), c(c(\overline{\kappa}(\overline{U}))))\big)\Big) \\
&= c\Big(\overline{\psi}\big(c(\kappa(\overline{U})), c(\overline{\kappa}(\overline{U}))\big)\Big) \\
&= c\Big(\overline{\psi}\big(\overline{\kappa}(U), \kappa(U)\big)\Big) \\
&= c\Big(\overline{\psi}\big(\kappa(U), \overline{\kappa}(U)\big)\Big) \\
&= c\Big(\mathsf{cr}_{\overline{\psi}}(U)\Big).
\end{aligned}$$

This measure can be use on insurance data for experience rating and in the group insurance, to helps in determining premiums by considering the claims experience of the entire group as well as the broader population. Also, cr_h can be useful to evaluate the risk of large claims and to set appropriate reinsurance premiums, in loss reserving and rate making in setting rates by blending historical data of a specific line of insurance with overall market trends. For example, determining the credibility of a new health insurance package by analyzing historical health claims data, adjusting premiums based on the reliability of past data, and ensuring that the new package's credibility value does not exceed equilibrium.

5 Conclusion

This paper advances the investigation of the $c-$credibility measure in fuzzy environment, extending its theoretical framework and establishing novel properties. We generalized this measure and proved some new properties of this measure. As monotonous and semi-continuous measure, it has adaptability to diverse applications. We defined new measure called $\inf -c-$credibility measure.In the future research, we shall continue to study more concepts in the framework of $c-$credibility space. Also, our attempt will be to give more practical applications of this measure in fuzzy environment (such as in article [18]) .

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The authors equally contributed in the present research, at all stages from the formulation of the problem to the final findings and solution.

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Conflicts of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.